Response functions and superfluid density in a weakly interacting Bose gas with nonquadratic dispersion

Jonathan Keeling

Department of Physics, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139, USA

(Received 19 May 2006; revised manuscript received 6 July 2006; published 27 October 2006)

Motivated by the experimental search for Bose condensation of quasiparticles in semiconductors, the response functions of a weakly interacting Bose gas, with isotropic but nonquadratic dispersion, are considered. Nonquadratic dispersion modifies the definition of particle current, and leads to modified sum rules for the current-current response function. The effect of these modifications on the Berezhinski-Kosterlitz-Thouless transition is discussed.

DOI: 10.1103/PhysRevB.74.155325 PACS number(s): 03.75.Hh, 47.37.+q, 71.35.Lk, 31.15.Lc

Recently, there has been increasing interest in Bose condensation of quasiparticles in solid state systems. Examples include indirect excitons in semiconductor quantum wells,[1] exciton-polaritons in semiconductor microcavities,[2–4] quantum Hall bilayer excitons,[5,6] and spin "triplons" in copper compounds.[7–9] In many of these cases, the composite nature of the quasiparticle leads to significant deviations from a quadratic dispersion. Such deviations mean that a current defined by $\mathbf{J}(x)=\psi^\dagger(x)i\nabla\psi(x)$ is no longer correct: such a current is not conserved, and so its correlation functions do not obey simple sum rules. Neither can this problem be extricated by working in terms of more fundamental fields, e.g., the photon/exciton fields for the polariton problem, as in such an example the photon current is not conserved, the Hamiltonian has terms by which photon current is transferred to exciton current and back again.

There is an obvious solution to this problem: the correct definition of current is the Noether current associated with invariance of the action under global phase rotations. Such a definition automatically leads to a conserved current, which for quadratic dispersion is just the standard definition. The definition of current and its response functions are of particular importance due to another common feature of these solid state systems in which condensation is sought: they are two dimensional, and therefore the transition is of the Berezhinski-Kosterlitz-Thouless (BKT) class.[10,11] Therefore, to find the critical conditions at which the transition should occur, it is necessary to find the superfluid stiffness, including effects of depletion by density fluctuations. This is most naturally achieved by finding the current response functions, and thus separating the current response into normal and superfluid components.[12–14] For the weakly interacting case, one may then perturbatively evaluate the current response functions. Such a perturbative evaluation relies on two properties of the current response: a sum rule on the longitudinal response function (a consequence of using a conserved current),[15] and an understanding of the effect of vertex corrections on the transverse response functions.[16–18]

The aim of this article is to discuss the correct generalization of response functions for nonquadratic, but isotropic, quasiparticle dispersion. Previous work on the BKT transition in a model of weakly interacting bosons with nonquadratic dispersion[19,20] did not generalize the current in this manner. As a result, the current in that work was not conserved, and so there is no sum rule relating the longitudinal response function to density. As the spectrum considered there was quadratic for small momenta, any formalism which recovers the standard form at low densities (i.e., when only low momentum particles are excited) will agree in this limit. However, at higher densities, when particles beyond the quadratic dispersion contribute to the current response, there are differences between the method described here and the method in those previous works, as will be shown below. The effect of nonquadratic dispersion on the transition temperature in three dimensions, in the context of triplons, has also been previously considered.[21]

To be precise, consider the following model of a weakly interacting Bose gas:

$$H=\sum_{\mathbf{k}}\epsilon_{\mathbf{k}}\psi^\dagger_{\mathbf{k}}\psi_{\mathbf{k}}+\frac{g}{2}\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}}\psi^\dagger_{\mathbf{k}+\mathbf{q}}\psi^\dagger_{\mathbf{k}'-\mathbf{q}}\psi_{\mathbf{k}'}\psi_{\mathbf{k}}, \tag{1}$$

where $\epsilon_{\mathbf{k}}$ is isotropic, and has a quadratic part as $k\to 0$, but is otherwise general. This Hamiltonian is invariant under global phase rotations, and so there is an associated Noether current $\mathbf{J}$ given by (Ref. 22):

$$J_i(x)=\frac{\delta S}{\delta\partial_i\psi^\dagger(x)}i\psi^\dagger(x)-\frac{\delta S}{\delta\partial_i\psi(x)}i\psi(x), \tag{2}$$

where S is the action from the Hamiltonian in Eq. (1). By definition, this current is conserved, so:

$$[H,\rho(\mathbf{q})]=\mathbf{q}\cdot\mathbf{J}(\mathbf{q}),\quad \rho(\mathbf{q})=\sum_{\mathbf{k}}\psi^\dagger_{\mathbf{k}+\mathbf{q}/2}\psi_{\mathbf{k}-\mathbf{q}/2}. \tag{3}$$

In the following, we will be interested in the static response of the system to an applied force that couples to such a current, described by the response function:

$$\chi_{ij}(\omega=0,\mathbf{q})=2\int_0^\beta d\tau\langle\langle J_i(\mathbf{q},\tau)J_j(-\mathbf{q},0)\rangle\rangle, \tag{4}$$

where double angle brackets indicate quantum and thermal averaging. For an isotropic system, the most general form of the response function is:

$$\chi_{ij}(\mathbf{q})=\chi_T(q)\left(\delta_{ij}-\frac{q_iq_j}{q^2}\right)+\chi_L(q)\frac{q_iq_j}{q^2}. \tag{5}$$

The standard rotating bucket argument still applies in dividing the response into a superfluid part that contributes only to χ_L and a normal part that contributes to both χ_L and χ_T. With a quadratic dispersion, the relevant quantity is $\rho_s/m = \lim_{q\to 0}[\chi_L(q)-\chi_T(q)]$. With nonquadratic dispersion mass is now q dependent, so the identification of ρ_s and m separately is not possible, but it is not necessary; the effective vortex action depends only on the well defined quantity:

$$\chi_s = \lim_{q\to 0}[\chi_L(q) - \chi_T(q)]. \tag{6}$$

Since the current used is by definition conserved, χ_L will be subject to a sum rule; a generalization of the sum rule that would for quadratic dispersion relate χ_L to the density. Below, this sum rule is evaluated, and thus χ_L and χ_T are calculated in a perturbative expansion.

Before evaluating this sum rule, it is first worth stressing why the above generalization gives the quantity appropriate to the BKT transition. The BKT transition is associated with the unbinding of vortex pairs. The conditions at which the transition occurs are therefore described by the effective vortex-vortex interaction, and the vortex fugacity. Starting from a microscopic model, these quantities both depend on the phase stiffness: the coefficient of $[\nabla\phi(x)]^2 \sim q^2\phi_q^2$ in the effective action. It is only this quadratic phase dispersion which matters: nonquadratic terms in the phase dispersion lead only to short range vortex interactions, while the quadratic term leads to a logarithmic confining potential. However, the phase stiffness is modified by density fluctuations, and the nonquadratic dispersion of density fluctuations can modify the phase stiffness. Nonquadratic dispersion matters because after integrating out density fluctuations, nonquadratic dispersion of density fluctuations modifies the coefficient of quadratic dispersion of phase fluctuations. It is technically easier to evaluate the current response functions than to directly integrate out density fluctuations, and the associated definitions of superfluid stiffness are equivalent (Ref. 18).

The sum rule for $\chi_L(q) = q_i q_j \chi_{ij}(\mathbf{q})/q^2$ follows from Eq. (4) and Eq. (3), and the standard procedure, as described for example in Ref. 15:

$$\chi_L(q) = \frac{2}{q^2}\int_0^\beta d\tau \langle\langle \mathbf{q}\cdot\mathbf{J}(\mathbf{q},\tau)\mathbf{q}\cdot\mathbf{J}(-\mathbf{q},0)\rangle\rangle$$

$$= \frac{1}{\mathcal{Z}q^2}\sum_n e^{-\beta E_n}\langle n|[\rho(\mathbf{q}),\mathbf{q}\cdot\mathbf{J}(\mathbf{q})]|n\rangle, \tag{7}$$

where one use has been made of the commutation relation Eq. (3). Writing the commutation relations explicitly in terms of the $\psi^\dagger$, ψ operators, one has:

$$\mathbf{q}\cdot\mathbf{J}(\mathbf{q}) = \sum_{\mathbf{k}} (\epsilon_{\mathbf{k}+\mathbf{q}/2} - \epsilon_{\mathbf{k}-\mathbf{q}/2})\psi^\dagger_{\mathbf{k}+\mathbf{q}/2}\psi_{\mathbf{k}-\mathbf{q}/2},$$

$$[\rho(\mathbf{q}),\mathbf{q}\cdot\mathbf{J}(\mathbf{q})] = \sum_{\mathbf{k}} (\epsilon_{\mathbf{k}+\mathbf{q}} + \epsilon_{\mathbf{k}-\mathbf{q}} - 2\epsilon_{\mathbf{k}})\psi^\dagger_{\mathbf{k}+\mathbf{q}/2}\psi_{\mathbf{k}-\mathbf{q}/2}.$$

In the limit $q\to 0$, the terms in parentheses are independent of the direction of k, so we may average over solid angles $d\Omega$ and thus have:

$$\lim_{q\to 0}\chi_L(q) = \sum_{\mathbf{k}} g_k \langle\langle \psi^\dagger_{\mathbf{k}}\psi_{\mathbf{k}}\rangle\rangle, \tag{8}$$

$$g_k = \int \frac{d\Omega}{\Omega}\lim_{q\to 0}\left(\frac{\epsilon_{\mathbf{k}+\mathbf{q}} + \epsilon_{\mathbf{k}-\mathbf{q}} - 2\epsilon_{\mathbf{k}}}{q^2}\right). \tag{9}$$

Since dispersion is isotropic, we may write it as $\epsilon_{\mathbf{k}} = f(k^2)$, thus $g_k = (4/d)k^2 f''(k^2) + 2f'(k^2)$, and as expected a quadratic dispersion, $f(x) = x/2m$, reduces to $g_k = 1/m$. In general, Eq. (8) can be considered as density weighted by effective inverse mass at each momentum. The longitudinal response is thus reduced to finding an approximation scheme for the occupation of each finite k mode, which will be discussed below.

As yet we have only written explicitly the longitudinal component of the current. To find correlations of the transverse component, it is convenient to write:

$$J_i(\mathbf{q}) = \sum_{\mathbf{k}} \Psi^\dagger_{\mathbf{k}+\mathbf{q}}\gamma_i(\mathbf{k}+\mathbf{q},\mathbf{k})\Psi_{\mathbf{k}}, \quad \Psi_{\mathbf{k}} = \begin{pmatrix}\psi_{\mathbf{k}} \\ \psi^\dagger_{-\mathbf{k}}\end{pmatrix}, \tag{10}$$

and from conservation of current, we have:

$$q_i\gamma_i(\mathbf{k}+\mathbf{q},\mathbf{k}) = \sigma_3(\epsilon_{\mathbf{k}+\mathbf{q}} - \epsilon_{\mathbf{k}}). \tag{11}$$

Thus, we know the projection of the vector γ_i onto one axis; what remains is to find its direction. This follows from the definition in Eq. (2), which shows that under the interchange $\psi_{\mathbf{k}} \leftrightarrow \psi^\dagger_{\mathbf{k}}$ the current changes as $J_i \to -J_i$. With a little algebra, it can be seen that this directly implies $\gamma_i(\mathbf{p},\mathbf{q}) = (p_i + q_i)f(\mathbf{p},\mathbf{q})\sigma_3$ where $f(\mathbf{p},\mathbf{q})$ is a scalar function chosen to satisfy Eq. (11).

From this definition of current, it is now possible to calculate the current response function perturbatively. As in the quadratic dispersion case, the leading order perturbative calculation relies on properties of the corrections to the current vertex γ_i that result from interactions. For clarity, the standard argument[15,18] is summarized here. A full calculation of the response would be given by

$$\lim_{q\to 0}\chi_{ij}(q) = 2\int \frac{d^dk}{(2\pi)^d}\mathrm{Tr}[\mathcal{G}(\mathbf{k})\Gamma_i(\mathbf{k},\mathbf{k})\mathcal{G}(\mathbf{k})\gamma_j(\mathbf{k},\mathbf{k})], \tag{12}$$

where $\mathcal{G}(\mathbf{k})$ is the Green's function in the Nambu representation indicated in Eq. (10), and Γ_i is the vertex γ_i including corrections. At one loop order, vertex corrections are necessary to make Eq. (12) satisfy the sum rule, Eq. (8). However, it can be seen that these required vertex corrections are of the form shown in Fig. 1.

Since these involve a vertex where current couples directly to the condensate, they involve $\gamma_i(\mathbf{q},0)$, which, due to the previous discussion of the direction of γ_i, is proportional to q_i. Such a correction therefore only changes the longitudinal response. Therefore, we may safely evaluate the transverse response at one-loop order without such corrections.

Having understood why vertex corrections can be ignored, the perturbative calculation of χ_T now follows directly; writing:

$$\gamma_i(\mathbf{k},\mathbf{k}) = k_i\lim_{q\to 0}\left(\frac{\epsilon_{\mathbf{k}+\mathbf{q}/2} - \epsilon_{\mathbf{k}-\mathbf{q}/2}}{\mathbf{k}\cdot\mathbf{q}}\right)\sigma_3 = 2k_i f'(k^2), \tag{13}$$

with $f(k^2) = \epsilon_{\mathbf{k}}$ as before for an isotropic mass, we thus have:

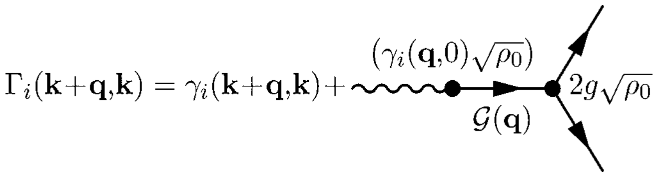

FIG. 1. Vertex corrections required at one-loop order.

$$\lim_{q\to 0}\chi_T(q) = \frac{2}{d}\int\frac{d^dk}{(2\pi)^d}k^2[2f'(k^2)]^2\mathrm{Tr}[\mathcal{G}(\mathbf{k})\sigma_3\mathcal{G}(\mathbf{k})\sigma_3]. \tag{14}$$

To complete the evaluation of χ_T then requires an explicit form for the Green's function. The Bogoliubov one-loop approximation for the condensed Green's function is:

$$\mathcal{G}(\omega,\mathbf{k}) = \frac{1}{\omega^2+\xi_k^2}\begin{pmatrix} -i\omega+\epsilon_k+\mu & -\mu \\ -\mu & i\omega+\epsilon_k+\mu \end{pmatrix}, \tag{15}$$

where $\xi_k=\sqrt{\epsilon_k(\epsilon_k+2\mu)}$ is the Bogoliubov quasiparticle energy. Thus, Eq. (14) becomes:

$$\mathrm{Tr}(\mathcal{G}\sigma_3\mathcal{G}\sigma_3) = \frac{1}{2}\sum_{\omega_n}\frac{\xi_k^2-\omega_n^2}{(\omega_n^2+\xi_k^2)^2} = -\frac{\beta}{2}n_B'(\xi_k), \tag{16}$$

$$\lim_{q\to 0}\chi_T(q) = -\frac{2}{d}\int\frac{d^dk}{(2\pi)^d}\frac{k^2[2f'(k^2)]^2}{2}n_B'(\xi_k). \tag{17}$$

Finally, to find χ_L requires evaluation of the average occupation of each k mode in Eq. (8). In evaluating Eq. (8), as the effects of fluctuations in the presence of a condensate are required, the condensate depletion due to fluctuations must be included in order to derive a consistent answer.[23,24] This means $\rho_0=\langle\langle\psi_0^\dagger\psi_0\rangle\rangle$ must include fluctuation corrections, determined by considering a chemical potential coupled only to $k=0$ modes, or equivalently by using the Hugenholtz-Pines relation at one-loop order:

$$\rho_0 = \frac{\mu}{g} - \sum_{\mathbf{k}}\left[2\langle\langle\psi_{\mathbf{k}}^\dagger\psi_{\mathbf{k}}\rangle\rangle + \frac{1}{2}(\langle\langle\psi_{\mathbf{k}}^\dagger\psi_{\mathbf{k}}^\dagger\rangle\rangle + \mathrm{H.c.})\right]. \tag{18}$$

Inserting this in Eq. (8) yields the final form:

$$\lim_{q\to 0}\chi_L(q) = g_0\frac{\mu}{T_{2B}} - \int\frac{d^dk}{(2\pi)^d}\times\left[n_B(\xi_k)\left(g_0\frac{2\epsilon_k+\mu}{\xi_k} - g_k\frac{\epsilon_k+\mu}{\xi_k}\right) + (2g_0-g_k)\frac{\epsilon_k+\mu-\xi_k}{2\xi_k} + \frac{g_0\mu}{2}\frac{\xi_k-\epsilon_k-\mu}{\xi_k(\epsilon_k+\mu)}\right]. \tag{19}$$

(To avoid the ultraviolet divergence associated with a delta-function interaction, the standard T-matrix renormalization[24] has been performed, thus T_{2B} is the renormalized two-body T-matrix corresponding to the bare interaction g.) In two dimensions, the BKT transition is found, as discussed above, by evaluating Eq. (17) and Eq. (19) at a fixed temperature, and finding the value of μ which satisfies $\lim_{q\to 0}[\chi_L(q)-\chi_T(q)]=\pi k_BT/2$.

The discussion up to now has been for a generic isotropic dispersion. In order to calculate a phase boundary—in order to see how large the differences between the method here, and direct application of the standard Landau formula for superfluid density—one must consider some specific dispersion. For illustration, I will therefore consider the dispersion of the lower polariton in CdTe, for which the results of the Landau formula have been considered in Refs. 19 and 20. However, several caveats must be raised about any comparison of such a phase boundary and experiments. The discussion here does not include the presence of an upper polariton, nor effects due to the composite nature of polaritons.[25–27] Both of these effects, and the deviation of the lower polariton from quadratic dispersion, will become relevant at similar temperature scales, as all of these scales are set by the Rabi splitting. In addition, except for very recent experiments,[28,29] polariton systems have not reached thermal equilibrium, and so dynamical condensation, with occupations set by balance of pumping and decay may need to be considered[30]—furthermore, even when distributions are close to equilibrium, pumping and decay may still have important effects.[31]

Figure 2 shows the critical density for a BKT transition at a given temperature, with the following dispersion:

$$f(k^2) = \frac{1}{2}\left[\frac{k^2}{2m_1} - \sqrt{\left(\frac{k^2}{2m_2}\right)^2 + \Omega_R^2}\right], \tag{20}$$

with $1/m_1=1/m_X+1/m_P$, and $1/m_2=1/m_X-1/m_P$. Parameters are chosen close to those of the experiments of Ref. 28 in CdTe: exciton mass $m_X=0.08m_e$, photon mass $m_P=2.58\times 10^{-5}m_e$, $\Omega_R=26$ meV, and $T_{2B}=13$ meV$/10^{11}$ cm^{-2}. Bearing in mind the above caveats, Fig. 2 should be taken as an illustration of the size of one specific effect—deviation of the lower polariton from quadratic dispersion—on the equilibrium phase boundary.

In the normal state, the transverse and longitudinal response functions should become equal. It is instructive to see how the expression for effective mass, weighting the density, appears in such a calculation. In the normal state, there are no condensate depletion effects to worry about, and so:

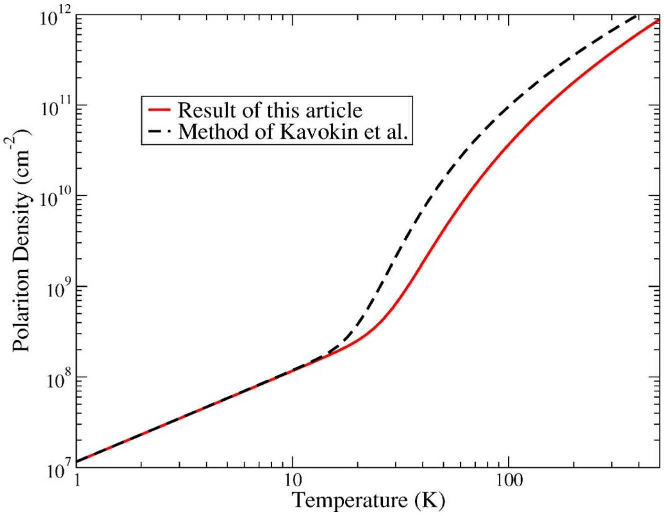

FIG. 2. (Color online) Comparison of calculation of BKT critical density vs temperature for the method discussed here and the method in Refs. 19 and 20. At low densities, both calculations agree, as nonquadratic effects are irrelevant, but where such effects matter, their predictions differ.

$$\chi_L(0) = \int \frac{d^dk}{(2\pi)^d} g_k n_B[f(k^2)]. \tag{21}$$

The one-loop transverse response is as in Eq. (17), but with $\xi_k \to \epsilon_k = f(k^2)$. To see that they agree, it is convenient to rewrite Eq. (17) with a change of integration variables. We first introduce $x=k^2$, so $d^dk = S_d x^{d/2-1} dx/2$, with S_d the surface of the d-dimensional hypersphere, and then change the integration variable again to $f(x)$, with $dx = df/f'(x)$, giving:

$$\begin{aligned}\chi_T(0) &= -\frac{2}{d}\int \frac{S_d df}{(2\pi)^d}\frac{x^{d/2-1}}{2f'(x)}\frac{x[2f'(x)]^2}{2}\frac{dn_B(f)}{df}\\ &= \frac{2}{d}\int \frac{S_d df}{(2\pi)^d}\left(\frac{1}{2f'(x)}\frac{d}{dx}[x^{d/2}2f'(x)]\right)n_B(f)\\ &= \frac{2}{d}\frac{d}{2}\int \frac{d^dk}{(2\pi)^d} g_k n_B[f(k^2)],\end{aligned} \tag{22}$$

where the second line is integration by parts, and the last used $\partial_x[2x^{d/2}f'(x)] = (d/2)x^{d/2-1}g(x)$.

In conclusion, a formalism for calculating the transverse and longitudinal response functions of a Bose gas with arbitrary isotropic dispersion has been presented. A sum rule relates the longitudinal response to density weighted by effective inverse mass at a given momentum. Using such a formalism recovers the equivalence of transverse and longitudinal responses in the normal state. This formalism allows a consistent formulation of the critical conditions for the BKT transition in a two-dimensional Bose gas.

ACKNOWLEDGMENTS

I would like to thank F. M. Marchetti and M. H. Szymańska for the suggestion that led to this work and P. B. Littlewood for helpful discussion, and to acknowledge financial support from the Lindemann Trust.

[1] L. V. Butov, J. Phys.: Condens. Matter **16**, R1577 (2004).
[2] L. S. Dang, D. Heger, R. André, F. Bœuf, and R. Romestain, Phys. Rev. Lett. **81**, 3920 (1998).
[3] H. Deng, G. Weihs, C. Santori, J. Bloch, and Y. Yamamoto, Science **298**, 199 (2002).
[4] M. Richard, J. Kasprzak, R. Romestain, R. André, and L. S. Dang, Phys. Rev. Lett. **94**, 187401 (2005).
[5] J. P. Eisenstein and A. H. MacDonald, Nature (London) **432**, 691 (2004).
[6] J. P. Eisenstein, Science **305**, 950 (2004).
[7] C. Rüegg, N. Cavadinin, A. Furrer, H.-U. Güdel, K. Krämer, H. Mukta, A. Wildes, K. Habicht, and P. Worderwisch, Nature (London) **423**, 62 (2003).
[8] M. Jaime, V. F. Correa, N. Harrison, C. D. Batista, N. Kawashima, Y. Kazuma, G. A. Jorge, R. Stern, I. Heinmaa, S. A. Zvyagin *et al.*, Phys. Rev. Lett. **93**, 087203 (2004).
[9] T. Radu, H. Wilhelm, V. Yushankhai, D. Kovrizhin, R. Coldea, Z. Tylczynski, T. Lühmann, and F. Steglich, Phys. Rev. Lett. **95**, 127202 (2005).
[10] J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973).
[11] D. R. Nelson and J. M. Kosterlitz, Phys. Rev. Lett. **39**, 1201 (1977).
[12] D. R. Nelson, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J. Lebowitz (Academic Press, London, 1983), Vol. 7, Chap. 1, p. 2.
[13] D. S. Fisher and P. C. Hohenberg, Phys. Rev. B **37**, 4936 (1988).
[14] H. T. C. Stoof and M. Bijlsma, Phys. Rev. E **47**, 939 (1993).
[15] L. P. Pitaevskii and S. Stringari, *Bose-Einstein Condensation* (Clarendon Press, Oxford, 2003).
[16] Y. Takahashi, Nuovo Cimento **6**, 371 (1957).
[17] J. C. Ward, Phys. Rev. **78**, 182 (1950).
[18] A. Griffin, *Excitations in a Bose-Condensed Liquid* (Cambridge University Press, Cambridge, 1994).
[19] A. Kavokin, G. Malpuech, and F. P. Laussy, Phys. Lett. A **306**, 187 (2003).
[20] G. Malpuech, Y. G. Rubo, F. P. Laussy, P. Bigenwald, and A. V. Kavokin, Semicond. Sci. Technol. **18**, S395 (2003).
[21] E. Ya Sherman, P. Lemmens, B. Busse, A. Oosawa, and H. Tanaka, Phys. Rev. Lett. **91**, 057201 (2003).
[22] N. Nagaosa, *Quantum Field Theory in Condensed Matter Physics* (Springer-Verlag, Berlin, 1999).
[23] J. Keeling, P. R. Eastham, M. H. Szymanska, and P. B. Littlewood, Phys. Rev. B **72**, 115320 (2005).
[24] U. AlKhawaja, J. O. Andersen, N. P. Proukakis, and H. T. C. Stoof, Phys. Rev. A **66**, 013615 (2002).
[25] P. R. Eastham and P. B. Littlewood, Solid State Commun. **116**, 357 (2000).
[26] J. Keeling, P. R. Eastham, M. H. Szymanska, and P. B. Littlewood, Phys. Rev. Lett. **93**, 226403 (2004).
[27] F. M. Marchetti, J. Keeling, M. H. Szymańska, and P. B. Little-

wood, Phys. Rev. Lett. **96**, 066405 (2006).
[28] J. Kasprzak and L. S. Dang (private communication).
[29] H. Deng, D. Press, S. Goetzinger, G. S. Solomon, R. Hey, K. H. Ploog, and Y. Yamamoto, cond-mat/0604394 (unpublished).
[30] T. D. Doan, H. T. Cao, D. B. Tran Thoai, and H. Haug, Phys. Rev. B **72**, 085301 (2005).
[31] M. H. Szymańska, J. Keeling, and P. B. Littlewood, Phys. Rev. Lett. **96**, 230602 (2006).