RAPID COMMUNICATIONS

PHYSICAL REVIEW E VOLUME 50, NUMBER 4 OCTOBER 1994

Langmuir wave instability in a dusty plasma

Jin-Xiu Ma* and M. Y. Yu
Institut für Theoretische Physik I, Ruhr-Universität Bochum, D-44780 Bochum, Germany
(Received 8 July 1994)

By means of a self-consistent theory for dust charging, it is shown that the dust-charge relaxation process leads to an instability of the Langmuir waves.

PACS number(s): 52.25.Vy, 52.35.Fp, 52.35.Lv, 52.35.Dm

The effect caused by the charge fluctuations of dust grains in a plasma has been of interest [1–16] for some time because of the many applications of dusty plasmas in space and technology. In the absence of other plasma motion, such as collective plasma oscillations, the charge on a dust grain tends to return to its equilibrium value when perturbed, corresponding to a pure relaxation process. However, when there exist other density and potential fluctuations [10–16], such as those associated with the collective oscillations, the currents at the dust grains can be affected, resulting in a change of the dust-charging process.

Recently, the interaction of dust-charging with collective plasma oscillations has been investigated [13–15]. In these studies, the electrostatic probe model [12–15,17,18] for the dust grain is used. The average dust grain is assumed to be a spherical conductor immersed in an equilibrium plasma. Electrons and ions can flow into the sphere according to the difference in the electrostatic potential between the local plasma and the grain surface. Existing results from electrostatic-probe theory [17,18] are then used. It has been found [13–15] that dust-charge relaxation can result in the damping of various electrostatic modes. In particular, it was found that instability can only arise in the presence of streaming of the electrons or ions with respect to the dust grains.

In the probe model, both the electrons and ions in the vicinity of the dust grain are assumed to be in local thermal equilibrium. One version assumes equilibrium electrons but cold, monoenergetic ions. In both cases, the currents, which shall be referred to as the grain currents, flowing into the dust grain are taken from the standard electrostatic-probe theory [17,18]. The effect of collective oscillations on dust charging is then included through the particle densities and the electrostatic potential appearing in these predetermined grain currents. Based on the argument that the dust grain is well shielded and that the steady-state particle distributions are not affected by the perturbations, the grain currents are obtained without taking into account self-consistently the plasma oscillations. Thus, for many applications of these models one must pay special attention to the difference in the electron and ion time scales of the predetermined grain currents, since the electron and ion densities and the potential appearing in the latter are not explicitly time scale dependent. For example, in considering high-frequency waves on the electron time scale (which is shorter than the charge-relaxation time) in weakly collisional dusty plasmas, the perturbation ion grain current could not be correctly represented by a formula obtained by assuming that the ions are in thermal equilibrium even in the perturbed state. It could also not be represented by the monoenergetic-ion model, since the perturbed ion motion could not be included in the (constant) flow speed appearing in the latter. In fact, from physical arguments there should not be any perturbation of the ion grain current since the ions cannot react to the high-frequency waves. Furthermore, since the electrons are involved in the oscillations, they depart from thermal equilibrium. It follows that the Debye shielding of the dust grain and the electron grain current must be dynamically affected by the high-frequency waves. In this paper, we consider the linear coupling of the Langmuir waves with the charge-relaxation process by neglecting the ion grain-current perturbations and including self-consistently the perturbations of the electron grain current in the presence of the Langmuir wave motion. It is shown that the Langmuir waves are unstable, and the rate of the dust-charging process differs from that obtained by assuming equilibrium electrons and ions.

Since the dust grains are much heavier than the ions and electrons, their dynamics is on a much longer time scale, so that they can be treated as an immobile but nonstationary background in which the dust charge can vary in space and time. The equilibrium (steady-state) grain currents are calculated assuming thermal-equilibrium electrons and ions. The perturbation electron grain current will be obtained using self-consistent kinetic theory. Although there exist steady-state ion grain currents, the ions do not respond at all to perturbations at the electron time scale $\omega_{pe}^{-1}=(4\pi e^2 n_{e0}/m_e)^{-1/2}$ [which is much shorter than the ion response time $\omega_{pi}^{-1}=(4\pi e^2 n_{e0}/m_i)^{-1/2}$] and the dust-charge relaxation time, so that there is no perturbation in the ion grain current. That is, for the perturbation the immobile background consists of the variable-charge dust grains and the fixed-charge ions.

In the probe model for the dust grain, the latter is charged by the the plasma currents at the grain surface. Thus we have [12–15]

$$\frac{\partial Q}{\partial t}=I_e+I_i, \tag{1}$$

where Q is the charge of the immobile dust grain. The electron and ion grain currents I_e and I_i are

*Permanent address: Shanghai Institute of Optics and Fine Mechanics, Academia Sinica, P.O. Box 800-211, Shanghai 210800, P.R. China.

$$I_e=I_{e0}+\tilde{I}_e$$
$$=-\pi a^2 e\sqrt{\frac{8T_e}{\pi m_e}}n_{e0}\exp\left[\frac{e(\phi_g-\phi_0)}{T_e}\right]+\tilde{I}_e(\tilde{n}_e,\tilde{\phi}), \quad (2)$$

and

$$I_i=I_{i0}=\pi a^2 e\sqrt{\frac{8T_i}{\pi m_i}}n_{i0}\left[1-\frac{e(\phi_g-\phi_0)}{T_i}\right], \quad (3)$$

where a is the dust-sphere radius, $m_{e,i}$, $T_{e,i}$, and $n_{e0,i0}$ are the mass, temperature, and equilibrium density of the electrons and ions, respectively. We have defined $\tilde{n}_e$, $\tilde{I}_e$, and $\tilde{\phi}$ as the perturbation electron density and grain current, and the perturbation potential, respectively. Furthermore, $\phi_g-\phi_0$ is the potential difference between the grain and the unperturbed local plasma, so that the unperturbed dust charge is $Q_0 = C(\phi_g-\phi_0)<0$ where C is the capacitance of the grain. In the absence of the perturbations, the electron and ion grain currents are equal and the grain surface is at the so-called floating potential. Since the calculation for the monoenergetic-ion model is basically similar, we shall not discuss it here.

Note that Q_0 is to be considered as the mean charge of the dust grains. In the earlier studies [12–15], ϕ_0 in (2) and (3) is replaced by the total local plasma potential $\phi=\phi_0+\tilde{\phi}$ and the perturbation currents $\tilde{I}_{e,i}$ are calculated by expanding the exponentials. This implicitly assumes that the ions and electrons can respond to any variation in the potential as if they are in thermal equilibrium, an assumption which is clearly not valid for perturbations on the Langmuir wave time scale unless the electron collision frequency is much larger than the plasma frequency. On the other hand, a similar difficulty does not arise for the spatial scaling, since the dust size is usually much smaller than the Debye length or wavelength.

The total mean charge Q consists of Q_0 and a fluctuating part $\tilde{Q}$, which is governed by [13–15,17]

$$\frac{\partial\tilde{Q}}{\partial t}+\eta\tilde{Q}=\tilde{I}_e=-e\int_{v_m}^{\infty}\tilde{f}v\sigma(v,Q_0)d^3v, \quad (4)$$

where $v_m=(-2eQ_0/m_eC)^{1/2}$ is the minimum speed an electron must have in order to arrive at the grain surface, and $\tilde{f}$ is the perturbation electron distribution function to be calculated self-consistently. The charging rate η, originating from the variations in the effective collision cross section due to the charge perturbations at the grain surface as experienced by the unperturbed particles, can be written as

$$\eta=e|I_{e0}|\left(\frac{1}{CT_e}+\frac{1}{CT_i-eQ_0}\right).$$

The effective collision cross section is

$$\sigma(v,Q_0)=\pi a^2\left(1+\frac{2eQ_0}{m_eCv^2}\right). \quad (5)$$

The perturbation electron distribution function $\tilde{f}$ is given by the solution of the Vlasov equation [15,18],

$$\tilde{f}=-\frac{ieE_x}{m_e}\frac{\partial_{v_x}f_0}{\omega-kv_x}, \quad (6)$$

where $E_x=-\partial_x\tilde{\phi}$ and perturbations of the form $\exp[i(\omega t-kx)]$ have been assumed. We have also assumed that the dust-charge variation is small compared to the total number of electrons in the plasmas.

The perturbation dust charge is then

$$\tilde{Q}=\frac{e^2E_x}{m_e(\omega-i\eta)}\int_{v_m}^{\infty}\frac{\partial_{v_x}f_0}{\omega-kv_x}v\sigma(v,Q_0)d^3v. \quad (7)$$

Thus the coupling of the dust-charge relaxation process with the collective plasma motion is effected through the plasma density as well as potential perturbations.

In equilibrium, there is overall charge neutrality, i.e., $en_{i0}-en_{e0}+Q_0n_{d0}=0$, where n_{d0} is the density of the immobile dust grains. The linearized Poisson's equation is then

$$\nabla^2\tilde{\phi}=4\pi(e\tilde{n}_e-n_{d0}\tilde{Q}), \quad (8)$$

which we note also contains the dust-charge perturbation $\tilde{Q}$ and no ion density perturbations.

One can easily obtain from (6)–(8) the linear dispersion relation for the Langmuir waves

$$1=-\chi_e+\frac{i\omega_{pe}^2}{k^2(\omega-i\eta)}\frac{n_{d0}}{n_{e0}}\int_{v_m}^{\infty}\frac{\partial_{v_x}f_0}{v_\phi-v_x}v\sigma(v,Q_0)d^3v, \quad (9)$$

where $v_\phi=\omega/k$ is the phase velocity and

$$\chi_e=\frac{\omega_{pe}^2}{k^2}\frac{1}{n_{e0}}\int_{-\infty}^{\infty}\frac{\partial_{v_x}f_0}{v_\phi-v_x}d^3v \quad (10)$$

is the electron susceptibility [18].

The integration in (9) can be performed using spherical coordinates for the velocity space. In the limit $v_\phi\gg v_{Te}=\sqrt{T_e/m_e}$ for propagating electron plasma waves, one obtains

$$1+\chi_e+\frac{i\beta\omega_{pe}^2}{(\omega-i\eta)\omega^2}=0, \quad (11)$$

where the electron susceptibility χ_e is given by

$$\chi_e=-\frac{\omega_{pe}^2}{\omega^2}\left(1+\frac{3k^2v_{Te}^2}{\omega^2}\right)+i\chi_L,$$
$$\chi_L\equiv-\frac{\sqrt{\pi}}{\sqrt{2}k^3\lambda_e^3}\exp\left(-\frac{3}{2}-\frac{1}{2k^2\lambda_e^2}\right), \quad (12)$$

and

$$\beta=\frac{8}{3}\pi a^2n_{d0}\left(\frac{T_e}{2\pi m_e}\right)^{1/2}\left(2-\frac{eQ_0}{CT_e}\right)\exp\left(\frac{eQ_0}{T_eC}\right), \quad (13)$$

where $\lambda_e = v_{Te}/\omega_{pe}$ is the electron Debye length.

The dispersion relation can be written as

$$(\omega^2-\omega_L^2+i\omega_{pe}^2\chi_L)(\omega-i\eta)=-i\beta\omega_{pe}^2, \quad (14)$$

where $\omega_L = (\omega_{pe}^2 + 3k^2 v_{Te}^2)^{1/2}$ is the Langmuir wave frequency.

Equation (14) is cubic in ω and has three roots. We note that if $a \rightarrow 0$, the roots would correspond to the usual Langmuir waves with $\omega = \pm\omega_L - i\omega_{pe}\chi_L/2$ and the purely damped charge-relaxation mode $\omega = i\eta$ ($\rightarrow 0$).

Although (14) can be solved analytically, it is instructive to make use of the contrasting magnitudes of the parameters involved and obtain accurate approximate solutions. Since $\omega_L \gg \eta, \beta$, we find

$$\omega_{1,2} = \pm\omega_L + \delta_{1,2}, \quad \omega_3 = i\eta + \delta_3, \tag{15}$$

where $\delta_{1,2,3}$ are given by

$$\delta_{1,2} = -i\frac{\beta}{2}\frac{\omega_{pe}^2}{\omega_L^2} + i\nu_L, \quad \delta_3 = i\beta\frac{\omega_{pe}^2}{\omega_L^2}, \tag{16}$$

where $|\delta_{1,2,3}| \ll \omega_L$, and $\nu_L = -\omega_{pe}\chi_L/2$ is the electron Landau damping.

Thus, if $\beta > 2|\nu_L|$, the Langmuir waves are unstable. For long-wavelength oscillations, the growth rate is $\gamma = (\beta/2)(\omega_{pe}^2/\omega_L^2)$, and the dust-charge relaxation rate is enhanced by $\beta(\omega_{pe}^2/\omega_L^2)$. Our result differs considerably from those of the earlier models [13–15], which assume that the electrons and ions near the dust grains maintain thermal equilibrium even in the presence of the high-frequency perturbations, so that they respond to the perturbations through the electrostatic potential with only a delay caused by the grain capacitance. The present instability exists because on the Langmuir wave time scale the ions cannot contribute to the grain current, and the electrons behave as a warm fluid in the perturbations.

From the condition $\beta > 2|\nu_L|$, one can obtain the range of unstable wave numbers. This condition can be rewritten as

$$\frac{4}{3}\left(1 - \frac{eQ_0}{2CT_e}\right)|I_{e0}|\frac{n_{d0}}{n_{e0}} > \frac{i\sqrt{\pi}\omega_{pe}}{\sqrt{2}k^3\lambda_e^3}\exp\left(-\frac{3}{2} - \frac{1}{2k^2\lambda_e^2}\right), \tag{17}$$

which has been expressed in terms of the known equilibrium current I_{e0}. As an example, we found that for $T_e = 10T_i = 1$ eV, $n_{e0} = 0.9n_{i0} = 10^{12}$ cm^{-3}, and $a = 1\mu$m, instability occurs for $k\lambda_e < 0.23$. Furthermore, the range of unstable wave numbers increases with the dust density, since the number of electrons participating in the damping is then reduced.

We have shown that the coupling of charge relaxation of the dust grains with Langmuir waves leads to an instability of the latter. That is, there can be energy transfer between the plasma waves and the dust-charging process. Similar instabilities are expected to occur for other high-frequency ($\omega \gg \omega_{pi}$) waves. The instability resembles certain dissipative instabilities such as those associated with ionization [19]. It is not clear how the present instability is saturated. Over longer times, we expect that effects involving collective ion motion can come into play (e.g., when the finite-amplitude Langmuir waves become modulated by the ion waves) and limit the instability evolution. However, during the short [20] growing stage, the enhanced Langmuir waves could have already altered the properties of the plasma through various other interactions. Besides the usual quasilinear and nonlinear mechanisms for instability saturation, here there is also the possibility of enhanced shielding of the dust grains.

We should emphasize that results such as the present one are strongly dependent on the model of the dusty plasma. The probe model used here and elsewhere is at best a rough representation. In particular, although the total charge is conserved, the total number of electrons and ions is not conserved (even in equilibrium) in the probe model, since unlike the probe itself, the dust grains are not connected to an external circuit [21], and can thus indefinitely absorb the charged particles. Furthermore, in reality most dust grains are not spherical and are made of dielectric rather than conducting material, so that they may also act as charged dipoles or multipoles in the presence of the electrostatic potential of the plasma oscillations. These would greatly affect the shielding properties of the plasma and thus the physics (in particular, the close-range interaction force among the grains) of the problem. Another important property not taken into consideration is the large variation in the size, mass, shape, composition, charge, and spatial distribution of the dust grains in the plasma. Such inhomogeneities are highly dynamic and can strongly affect the normal modes as well as other plasma phenomena.

This work has been partially supported by the Sonderforschungsbereich 191 Niedertemperaturplasmen. One of the authors (J.X.M.) would like to thank the Alexander von Humboldt Foundation for financial support, and K. Elsässer and G. Wunner for hospitality.

[1] T. G. Northrop and J. R. Hill, J. Geophys. Res. **88**, 1 (1983).
[2] T. G. Northrop and G. E. Morfill, Adv. Space Res. **4**, 63 (1984).
[3] E. C. Whipple, T. G. Northrop, and D. A. Mendis, J. Geophys. Res. **90**, 7405 (1985).
[4] K. M. Leung, Phys. Rev. A **73**, 2461 (1986).
[5] P. Notni and H. Tiersch, Astron. Astrophys. **187**, 796 (1987).
[6] C. K. Goertz, Rev. Geophys. **27**, 271 (1989).
[7] G. Selwyn, J. E. Heidenreich, and K. Haller, Appl. Phys. Lett. **57**, 1867 (1990).
[8] G. Selwyn, J. S. McKillop, K. Haller, and J. J. Wu, J. Vac. Sci. Technol. **18**, 1726 (1990).
[9] G. S. Selwyn, J. Vac. Sci. Technol. B **9**, 3487 (1991).
[10] N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. **38**, 543 (1990).
[11] A. E. Lonngren, Planet. Space Sci. **38**, 1457 (1990).
[12] M. S. Barnes, J. H. Keller, J. C. Forster, J. A. O'Neill, and D. K. Coultas, Phys. Rev. Lett. **68**, 313 (1992).
[13] R. K. Varma, P. K. Shukla, and V. Krishan, Phys. Rev. E **47**, 3612 (1993).

[14] M. R. Jana, A. Sen, and P. K. Kaw, Phys. Rev. E **48**, 3930 (1993).
[15] V. N. Tsytovich and O. Havnes, Comments Plasma Phys. Controlled Fusion **15**, 267 (1993).
[16] Yinhau Chen and M. Y. Yu, Phys. Lett. A **185**, 475 (1994).
[17] F. F. Chen, in *Plasma Diagnostic Techniques*, edited by R. H. Huddlestone and S. L. Leonard (Academic, New York, 1965), Chap. 4.
[18] F. F. Chen, *Introduction to Plasma Physics* (Academic, New York, 1965).
[19] L. Pekarek, Usp. Fiz. Nauk **94**, 463 (1968) [Sov. Phys. Usp. **11**, 188 (1968)].
[20] Say, less than 30 ω_{pe}^{-1} for a dusty hydrogen plasma, taking into consideration that the electron density can be down to about half of that of the ions if the plasma is sufficiently dusty [12].
[21] For the same reason, the result here does not imply that a diagnostic probe would lead to unstable Langmuir waves in a plasma. On the other hand, our conclusion should apply to the plasma environment near artificial satellites.