**ARTICLE IN PRESS** 

No. of Pages 6, Model 5G

Optics Communications xxx (2008) xxx-xxx

Contents lists available at ScienceDirect



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**Optics Communications** 

journal homepage: www.elsevier.com/locate/optcom

#### Double grating systems with one steel tape grating 2

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## ARTICLE INFO

9	
8	Article history:
9	Received 5 February 2008
10	Received in revised form 27 June 2008
11	Accepted 11 August 2008
12	Available online xxxx
13	PACS:
14	42.25p
15	42.25.Fx
16	42.25.Hz
17	42.79.Dj

#### 33 1. Introduction

Double grating systems are used in numerous applications such 34 as metrology, interferometry [1–3], and spectrometry [4,5], exist-35 ing several common configurations [6,7]. In Moiré configuration, 36 37 a plane wave illuminates a system formed by two gratings of the 38 same period [8]. Then fringes are observed just after the second grating. The well-known Talbot effect appears, that is, a periodic 39 modulation of contrast in terms of the distance between gratings. 40 The period of this modulation is the well-known Talbot distance 41 42  $z_t = p^2 / \lambda$ , where *p* is the period of the gratings and  $\lambda$  the wavelength. 43 In so-called Lau configuration, on the other hand, a point source is not required. [9,10]. The observation plane is located at infinite ΔΔ 45 and, in practice, a lens is used to detect fringes at its focal plane [11]. In Generalized Grating Imaging configuration the two grat-46 ings may present equal or different periods and fringes are ob-47 tained at finite distances from the second grating [12-15]. As a 48 consequence, the devices which use this configuration are more 49 compact and robust, since no lenses are required. 50

51 In most applications, chrome on glass gratings is used. They are 52 easily manufactured and their period can be very small. However, 53 glass gratings are not appropriate for measuring displacements longer than 3 m, since they are difficult to manufacture and handle. 54 55 In these cases, steel tape gratings are used. This kind of gratings 56 can be much more easily manufactured for longer lengths than chrome on glass gratings. Besides, their handling is not critical. 57 However, some disadvantages appear. The period of steel tape 58

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## ABSTRACT

Steel tape gratings are used in different metrology applications. As the period of these gratings was large (around 100  $\mu$ ), its analytical study has been performed, up to date, using a geometrical approach. Nowadays, steel tape gratings can be manufactured with lower periods, around  $20-40 \,\mu$ , and diffractive effects must be taken into account. Also, due to the roughness of the surface, statistical techniques need to be considered to analyze their behavior. In this work, an analysis of the pseudo-imaging formation in a double grating system including one steel tape grating is performed. In particular Moiré and Lau configurations are analyzed. We have found that roughness significantly affects to Moiré configuration. However, its effect is negligible in Lau configuration. Generalized grating imaging configuration is also studied in depth. It is shown that roughness does not affect to the contrast of pseudoimages, but it modifies their depth of focus.

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gratings is larger. Traditionally, the standard periods for steel tape gratings were around 100 µ, and a geometrical analysis was enough to determine the main characteristics of the fringes. Nowadays, periods of 20–40  $\mu$  are available for commercial steel tape gratings [16]. With this range of periods, the diffractive behavior of the gratings must be taken into account. Also, steel tape gratings are not ideal, because their surface presents a certain roughness due to the fabrication process and to the nature of the substrate. This roughness produces adverse effects in the self-imaging process [17,18].

In this work, we analyze double grating systems when one of the gratings is a steel tape grating. In particular, we have analyzed the fringe formation when the first grating is a steel tape grating and the second grating is an amplitude grating (for example, a chrome on glass grating). A scalar Fresnel approach is used for the propagation calculations since the period of the gratings is much larger than the wavelength of the light used. Also, owing to the rough surface, statistical techniques need to be used to determine the average intensity distribution at the observation plane. The case where the first grating is an amplitude grating and the second grating is a steel tape grating can be easily derived from this work.

## 2. Theoretical approach

The general configuration for a double grating system is shown 82 in Fig. 1a, where the first grating is a steel tape grating and the sec-83 ond one is an amplitude grating (chrome on glass grating). Let us 84 consider a monochromatic light source with wavelength  $\lambda$  and lat-85 eral size S. The periods of the gratings,  $p_1$  and  $p_2$ , respectively, are 86 assumed much larger than the wavelength and then a scalar ap-87

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**Fig. 1.** (a) Standard set-up for a double grating system showing the parameters involved. (b) Set-up when a steel tape grating is used. Since the first grating is opaque, a beam splitter is required for the illumination.

88 proach is acceptable. As shown in Fig. 1, the distances between the 89 source, gratings, and observation plane are, respectively, denoted by  $z_0$ ,  $z_1$  and  $z_2$ . Since the system is symmetrical along the y-axis, 90 91 a 2-D analysis can be performed. The steel tape grating is opaque 92 and then the configuration depicted in Fig. 1a is only valid for the-93 oretical purposes. On the other hand, a practical set-up is shown in 94 Fig. 1b where a beam splitter is used to illuminate the grating. This 95 set-up can be used to analyze all the double grating configurations. 96 The light source is made up of point-like emitters which incoher-97 ently generate divergent spherical waves. By the moment, let us 98 consider one of these point emitters placed at a distance  $x_0$  from 99 the axis. The amplitude just before the first grating is given by the Fresnel propagation of a single point source 100

$$U_1(x_1, z_0) = \frac{A_0}{\sqrt{i\lambda z_0}} \exp\left[\frac{ik}{2z_0}(x_1 - x_0)^2\right],$$
(1)

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103  $A_0$  being the amplitude,  $k = 2\pi/\lambda$ , and  $x_1$  the transversal coordinate at the first grating plane. Light is reflected by the first grating (steel 104 105 tape grating). As it is shown in [17], the steel tape grating presents 106 two roughness levels. The high roughness level scatters light in all 107 directions and its contribution to the diffraction pattern is a con-108 stant background intensity. Then, the reflectance of the steel tape 109 grating can be mathematically described as the product of two 110 terms  $T(x_1) = g_1(x_1)t(x_1)$ . The first term is a binary amplitude grating 111 whose infinite Fourier series is  $g_1(x_1) = \sum_n a_n \exp(inq_1x_1)$ , where 112  $q_1 = 2\pi/p_1$ , and  $a_n$  is the *n*th coefficient of the grating with *n* integer.

The second term,  $t(x_1)$ , includes the topography of the rough surface. It is defined using a stochastic function,  $\zeta(x_1)$ , whose average height is zero  $\langle \zeta(x_1) \rangle = 0$ . Taking the thin element approach, the reflectance due to roughness is given by  $t(x_1) = \exp[-2ik\zeta(x_1)]$ , [19,20]. Then, the amplitude of the light field after the grating results

$$\hat{J}(x_1, z_0) = \frac{A_0}{\sqrt{i\lambda z_0}} \exp\left[\frac{ik}{2z_0}(x_1 - x_0)^2\right] t(x_1) \sum_n a_n \exp(inq_1 x_1).$$
 (2) 120

The next step is to propagate the field up to the second grating,121placed at a distance  $z_1$  from the first grating122

$$U(x_{2}, z_{1}) = \frac{A_{0}}{i\lambda\sqrt{z_{0}z_{1}}} \int_{-\infty}^{\infty} \sum_{n} a_{n} \exp(inq_{1}x_{1}) \exp\left[\frac{ik}{2z_{0}}(x_{1} - x_{0})^{2}\right] t(x_{1})$$
$$\times \exp\left[\frac{ik}{2z_{1}}(x_{2} - x_{1})^{2}\right] dx_{1}.$$
(3) 124

The second grating is a binary amplitude grating with period  $p_2$  125 which is also described by its Fourier Series expansion 126  $g_2(x_2) = \sum_m b_m \exp(imq_2x_2)$ , where  $b_m$  is the *m*th coefficient of the 127 grating with *m* integer,  $q_2 = 2\pi/p_2$ , and  $x_2$  is the transversal coordinate at the second grating plane. Thus, the amplitude of the light 129 field after the second grating is given by 130

$$\begin{aligned} (x_{2}, z_{1}) &= g_{2}(x_{2})U(x_{2}, z_{1}) \\ &= \frac{A_{0}}{i\lambda\sqrt{z_{0}z_{1}}} \int_{-\infty}^{\infty} \sum_{n} a_{n} \exp(inq_{1}x_{1}) \sum_{m} b_{m} \exp(imq_{2}x_{2}) \\ &\times \exp\left[\frac{ik}{2z_{0}}(x_{1} - x_{0})^{2}\right] t(x_{1}) \\ &\times \exp\left[\frac{ik}{2z_{1}}(x_{2} - x_{1})^{2}\right] dx_{1}. \end{aligned}$$
(4)

Finally, light propagates along a distance  $z_2$  from the second grating 133 up to the location of the photodetector or the observation plane and 134 the amplitude is 135

$$U(x_{3}, z_{2}) = \frac{A_{0}}{(i\lambda)^{3/2} \sqrt{z_{0} z_{1} z_{2}}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sum_{n} a_{n} \exp(inq_{1} x_{1})$$

$$\times \sum_{m} b_{m} \exp(imq_{2} x_{2}) \exp\left[\frac{ik}{2z_{0}} (x_{1} - x_{0})^{2}\right] t(x_{1})$$

$$\times \exp\left[\frac{ik}{2z_{1}} (x_{2} - x_{1})^{2}\right] \exp\left[\frac{ik}{2z_{2}} (x_{3} - x_{2})^{2}\right] dx_{1} dx_{2}.$$
(5) 137

An exact equation for the field at the observation plane cannot 138 be determined since the topography  $\zeta(x_1)$  is stochastic and thus the 139 reflectance coefficient  $t(x_1)$  is unknown. Nevertheless, the average 140 intensity at the observation plane  $(x_3, z_2)$  can be calculated from the 141 amplitude using an averaging process,  $\langle I(x_3, z_2) \rangle = \langle U(x_3, z_3) \rangle$ 142  $z_2 U^*(x_3, z_2)$ , where  $\langle \bullet \rangle$  represents the average over a hypothetical 143 ensemble of rough surfaces. We will assume that roughness is sta-144 tionary and, therefore, the amplitude correlation of the speckle 145 field is stationary too. The only stochastic factor in the intensity 146 equation is  $\langle t(x_1)t^*(x'_1)\rangle$ , which is known as the autocorrelation 147 function of the surface [19]. In many theoretical and experimental 148 works on roughness a Gaussian function is used to represent the 149 autocorrelation function  $\langle t(x_1)t^*(x'_1)\rangle = \exp[-(x_1 - x'_1)^2/T_0^2], T_0$ 150 being the correlation length of the field. The correlation length of 151 the field is related to the roughness parameters according to 152  $T_0 = \lambda T / 4\pi\sigma$ , where T is the correlation length of the roughness 153 and  $\sigma$  is the standard deviation in heights [21]. After a straightfor-154 ward calculation, the average intensity at the observation plane 155 results 156

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 $A_{0}^{2}$ 

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$$\langle I(x_{3}, z_{2}) \rangle = \frac{1}{\lambda Z_{T}} \sum_{n, n', m, m'=-\infty} a_{n} a_{n'}^{*} b_{m} b_{m'}$$

$$\times \exp \left\{ - \left[ z_{0} \frac{(n-n')q_{1}z_{12} + (m-m')q_{2}z_{2}}{kT_{0}z_{T}} \right]^{2} \right\}$$

$$\times \exp \left\{ -i \left[ (n^{2} - n'^{2}) \frac{q_{1}^{2}}{2k} \frac{z_{0}z_{12}}{z_{T}} + (m^{2} - m'^{2}) \frac{q_{2}^{2}}{2k} \frac{z_{2}z_{01}}{z_{T}} \right] \right\}$$

$$\times \exp \left\{ -i \left[ (n'-n)q_{1} \frac{(x_{3}z_{0} + x_{0}z_{12})}{z_{T}} + (m'-m)q_{2} \frac{(x_{3}z_{01} + x_{0}z_{2})}{z_{T}} \right] \right\}$$

$$\times \exp \left\{ -i \left[ (mn - m'n') \frac{q_{1}q_{2}}{k} \frac{z_{0}z_{2}}{z_{T}} \right] \right\}, \qquad (6)$$

160 being  $z_T = z_0 + z_1 + z_2$ ,  $z_{12} = z_1 + z_2$ , and  $z_{01} = z_0 + z_1$ . The effect of roughness appears in the first Gaussian term of (6) and produces 161 162 a decreasing of the average intensity at the observation plane. This reduction depends on the correlation length of the field  $T_0$ , the dis-163 164 tances involved, the periods of the gratings, and the wavelength of the incident beam. From this general approach, some important 165 166 cases can be analyzed, such as Moiré, Lau or Generalized Grating 167 Imaging configurations.

## 168 2.1. Moiré configuration

169 Moiré effect can be derived from Eq. (6) considering that both 170 gratings have the same period  $p_1 = p_2 = p$  and placing the light 171 source at infinite,  $z_0 \rightarrow \infty$ . Then, the mean intensity for Moiré con-172 figuration results

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$$\langle I(x_{3}) \rangle \propto \sum_{n,n',m,m'=-\infty}^{\infty} a_{n} a_{n'}^{*} b_{m} b_{m'}^{*} \times \exp\left\{-\left[q \frac{(n-n')z_{12} + (m-m')z_{2}}{kT_{0}}\right]^{2}\right\} \times \exp\left\{-i \frac{q^{2}}{2k} \left[(n^{2} - n'^{2})z_{12} + (m^{2} - m'^{2})z_{2}\right]\right\} \times \exp\left\{iqx_{3}[(n-n') + (m-m')]\right\} \times \exp\left[-i \frac{q^{2}}{k}(mn - m'n')z_{2}\right].$$
(7)

176 Classically, Moiré effect is analyzed when the observation plane 177 coincides with the second grating plane,  $z_2 \rightarrow 0$ . Thus, Eq. (7) simpli-178 fies to

$$\langle I(\mathbf{x}_{3}) \rangle \propto \sum_{n,n',m,m'=-\infty}^{\infty} a_{n} a_{n'}^{*} b_{m} b_{m'}^{*}$$

$$\times \exp\left\{-\left[q \left(\frac{(n-n')z_{1}}{kT_{0}}\right)^{2}\right\} \right\}$$

$$\times \exp\left\{-i \frac{q^{2}}{2k}\left[(n^{2}-n'^{2})z_{1}\right]\right\}$$

$$\times \exp\left\{iqx_{3}\left[(n-n')+(m-m')\right]\right\}.$$

$$(8)$$

181 In Moiré configuration, the presence of roughness produces a 182 Gaussian decreasing of the intensity in terms of  $z_1$ , which is shown 183 in Fig. 2. The relative displacement  $\Delta x$  between gratings can be in-184 cluded in the equations using the following change:  $a_n \rightarrow$ 185  $a_n \exp(iqn\Delta x)$ . Then the average intensity distribution results



**Fig. 2.** Fringes obtained with a Moiré configuration when the wavelength is  $\lambda = 0.68 \ \mu\text{m}$  and the period of both gratings is  $p = 20 \ \mu\text{m}$ . (a) The first grating is a chrome on glass grating. (b) The first grating is a steel tape grating whose roughness parameters are  $\sigma = 0.1 \ \mu\text{m}$ ,  $T = 50 \ \mu\text{m}$ .

$$\langle I(x_3) \rangle_M \propto \sum_{n,n',m,m'=-\infty}^{\infty} a_n a_{n'}^* b_m b_{m'}^* \exp\left[iq(n-n')\Delta x\right] \\ \times \exp\left\{-\left[q\frac{(n-n')z_1}{kT_0}\right]^2\right\} \exp\left\{-i\frac{q^2}{2k}[n^2-n'^2)z_1\right]\right\} \\ \times \exp\left\{iqx_3[(n-n')+(m-m')]\right\}.$$
(9) 187

When  $z_1 \gg kT_0/q$ , the only significant terms are those that fulfill n = n'. Then the information about the relative displacement between gratings disappears and the second grating receives a constant field. On the other hand, when roughness is zero,  $T_0 \rightarrow \infty$ , the classical expression of Moiré effect is recovered.

### 2.2. Lau configuration

Lau effect can also be obtained from Eq. (6) considering that the size of the source is infinite,  $S \to \infty$ , assuming  $p_1 = p_2 = p$ , and placing the observation plane at infinite,  $z_2 \to \infty$ . We will consider first a source of finite size *S* and perform an integration in  $x_0$  with the following limits:  $-S/2 < x_0 < S/2$ . Then the average intensity is

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$$\langle I(\theta) \rangle \propto \sum_{n,n',m,m'=-\infty}^{\infty} a_n a_{n'}^* b_m b_{m'}^* \\ \times \exp\left\{ -\left[ \frac{z_0 q}{kT_0} (n - n' + m - m') \right]^2 \right\} \exp\left[ -i \frac{q^2}{2k} (n^2 - n'^2) z_0 \right] \\ \times \exp\left[ -i \frac{q^2}{2k} (m^2 - m'^2) z_{01} \right] \exp\left\{ i q \theta [(n - n') z_0 + (m - m') z_{01}] \right\} \\ \times \exp\left[ -i \frac{q^2}{k} (mn + m'n') z_0 \right] \sin c \left[ \frac{qS}{2} (n - n' + m - m') \right],$$
(10)

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where we have used  $\theta = x_3/z_2$  and  $\operatorname{sinc}(x) = \operatorname{sin}(x)/x$ . For an infinite source we need to consider the limit  $S \to \infty$ , and then the sinc function tends to a Kronecker delta function  $\delta(n - n' + m - m')$ . Thus, the average intensity simplifies to

$$\langle I(\theta) \rangle_L \propto \sum_{n,n',m'=-\infty}^{\infty} a_n a_{n'}^* b_{(-n+n'+m')} b_{m'}^*$$

$$\times \exp\left[-\mathrm{i}\frac{q^2}{2k}(n-n')^2 z_1\right] \exp\left[-\mathrm{i}\frac{q^2}{k}(n'-n)m' z_1\right]$$

$$\times \exp\left[-\mathrm{i}q\theta(n-n')z_1\right].$$

$$(11)$$

Roughness dependence disappears from the equation and the expression obtained corresponds to the classical expression for the Lau effect [22].

## 211 2.3. Generalized grating imaging

212 Generalized grating imaging configuration has become very 213 common because, for it, lenses are not required to obtain fringes. 214 Due to this, this kind of devices is more compact and robust. The 215 period of the gratings can be equal or different and the light source has finite size, S. Fringes are formed at finite distances from the 216 second grating. The expression for generalized grating imaging 217 218 when a finite source is considered can be determined from Eq. 219 (6). The light source can be considered as a sum of incoherent point 220 sources. Then the average intensity can be obtained as an integra-221 tion of Eq. (6) between  $-S/2 < x_0 < S/2$ , resulting in 222

$$\langle I(x_3, z_2) \rangle \propto \sum_{n,n',m,m'=-\infty}^{\infty} a_n a_{n'}^* b_m b_{m'}^* \\ \times \exp\left\{-\left\{\frac{z_0}{kT_0 z_T}[(n-n')q_1 z_{12} + (m-m')q_2 z_2]\right\}^2\right) \\ \times \exp\left\{-i\frac{1}{2kz_T}[(n^2 - n'^2)q_1^2 z_0 z_{12} + (m^2 - m'^2)q_2^2 z_2 z_{01}]\right\} \\ \times \exp\left\{-\frac{ix_3}{z_T}[(n'-n)q_1 z_0 + (m'-m)q_2 z_{01}]\right\} \\ \times \exp\left\{-i\left[(mn - m'n')\frac{q_1 q_2}{k}\frac{z_0 z_2}{z_T}\right]\right\} \\ \times \sin c\left\{\frac{S}{2z_T}[(n-n')q_1 z_{12} + (m-m')q_2 z_2]\right\}.$$
(12)

The intensity distribution depends on the correlation length of 225 the field  $T_0$ , as it is shown in Fig. 3. When the distance between 226 227 the light source and the first grating  $z_0$  is zero, the exponential 228 term associated to roughness disappears and (12) becomes the 229 standard equation for the generalized grating imaging phenome-230 non with finite source [14]. When the light source is infinite 231  $(S \rightarrow \infty)$ , the sinc function tends to a Kronecker delta function 232  $\delta[(n - n')q_1z_{12} + (m - m')q_2z_2]$ . In this case, the roughness effect 233 also disappears for any distance  $z_0$  and the mean intensity is given 234 by



**Fig. 3.** Self-images obtained in generalized grating imaging (a) without considering roughness, (b) considering roughness,  $\sigma = 0.5 \mu m$ ,  $T = 10 \mu m$ . In both cases the wavelength is  $\lambda = 0.68 \mu m$ , the source size is  $S = 300 \mu m$ , the period of the gratings is  $p_1 = p_2 = 20 \mu m$ , and  $z_0 = 5 mm$ .

$$I(x_{3})\rangle \propto \sum_{n,n',m,m'=-\infty}^{\infty} a_{n}a_{n'}^{*}b_{m}b_{m'}^{*}\exp ix_{3}[(n'-n)q_{1} + (m'-m)q_{2}]\exp\left[-i(m+m')(n-n')\frac{q_{1}q_{2}}{2k}z_{1}\right] \times \delta[(n-n')q_{1}z_{12} + (m-m')q_{2}z_{2}],$$
(13) 236

which is equivalent to that obtained by Swanson and Leith [12]. 237 However, when roughness is present, the width of the pseudoimages decreases, thus reducing the tolerances of optical devices 239

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images decreases, thus reducing the tolerances of optical devices based in generalized grating imaging systems. To analyze this effect, it is interesting to perform the following change of variables in (12): N = n - n', M = m - m', u = n - N/2, and v = m - M/2. As a result, the mean intensity is

$$\langle I(x_3, z_2) \rangle \propto \sum_{N,M=-\infty}^{\infty} \exp\left\{-\frac{ix_3}{z_T} [Nq_1 z_0 + Mq_2 z_{01}]\right\} \times \sin c \left\{\frac{S}{2z_T} [Nq_1 z_{12} + Mq_2 z_2]\right\} \times \exp\left(-\left\{\frac{z_0}{kT_0 z_T} [Nq_1 z_{12} + Mq_2 z_2]\right\}^2\right) \sum_{u,v=-\infty}^{\infty} a_n a_{n'}^* \times \exp 2\pi i [u(N\gamma_{11} + M\gamma_{12}) + v(M\gamma_{22} + N\gamma_{12})],$$
(14) 246

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247 where  $\gamma_{11} = z_0 z_{12}/z_{\lambda} z_T$ ,  $\gamma_{22} = z_2 z_{01} R^2/z_{\lambda} z_T$ ,  $\gamma_{12} = z_0 z_2 R/z_{\lambda} z_T$ , R = p1/p2, and  $z_{\lambda} = p_1^2/\lambda$ . A given pseudoimage (*N*,*M*) presents a maximum value when the argument of the sinc function in Eq. (14) is zero, resulting

$$z_2 = \frac{1}{RQ - 1} z_1, \tag{15}$$

where Q = -M/N. At the exact locations of the pseudoimages, the sinc term and the Gaussian term are unity and the intensity does not depend on the roughness parameters.

For the usual distances (millimeters-centimeters), pseudoimages are quite narrow and they do not overlap (pseudoimage isola-



**Fig. 4.** (a) Width of the pseudoimage (1,-2) at  $z_2 = 2.4$  mm, defined as Eq. (16), for different values of  $\sigma$ : 0.05 µm, 0.1 µm, 0.25 µm, 0.5 µm, 0.75 µm, and 1 µm. The wavelength is  $\lambda = 0.68$  µm, the source size is S = 300 µm, the period of the gratings is  $p_1 = p_2 = 20$  µm, and  $z_1 = 5$ mm. (b) Profile of the pseudoimage (1,-2) for different values of the correlation length  $T_0$  for the same conditions of (a) when  $T_0 = 5$  µm (dashed),  $T_0 = 10$  µm (solid), and  $T_0 = 50$  µm (dashed-dot).

tion, regime 3 of Ref. [23]). The sinc term and the Gaussian term control the width of the pseudoimage, being both terms competitive. We will define the width of a given pseudoimage as

$$\omega_{N,M}^{2} = \frac{\int (z_{1} - \bar{z})^{2} \langle \operatorname{Amp}(z_{2}) \rangle dz_{1}}{\int \langle \operatorname{Amp}(z_{2}) \rangle dz_{1}},$$
(16)

where  $\langle Amp(z_2) \rangle = \max \langle I_{N,M}(x_3, z_2) \rangle - \min \langle I_{N,M}(x_3, z_2) \rangle$  and  $\overline{z} = \int z_1 \langle Amp(z_2) \rangle dz_1 / \int \langle Amp(z_2) \rangle z_1$ . The width of a given pseudoimage (*N*,*M*) is dependent on the correlation length of roughness *T*<sub>0</sub>, as it is shown in Fig. 4.

For low values of  $T_0$ , the Gaussian term controls the width of the pseudoimage which increases linearly as

$$\omega_{N,M} \approx \frac{kT_0 z_T}{\sqrt{2} z_0 N q_1} \tag{17}$$

However, when roughness is very low, the width of the pseudoimage is controlled by the sinc function, resulting in

$$\omega_{N,M} \approx \frac{2.6 z_{\rm T}}{5 N q_1} \tag{18}$$

This effect can be observed in Fig. 4a. For low values of  $T_0$ , the width of the pseudoimage presents a linear dependence with  $T_0$ . On the other hand, when  $T_0$  is large, then the width is constant. We can also see in Fig. 4b that, depending on the value of  $T_0$ , the shape of the pseudoimage varies from a Gaussian shape for low values of  $T_0$  up to a maximum with several lobes when roughness is null.

## 3. Conclusions

## In this work, we have performed an analysis of the behavior of a double grating system with a steel tape grating. Moiré configuration has been shown to strongly depend on the roughness of the grating and the self-imaging process eventually disappears as the distance between the two diffraction gratings increases. On the contrary, for the Lau and Generalized grating imaging configurations, the roughness of the steel tape grating does not affect the self-imaging process, although it affects the depth of focus of the self-images.

## Acknowledgements

The authors thank Agustin Gonzalez-Cano for his invaluable help. This work was supported by the DPI2005-02860 project of the Ministerio de Educación y Ciencia of Spain and the "Tecnologías avanzadas para los equipos y procesos de fabricación de 2015: e-eficiente, e-cológica, e-máquina" CENIT project of the Ministerio de Industria, Turismo y Comercio. During the realization of this work Sanchez-Brea was contracted by the Universidad Complutense de Madrid under the "Ramón y Cajal" research program of the Ministerio de Educación y Ciencia of Spain.

## References

- [1] G.W.R. Leibbrandt, G. Harbers, P.J. Kunst, App. Opt. 35 (1996) 6151.
- [2] Y. Xu, O. Sasaki, T. Suzuki, Opt. Lett. 28 (2003) 1751.
- [3] Y. Xu, O. Sasaki, T. Suzuki, App. Opt. 43 (2004).
- [4] I.B. Gornushkin, N. Omenetto, B.W. Smith, J.D. Winefordner, App. Spectrosc. 58 (2004).
- [5] S. Grabarnik, R. Wolffenbuttel, A. Emadi, M. Loktev, E. Sokolova, G. Vdovin, Opt. Express 15 (2007) 3581.
- [6] K. Patorski, The self-imaging phenomenon and its applications, in: E. Wolf (Ed.), Progress in Optics 27, North Holland, Amsterdam, 1989.
- [7] A.W. Lohmann, D.E. Silva, Opt. Commun. 2 (1971) 413.
- [8] K. Patorski, Moirè Metrology, Pergamon, New York, 1998.
- [9] E. Lau, Ann. Phys. 6 (1948) 417.
- [10] D. Crespo, J. Alonso, T. Morlanes, E. Bernabéu, Opt. Eng. 39 (2000) 817.
- [11] J. Jahns, W. Lohmann, Opt. Commun. 28 (1979) 263.

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- F.J. Torcal-Milla et al./Optics Communications xxx (2008) xxx-xxx
- 318 [12] G.J. Swanson, E.N. Leith, J. Opt. Soc. Am. A 2 (1985) 789. 319

2 September 2008 Disk Used

- [13] S.C. Som, A. Satpathi, J. Mod. Opt. 37 (1990) 1215.
  [14] D. Crespo, J. Alonso, E. Bernabeu, J. Opt. Soc. Am. A 17 (2000) 1231. 320
- 321
- [15] D. Crespo, J. Alonso, E. Bernabeu, Appl. Opt. 41 (7) (2002) 1223. 322
- [16] Fagor Automation S. Coop. (www.fagorautomation.com), Heidenhain 323 Coorporation (www.heidenhain.com), Renishaw Group (www.renishaw.com).
- 324 [17] F.J. Torcal-Milla, L.M. Sanchez-Brea, E. Bernabeu, Appl. Opt. 46 (2007) 3668.
- 325 [18] L.M. Sanchez-Brea, F.J. Torcal-Milla, E. Bernabeu, J. Opt. Soc. Am. A 25 (2008) 326 828.
- [19] P. Beckmann, The Scattering of Electromagnetic Waves from Rough Surfaces, Artech House INC, 1987.
- [20] J.A. Ogilvy, Theory of Wave Scattering from Random Rough Surfaces, IOP, Bristol, 1991.
- [21] F. Perez-Quintián, A. Lutenberg, M.A. Rebollo, Appl. Opt. 45 (2006) 4821.
- [22] G.J. Swanson, E.N. Leith, J. Opt. Soc. Am. A 72 (1972) 552. [23] L. Garcia-Rodriguez, J. Alonso, E. Bernabeu, Opt. Express 12 (2004) 2529.