

Spin-orbit interaction of electrons on a curved surface

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Spin-orbit (SO) interaction of electrons confined near a curved semiconductor surface is studied. The curvature of the surface determines an interesting mechanism of SO interaction which complements the known mechanisms occurring in planar systems. The specific cases of nanosphere and nanotube are considered and the electron spin-split spectra in these systems are found.

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I. INTRODUCTION

It is well known that the Hamiltonian for electrons confined near an oriented planar two-dimensional system contains linear in lateral momentum terms which describe the spin-orbit (SO) interaction.^{1,2} The surface orientation may be conditioned by the asymmetry of the confining potential (the Rashba model¹) or by the inversion asymmetry of the initial crystal together with the spatial quantization (the Dresselhaus model^{3,4,2}).

In the first case the SO Hamiltonian has the form

$$\hat{H}_{so} = \alpha_R (\boldsymbol{\sigma}[\mathbf{p} \times \mathbf{n}]), \quad (1)$$

where $\boldsymbol{\sigma}$ are Pauli matrices, \mathbf{p} is the two-dimensional (2D) electron momentum, and \mathbf{n} is the normal to the plane of the system.

The Hamiltonian (1) leads to linear in \mathbf{p} spin splitting of the electron energy bands. This Hamiltonian has been widely applied to different problems of low-dimensional physics. In particular, it determines such phenomena as electric-field-induced spin-flip transitions,² phase decoherence time in weak localization theory,^{5,6} beating of the Shubnikov-de Haas oscillations,⁷⁻⁹ mixed spin-plasmon polaritons,¹⁰ and spin orientation by the lateral electric field.¹¹⁻¹³

The occurrence of the term (1) requires the inversional asymmetry of the system. The Rashba mechanism vanishes in symmetric quantum wells, such as double heterostructures with symmetric band offsets or δ -doped layers. What is more, the confinement of an electron even by the asymmetric quantum well does not result in linear terms unless a crystal structure is taken into account. This follows from the fact that the mean force vanishes in a bound state. Consequently, the effective SO interaction disappears in the envelope function approximation and remains relatively weak if to allow for the crystal structure.

Curved 2D systems have been the subject of extensive recent interest (see, e.g., Refs. 14-16). In particular, the Rashba and Dresselhaus models of SO interaction have been exploited for a study of electrons located on a cylindrical surface.¹⁷

In the present paper we find the effective Hamiltonian of spin-orbit interaction for electrons confined near the curved surface. We show that surface curvature marks the surface orientation and therefore causes an additional SO interaction. This idea can be accented by comparison of an atom and the

above-mentioned empty-lattice planar confining system. In the first case linear SO level splitting appears, while in the second case it does not. The splitting is retained if an atom is replaced by a potential well concentrating electrons near a spherical shell. As the shell radius grows, the shell flattens and the splitting falls down proportionally to the ratio of the quantum well width d to the radius of curvature R . Thus the curved system has an additional effective SO interaction due to curvature.

II. SCHRÖDINGER EQUATION ON THE CURVED SURFACE

The effective Hamiltonian for electrons on curved surface with no SO interaction has been obtained in Ref. 18 and 19. We shall study the spinor problem using the same approach, starting from the Schrödinger equation (SchE) for an electron with quadratic nondegenerate spectrum:

$$\left(-\frac{1}{2m} \Delta + V(\mathbf{r}) + \hat{\mathcal{H}}_{so} \right) \Psi = E \Psi, \quad (2)$$

$$\hat{\mathcal{H}}_{so} = -i\alpha [\boldsymbol{\sigma}, \nabla V] \nabla.$$

Here and below we set $\hbar = 1$; m is the effective electron mass, $\hat{\mathcal{H}}_{so}$ is the Hamiltonian of the basic SO interaction, α is the SO coupling constant. For free electrons $\alpha = 1/4(m_0 c)^2$, where m_0 is the electron mass, c is the velocity of light. In A_3B_5 crystals $\alpha = (2E_g m)^{-1} [\Delta(2E_g + \Delta)/(E_g + \Delta)(3E_g + 2\Delta)]$, where E_g is the width of the forbidden band and Δ is the SO splitting of the valence band.

Let $\mathbf{a}(u^1, u^2)$ parametrically specify the surface in 3D space. The parameters u^1 and u^2 may be considered as curvilinear coordinates on the surface (we use the traditional notations with super- and subscripts referring to the contra- and covariant components of tensors). Any point in some finite layer surrounding the surface can be expressed as follows:

$$\mathbf{r}(u^1, u^2, w) = \mathbf{a}(u^1, u^2) + w \mathbf{n}, \quad |w| \leq \delta, \quad (3)$$

where $\mathbf{n} = \partial_1 \mathbf{a} \times \partial_2 \mathbf{a} / |\partial_1 \mathbf{a} \times \partial_2 \mathbf{a}|$ is the normal to the surface, $\partial_i \equiv \partial / \partial u^i$, the layer width δ is assumed to be less than the curvature radii. The triad $\{u^1, u^2, w\}$ forms a curvilinear coordinate system in 3D space.

We shall consider an electron confined near the surface $w = 0$ by a potential well $V(w)$ with minimum at $w = 0$. The

bottom of the well $w=0$ is an equipotential surface. The well width $d < \delta$ and the transversal size of the electron wave function are assumed to be much less than geometric sizes of the curved surface, in particular, the curvature radii of the surface. Since the wave function is small enough when $w \sim \delta$, we impose zero conditions $\Psi=0$ at $w \rightarrow \pm\infty$. If $d/R \ll 1$, the well is locally planar, so that in the first approximation the motion of an electron along the well is adiabatic. It means conservation of the transversal quantum number n and the kinetic energy of longitudinal motion.

In the curvilinear coordinates u^i the space and surface (at $w=0$) intervals are defined by $ds^2 = G_{ij} du^i du^j + dw^2$ and $ds^2 = g_{ij} du^i du^j = \partial_i \mathbf{a} \cdot \partial_j \mathbf{a} du^i du^j$, respectively. The space and surface metrics are linked together:

$$G_{ij} = g_{ij} - 2wh_{ij} + w^2 \partial_i \mathbf{n} \cdot \partial_j \mathbf{n}, \quad (4)$$

where $h_{ij} = h_{ji} = -\partial_i \mathbf{a} \cdot \partial_j \mathbf{n}$ are the coefficients of the second fundamental form. The volume and area differentials can be written as $dV = \sqrt{G} du^1 du^2 dw = f dS dw$ and $dS = \sqrt{g} du^1 du^2$, where $G = \det(G_{ij})$, $g = \det(g_{ij})$, and $f = \sqrt{G/g}$. For small w , we have

$$f = 1 + w \text{Tr}(\hat{\beta}) + w^2 \det(\hat{\beta}) + O(w^3), \quad (5)$$

where $\hat{\beta} = -(\hat{h}g^{-1})$.

Using curvilinear coordinates, it is convenient to renormalize the wave function $\Psi = \chi/\sqrt{f}$. Then the SchE takes the form

$$(\hat{H}_w + \hat{H}_{u;w})\chi + \sqrt{f}\hat{\mathcal{H}}_{so}\left(\frac{\chi}{\sqrt{f}}\right) = E\chi. \quad (6)$$

The nonrelativistic Hamiltonian $\hat{H}_w + \hat{H}_{u;w}$ is separated on sharply (\hat{H}_w) and smoothly ($\hat{H}_{u;w}$) w -dependent parts, where \hat{H}_w corresponds to transversal motion and $\hat{H}_{u;w}$ does not contain derivatives of χ with respect to w :

$$\hat{H}_w = -\frac{1}{2m} \partial_w^2 + V(w), \quad (7)$$

$$\hat{H}_{u;w}\chi = -\frac{1}{2m} \sqrt{f} \hat{\mathcal{D}} \left(\frac{\chi}{\sqrt{f}} \right) - \frac{1}{8mf^2} [(\partial_w f)^2 - 2f \partial_w^2 f] \chi. \quad (8)$$

Here $\hat{\mathcal{D}}$ is the ‘‘surface’’ part of the Laplacian

$$\hat{\mathcal{D}} = \frac{1}{\sqrt{G}} \partial_i \sqrt{G} G^{ij} \partial_j. \quad (9)$$

The SO interaction transforms to

$$\hat{\mathcal{H}}_{so} = i\alpha \frac{1}{\sqrt{G}} (\partial_w V) \epsilon^{ij} \boldsymbol{\sigma}(\partial_i \mathbf{r}) \partial_j, \quad (10)$$

where ϵ^{ij} is the Levi-Civita symbol: $\epsilon^{11} = \epsilon^{22} = 0$, $\epsilon^{12} = -\epsilon^{21} = 1$.

Owing to SO interaction weakness, we search an effective Hamiltonian up to the order $\alpha(d/R)$. The nonrelativistic Hamiltonian of the lowest order in w becomes $\hat{H}_w + \hat{H}_{u;w=0}$, where

$$\hat{H}_{u;0} = -\frac{1}{2m} \hat{\mathcal{D}}_0 + V_g(u), \quad \hat{\mathcal{D}}_0 = \frac{1}{\sqrt{g}} \sum_{i,j=1}^2 \partial_i \sqrt{g} g^{ij} \partial_j. \quad (11)$$

The geometric potential $V_g \sim 1/mR^2$ results from the last term in Eq. (8). It depends on the surface coordinates $u \equiv \{u^1, u^2\}$ only and can be expressed via principal curvatures of the surface $\kappa_{1,2}$:¹⁹

$$V_g(u) = -\frac{1}{8m} [(\text{Tr} \hat{\beta})^2 - 4 \det \hat{\beta}] \equiv -\frac{1}{8m} (\kappa_1 - \kappa_2)^2. \quad (12)$$

The Hamiltonian (11) is separable, and $\chi = \phi_n(w) \psi(u)$, where $(\hat{H}_w - E_n) \phi_n = 0$; n is the number of the transversal quantization subband ($n \geq 0$). This is a zero approximation to the solution of Eq. (6). With desirable accuracy we have

$$\sqrt{f} \hat{\mathcal{D}} \left(\frac{\chi}{\sqrt{f}} \right) \approx \hat{\mathcal{D}}_0 \chi + w \hat{\mathcal{D}}_1 \chi, \quad (13)$$

$$\hat{\mathcal{D}}_1 = -\{\hat{\mathcal{D}}_0, \text{Sp}(\hat{\beta})\} - \frac{1}{\sqrt{g}} \partial_i \sqrt{g} [2(\hat{h}^{-1})_{ij} \det(\hat{\beta}) + g^{ij} \text{Sp}(\hat{\beta})] \partial_j. \quad (14)$$

The notation $\{\hat{A}, \hat{B}\}$ means $(\hat{A}\hat{B} + \hat{B}\hat{A})/2$. The expansion of SO Hamiltonian yields

$$\sqrt{f} \hat{\mathcal{H}}_{so} \left(\frac{\chi}{\sqrt{f}} \right) = \partial_w V(\hat{L} + w \hat{M}) \chi, \quad (15)$$

$$\hat{L} = \frac{i\alpha}{\sqrt{g}} \epsilon^{ij} (\boldsymbol{\sigma} \partial_i \mathbf{a}) \partial_j, \quad (16)$$

$$\hat{M} = \frac{i\alpha \epsilon^{ik}}{\sqrt{g}} [\beta_{ij} (\boldsymbol{\sigma} \partial_j \mathbf{a}) \partial_k - \{\text{Sp}(\hat{\beta}), (\boldsymbol{\sigma} \partial_i \mathbf{a}) \partial_k\}]. \quad (17)$$

In the zero order in w , the well is effectively planar and SO interaction vanishes. Hence the SchE should be expanded up to the first nonvanishing terms. Rewriting the total SchE in the form $(\hat{H}_w + \hat{H}_{u;0} + \hat{U} - E)\chi = 0$, we expand the wave function on the basis of $\phi_n(w)$:

$$\chi = \sum_n \psi_n(u) \phi_n(w),$$

$$(E_n + \hat{H}_{u;0} - E) \psi_n + \sum_{n' \neq n} \hat{U}_{nn'} \psi_{n'} = 0. \quad (18)$$

For any operator \hat{A} the notation $\hat{A}_{nn'}$ stands for $\hat{A}_{nn'} = \int dw \phi_n(w) \hat{A} \phi_{n'}(w)$. Note, that $\hat{A}_{nn'}$ is still the operator with respect to u .

Thanks to the small parameter d/R , the off-diagonal elements of $\hat{U}_{nn'}$ are less than \hat{U}_{nn} , and the wave function of some transversal state n has a small admixture of the other states.

The effective Hamiltonian can be found similarly to the near-degenerate perturbation theory. We express the ‘‘small’’ components $\psi_{n'}(u)$ via the ‘‘large’’ components $\psi_n(u)$ and substitute them into the equation for $\psi_n(u)$. Terminating the iteration, we get with a prescribed accuracy

$$(E_n + \hat{H}_{u;0} + \hat{U}_{nn} - E) \psi_n - \sum_{n' \neq n} \hat{U}_{nn'} \frac{1}{E_{n'} - E_n} \hat{U}_{n'n} \psi_{n'} = 0. \quad (19)$$

The perturbation to the Hamiltonian consists of w - and SO-dependent parts.

Collecting Eqs. (13), (15), and (19), we find the effective Hamiltonian for the n th subband:

$$\hat{H} = \hat{H}_{u;0} + \hat{H}_c + \hat{H}_{so}, \quad \hat{H}_c = -\frac{1}{2m} w_{nn} \hat{D}_1, \quad (20)$$

$$\hat{H}_{so} = \frac{1}{m} \sum_{n' \neq n} \frac{w_{n'n} (\partial_w V)_{nn'}}{E_{n'} - E_n} \{ \hat{L}, \hat{D}_1 \} + (w \partial_w V)_{nn} \hat{M}. \quad (21)$$

The first and second terms in Eq. (20) represent the nonrelativistic Hamiltonian. The applicability range of this Hamiltonian limited by the only inequality $d \ll R$ extends the range of Refs. 18 and 19. The latter requires an extra inequality $E - E_0 \ll E_1 - E_0$ corresponding to the strict 2D case.

The additional term \hat{H}_c is of centrifugal origin. This can be accounted for a change of the electron potential energy caused by the inertial shift of an electron to the outer side of the curved quantum well. Unlike the geometric potential, \hat{H}_c depends on the longitudinal motion of electrons, and vanishes if the energy coincides with a subband bottom. The Hamiltonian \hat{H}_c also vanishes in symmetric wells, for example, parabolic or hard-wall wells. Notice that in the limit $R \rightarrow \infty$ $\hat{H}_c \sim 1/R$ exceeds the geometric potential $V_g \sim 1/R^2$.

The effective spin-orbit Hamiltonian can be simplified using the identities

$$\sum_{n' \neq n} w_{n'n} (\partial_w V)_{nn'} (E_n - E_{n'})^{-1} = -1/2$$

and $(w \partial_w V)_{nn} = 2T_n$, where $T_n = -1/(2m) (\partial_w^2)_{nn}$ is the transversal kinetic energy in the well. The result is

$$(\hat{H}_{so})_n = 2T_n \hat{M} + \frac{1}{2m} \{ \hat{L}, \hat{D}_1 \}. \quad (22)$$

The first term in Eq. (22) depends on the transversal state of electron, whereas the second does not. The ratio of the sec-

ond contribution to the first one is proportional to $(E - E_n)/T_n$. If the energy is close to the subband bottom, the second term is negligible.

Physically, the SO term proportional to T_n originates from the difference of the electron tangential velocity \mathbf{v} on the inner and the outer well boundaries due to conservation of angular momentum. In a frame of reference accompanying the moving electron the boundary field ∇V transforms to a magnetic field $\mathbf{B}' = [\mathbf{v} \times \nabla V]/ec$ which has different values on two boundaries. As the electron moves across the well, its spin averages the magnetic field. This leads to a finite value proportional to the velocity.

The second contribution to SO interaction arises from the centripetal force mv^2/R , turning the electron along the curved well. This force, transformed to the moving frame, produces another component of the magnetic field $\sim -[\mathbf{v} \times \mathbf{R}]mv^2/ecR^2$.

The value of T_n can be found in specific models of transversal states. In particular, for a parabolic quantum well $V(w) = m\omega^2 w^2/2$, and $T_n = \omega(2n+1)/4$. For a well with hard walls $T_n = \pi^2(n+1)^2/2md^2$. For the Stern-Howard trial function²⁰ $\phi = 2(b/2)^{3/2} w e^{-bw/2}$, appropriate to the lowest self-consistent state on interface, $T_0 = b^2/8m = E_0/5$.

If the longitudinal energy is high enough, so that the wavelength is less than R , one can neglect the action of derivatives on the geometric parameters of the surface. In the natural coordinates u , which locally coincide with Cartesian coordinates in the tangent plane, Eq. (10) can be written as

$$\hat{H}_{so} = -2\alpha \left[T_n (\sigma_x \hat{p}_y \kappa_y - \sigma_y \hat{p}_x \kappa_x) - \frac{1}{2m} (\kappa_x \hat{p}_x^2 + \kappa_y \hat{p}_y^2) (\sigma_x \hat{p}_y - \sigma_y \hat{p}_x) \right]. \quad (23)$$

The axes x and y are chosen along the principal directions of curvature. Despite the smallness of the derivatives of κ , we retain the symmetrization in Eq. (23) to conserve Hermiticity of the Hamiltonian. If the electron state has a small size along the surface, compared with R , the curvatures can be considered as constant quantities. Moreover, if the momentum is a classical variable, the Hamiltonian (23) is easily diagonalized. The eigenvalues of (23) are

$$\pm 2 \frac{\alpha}{2m} \sqrt{(2mT_n \kappa_x - Q)^2 p_x^2 + (2mT_n \kappa_y - Q)^2 p_y^2}, \quad (24)$$

$$Q = \kappa_x p_x^2 + \kappa_y p_y^2.$$

At relatively small carrier concentrations, electrons occupy the bottom of the first transversal subband only, and the transversal terms in Eqs. (22) and (23) dominate, whereas if the concentration increases, the longitudinal terms take effect.

III. SPHERICAL AND CYLINDRICAL QUANTUM WELLS

As examples we consider spherical and cylindrical quantum wells (SQW's and CQW's). In these cases spherically or

axially symmetric potentials confine electrons near some radius R .

Using the lateral momentum on the sphere surface $\hat{\mathbf{p}}$ or angular momentum $\hat{\mathbf{I}}$ the SO Hamiltonian for SQW's in spherical coordinates can be written in two forms:

$$\begin{aligned}\hat{H}_{so} &= -\frac{2\alpha}{R} \left(T_n - \frac{\hat{p}^2}{2m} \right) (\hat{\boldsymbol{\sigma}}[\mathbf{n}_R \times \hat{\mathbf{p}}]) \\ &= -\frac{2\alpha}{R^2} \left(T_n - \frac{\hat{\mathbf{I}}^2}{2mR^2} \right) (\hat{\boldsymbol{\sigma}}\hat{\mathbf{I}}).\end{aligned}\quad (25)$$

Here $\hat{\mathbf{p}} = \hat{p}_\theta \mathbf{n}_\theta + \hat{p}_\varphi \mathbf{n}_\varphi$, $\hat{p}_\theta = -(i/R)\partial_\theta$, $\hat{p}_\varphi = -(i/R \sin\theta)\partial_\varphi$. The vectors $\mathbf{n}_\theta, \mathbf{n}_\varphi, \mathbf{n}_R = \mathbf{R}/R$ compose the orthogonal basis of the spherical coordinates. The first form of the SO Hamiltonian looks like the Rashba Hamiltonian, Eq. (1), for a planar system. In fact, in a small neighborhood of some point on the sphere the vector \mathbf{n}_R is almost constant, the sphere can be considered as a plane, and $\hat{\mathbf{p}}$ as a translational momentum along the plane. The difference of Eq. (25) and the Rashba Hamiltonian, Eq. (1), lies in the momentum dependence on the SO coupling constant. The second form is a conventional SO interaction in a spherical symmetric system. The energy levels with given orbital l and total $j = l \pm 1/2$ angular momenta are

$$\begin{aligned}\varepsilon_{nlj} &= \frac{l(l+1)}{2mR^2} + \frac{\alpha}{2mR^4} [2mR^2 T_n - l(l+1)] \\ &\times [j(j+1) - l(l+1) - 3/4].\end{aligned}$$

Let the longitudinal momentum be a classical variable. Equation (25) shows that the effective SO interaction of Fermi electrons changes its sign when the subband filling increases. If the only lowest subband is occupied up to the Fermi level E_F , the change of sign occurs at $E_F - E_0 = \pi^2/2md^2$ for the hard-wall well, at $E_F - E_0 = \omega/4$ for the parabolic well, and at $E_F - E_0 = E_0/5$ for the self-consistent interface states.

In this limit the surface motion of an electron is a rotation along the great circle with angular velocity $\boldsymbol{\omega} = [\mathbf{n}_R, \mathbf{p}]/mR$. Then commutation of the operator of spin \mathbf{S} with \hat{H}_{so} gives the equation of motion for the spin $\dot{\mathbf{S}} = 4m\alpha(T_n - p^2/2m) \times [\boldsymbol{\omega}, \mathbf{S}]$, which shows that the ratio of the spin rotation frequencies to the angular velocity $\omega_1/\omega = 4m\alpha(T_n - p^2/2m)$ is independent of the sphere radius.

The case of CQW can be considered in a similar way. Using cylindrical coordinates (ρ, φ, z) with axis z directed along the cylinder axis and notation $\hat{p}_\varphi = -(i/R)\partial_\varphi$ we find

$$\hat{H}_{so} = \frac{2\alpha}{R} \left[\left(\frac{\hat{p}_\varphi^2}{2m} - T_n \right) \hat{p}_\varphi \hat{\sigma}_z + \left[[\mathbf{n}_\rho, \hat{\boldsymbol{\sigma}}]_z, \frac{\hat{p}_\varphi^2}{2m} \right] \hat{p}_z \right]. \quad (26)$$

The energy levels of electrons in CQW with given projections of the total momentum $j_z = \pm 1/2, \pm 3/2, \dots$, and longitudinal momentum p_z are

$$\begin{aligned}\varepsilon_{np_z j_z} &= \frac{1}{2mR^2} \left\{ k^2 + j_z^2 + \frac{1}{4} - \frac{\bar{\alpha}}{2} \left(\frac{1}{4} + 3j_z^2 - t_n \right) \right. \\ &\quad \left. \pm \sqrt{j_z^2 \left[1 + \bar{\alpha} t_n - \bar{\alpha} \left(j_z^2 + \frac{3}{4} \right) \right]^2 + \bar{\alpha}^2 k^2 \left(j_z^2 + \frac{1}{4} \right)^2} \right\}.\end{aligned}\quad (27)$$

Here $\bar{\alpha} = 2\alpha/R^2$, $t_n = 2mR^2 T_n$ and $k = p_z R$.

Generally speaking, the curvature-induced SO Hamiltonian should be supplemented by the other (mentioned in the Introduction) terms that are not associated with the curvature. Let us consider the crossover between curvature-induced and Rashba SO interaction. We restrict ourselves to the case of states in CQW near the bottom of the lowest transversal subband. The generalization of Eq. (27) for the subband $n=0$ reads

$$\begin{aligned}\varepsilon_{0p_z j_z} &= \frac{1}{2mR^2} \left(k^2 + j_z^2 + \frac{1 - 2\bar{\alpha}_R + 2\bar{\alpha} t_0}{4} \right. \\ &\quad \left. \pm \sqrt{j_z^2 (1 - \bar{\alpha}_R + \bar{\alpha} t_0)^2 + \bar{\alpha}_R^2 k^2} \right).\end{aligned}\quad (28)$$

Here $\bar{\alpha}_R = 2m\alpha_R R$. Depending on a parameter $\alpha T_0/R\alpha_R$ one mechanism or another will prevail. For large R Eq. (28) goes to Eq. (33) from Ref. 17, for small R gives Eq. (27). This estimate is evidently valid not for CQW only but for any surface as well.

An estimation of the Rashba constant using the mean force, which acts on an electron, (see, e.g., Ref. 9) gives $\alpha_R \sim \alpha T_0^{3/2}$ and $\alpha T_0/R\alpha_R \sim d/R$. Strictly speaking, this way is incorrect due to vanishing the mean force mentioned above, but may be considered as an upper limit for α_R . In fact, the correct calculation of α_R goes beyond the envelope-function approximation resulting in an additional small parameter of the order of T_0/E_g . So we expect the ratio $\alpha T_0/R\alpha_R$ to be no less than $E_g d/T_0 R$.

Note that the degree of asymmetry and therefore the ratio of different mechanisms can be controlled by a gate electrode. For smaller radii or lower electron concentration the curvature contribution becomes more essential.

The obtained Hamiltonian together with spin-independent terms solves the problem of 3D SchE reduction to the effective surface problem. It is not difficult to complete the problem either by boundary conditions, if lateral confinement is considered, or by inclusion of an additional potential smoothly depending on the surface coordinates. The magnetic field can also be included in a gauge-invariant manner.

In conclusion, we have derived the effective spin-orbit Hamiltonian for electrons in the curved quantum well. Like known planar mechanisms, this Hamiltonian starts from the linear terms in a longitudinal momentum. The curvature-induced spin-orbit coupling is determined by simple expressions, containing surface curvatures and energies of transversal quantization. The strengths of the curvature-induced and the Rashba SO interactions may be comparable.

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