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Spin Hall conductivity of a disordered two-dimensional electron gas with Dresselhaus spin-orbit interaction

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The spin Hall conductivity of a disordered two-dimensional electron gas has been investigated for a general spin-orbit interaction. We have found that in the diffusive regime of electron transport, the dc spin-Hall conductivity of a homogeneous system is zero due to impurity scattering when the spin-orbit coupling contains only the Rashba interaction, in agreement with existing results. However, when the Dresselhaus interaction is taken into account, the spin-Hall current is not zero. We also considered the spin-Hall currents induced by an inhomogeneous electric field. It is shown that a time-dependent electric charge induces a vortex of spin-Hall currents.

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Spintronics is a fast developing area using the electron spin degrees of freedom in electronic devices.[1–4] One of the most challenging goals of spintronics is to find a method to manipulate spins by electric fields. The spin-orbit interaction (SOI), which couples the electron momentum and spin, can serve as a spin-charge mediator. There have been several suggestions to use the SOI in semiconductor quantum wells (QW) to create the electron and hole spin currents and to accumulate the spin polarization by applying an electric field parallel[5–8] or perpendicular[9,10] to the QW. The spin current induced by the parallel electric field and flowing perpendicular to it has been named the spin-Hall effect (see also Ref. 11). Since the prediction of this effect by Murakami *et al.*[5] and Sinova *et al.*,[6] there have been much discussions concerning the effect of nonmagnetic impurity scattering on the spin-Hall conductivity in systems with Rashba spin-orbit coupling. Some groups predicted that the impurity scattering should suppress the spin-Hall effect induced by a homogeneous and static electric field,[12–15] even if the mean scattering time τ is much longer than $1/\Delta$, where Δ is the spin-orbit splitting of the electron energy (we set $\hbar=1$). This result was confirmed by an analysis of the sum rules in Ref. 16. Yet some other groups came to different conclusions.[17–19]

In the present paper we use the diffusion approximation to derive an expression of the spin Hall conductivity for a general SOI, including both Rashba and Dresselhaus terms. For pure Rashba SOI, as well as for linear Dresselhaus interaction, we found that the dc spin-Hall conductivity of the homogeneous system becomes zero even for a weak disorder scattering, confirming thus the results of Refs. 12–16. On the other hand, when the cubic terms of Dresselhaus SOI is included, a finite spin current is produced. In order to study the effect of a spatially inhomogeneous electric field, our analysis keeps finite frequency Ω and wave number $\boldsymbol{Q}$ of the electric field. We found that for $\Omega \ll DQ^2$, where D is the electron diffusion constant, the flow of the spin-Hall currents is dominated by the screening effects. Similar to formation of an electron screening cloud around an external charge, the spin Hall currents form a vortex.

We consider a typical III-V semiconductor QW with only the lowest subband occupied. The spin-orbit coupling of conduction electrons has the form

$$H_{so} = \boldsymbol{h_k} \cdot \boldsymbol{\sigma}, \tag{1}$$

where $\boldsymbol{\sigma} \equiv (\sigma^x, \sigma^y, \sigma^z)$ is the Pauli matrix vector, and $\boldsymbol{h}_k$ a function of the two-dimensional wave vector $\boldsymbol{k}$. In general, $\boldsymbol{h_k}$ contains both the Dresselhaus and the Rashba terms. The former exists also in bulk crystals,[20] while the latter appears only in asymmetric QWs.[21] For a QW grown along the [001] direction, which is set as the z axis, the Dresselhaus SOI is given by[22]

$$h^x_{\boldsymbol{k}} = \beta k_x (k_y^2 - a^2),$$

$$h^y_{\boldsymbol{k}} = -\beta k_y (k_x^2 - a^2), \tag{2}$$

where the parameter a^2 is the average of the operator $-(\partial/\partial z)^2$ with respect to the lowest subband wave function. The Dresselhaus SOI in Eq. (2) contains terms both linear and cubic in $\boldsymbol{k}$. Usually, in heavily doped QWs, for electrons at the Fermi energy both terms are of the same order of magnitude.[23] The Rashba interaction has the form[21]

$$h^x_{\boldsymbol{k}} = \alpha k_y, \; h^y_{\boldsymbol{k}} = -\alpha k_x. \tag{3}$$

Let us apply an electric field along the x axis, and express it as the gradient of a scalar electric potential $\boldsymbol{E} = -\boldsymbol{\nabla} V$. This gauge is more convenient for studying the case of finite wave numbers $\boldsymbol{Q}$ in the Fourier expansion of $\boldsymbol{E}$. The one-particle spin-current operator is $J^i_j = (\sigma^i v^j + v^j \sigma^i)/4$, where the particle velocity is

$$v^i = \frac{k^i}{m^*} + \frac{\partial}{\partial k^i}(\boldsymbol{h_k} \cdot \boldsymbol{\sigma}). \tag{4}$$

This definition has to be used with caution, since the spin current is not conserving in systems with SOI, as discussed in Ref. 24. We are interested in calculating the spin current polarized in the z direction and flowing in y direction. Since $\boldsymbol{h_k}$ in Eqs. (3) and (2) has no z components the spin-current operator is $J^z_y = \sigma^z k^y/(2m^*)$. We will calculate the corresponding spin Hall current within the standard linear-response theory[25] and denote it as J. So, the initial expression for J is

$$J = -ie\Omega \sum_{\boldsymbol{k},\boldsymbol{k}'} \int \frac{d\omega}{2\pi}\frac{\partial n_F(\omega)}{\partial \omega}\langle Tr[G^a(\boldsymbol{k}_-,\boldsymbol{k}'_-,\omega) \times J^z_y G^r(\boldsymbol{k}'_+,\boldsymbol{k}_+,\omega+\Omega)]\rangle V(\Omega,\boldsymbol{Q}), \tag{5}$$

where $\boldsymbol{k}_\pm \boldsymbol{k} \pm \boldsymbol{Q}/2$, and $n_F(\omega)$ is the Fermi distribution function. In Eq. (5) the trace runs through the spin variables, and the angular brackets denote the average over the random distribution of impurities. The terms containing the products of the form G^aG^a and G^rG^r are neglected since their contribution to the spin Hall current is small.[15] For simplicity we assume that in the vicinity of the Fermi energy E_F, the amplitude of impurity elastic scattering is isotropic and momentum independent. In the quasiclassical approximation, when $E_F\tau \gg 1$, the average of the product of the retarded and advanced Green functions G^r and G^a can be calculated perturbatively. If we ignore weak localization effects, the perturbation expansion of Eq. (5) consists of the so-called ladder diagrams.[25,26] For small Ω and Q these diagrams describe the particle and spin diffusion processes. The spin diffusion also includes the D'yakonov-Perel spin relaxation.[27] Therefore, the spin-Hall current (5) is determined by the combination of spin and particle diffusion propagators.

To calculate and to combine these propagators for arbitrary $\boldsymbol{h}_{\boldsymbol{k}}$, we will follow the formalism of Refs. 28 and 29. In Eq. (5) the spin-current vertex J^z_y is coupled to the spin-independent potential V. Such a spin-charge coupling has two channels. In the first channel, J^z_y and V are coupled via the spin-independent particle diffusion propagator. This contribution to the spin-Hall current is denoted as J_1. For $\Omega \ll 1/\tau$ and $v_F Q \ll 1/\tau$, where v_F is the Fermi velocity, from Eq. (5) we obtain

$$J_1 = i\frac{e\Omega}{2\pi}\Psi D(\Omega,\boldsymbol{Q})V(\Omega,\boldsymbol{Q}), \tag{6}$$

where $D(\Omega,Q)=[\tau(-i\Omega+DQ^2)]^{-1}$ is the particle diffusion propagator.[26] The vertex Ψ is

$$\Psi = \sum_k \mathrm{Tr}[G^r(\boldsymbol{k}_+,E_F+\Omega)G^a(\boldsymbol{k}_-,E_F)J^z_y], \tag{7}$$

where $G^{r,a}(\boldsymbol{k},E)$ are the Green functions averaged over random impurity positions.

The second coupling channel is more complicated. The spin current couples first to the spin diffusion-relaxation propagator, which couples to V via the mixing of charge and spin diffusion processes. The mixing of these diffusion processes was pointed out explicitly by Burkov *et al.*[17] The spin Hall current due to this channel is denoted as J_2, and is obtained as

$$J_2 = i\frac{e\Omega}{2\pi}\Psi^l D^{lj}(\Omega,\boldsymbol{Q})M^j D(\Omega,Q)V(\Omega,\boldsymbol{Q}), \tag{8}$$

with the vertices

$$\Psi^l = \sum_k \mathrm{Tr}[G^r(\boldsymbol{k}_+,E_F+\Omega)\sigma^l G^a(\boldsymbol{k}_-,E_F)J^z_y]. \tag{9}$$

In Eq. (8) the superscripts l and j are summed over x, y, and z. The spin diffusion-relaxation propagator $D^{ij}(\Omega,\boldsymbol{Q})$ describes diffusion and relaxation of a spin density packet. Therefore, this propagator satisfies the spin diffusion equation for spins polarized in the j direction when a source creates spins polarized in the i direction. M^j is the spin-charge mixing, defined as

$$M^j = \frac{1}{4\pi\tau N_0}\sum_k \mathrm{Tr}[G^r(\boldsymbol{k}_+,E_F+\Omega)G^a(\boldsymbol{k}_-,E_F)\sigma^j], \tag{10}$$

where $N_0=m^*/(2\pi)$ is the two-dimensional (2D) density of states. M^j makes the diffusion of spins polarized in the j-direction dependent on the charge density distribution.[17,13] This spin-charge coupling is weak and is proportional to the small parameter h_k/E_F. Therefore, in Eq. (8) we keep only the terms linear in M^j. It should be noticed[8] that J_2 is closely related to the electric field induced accumulation of the in-plane polarized spin density S^l. For example, it can be shown that $J_2=\Psi^l S^l/2\pi\tau N_0$.

After averaging over the impurity positions, the retarded and advanced Green functions are obtained as

$$G^r(\boldsymbol{k},E) = [G^a(\boldsymbol{k},E)]^\dagger = (E - E_{\boldsymbol{k}} - \boldsymbol{h}_{\boldsymbol{k}}\cdot\boldsymbol{\sigma} + i\Gamma)^{-1}, \tag{11}$$

where $\Gamma=1/(2\tau)$ and $E_{\boldsymbol{k}}=k^2/(2m^*)$. For the case of short-range impurities and the constant density of states near E_F, the scattering rate Γ is independent of momentum.[25] Using Eq. (11) , for small Ω and $\boldsymbol{Q}$, one gets from Eqs. (7), (9), and (10)

$$\Psi = \frac{i\pi N_0}{2\Gamma}\epsilon^{lmz}Q^n\overline{(\nabla^n_{\boldsymbol{k}}h^l_{\boldsymbol{k}})h^m_{\boldsymbol{k}}v^y Z_{\boldsymbol{k}}},$$

$$\Psi^l = -\pi N_0\epsilon^{lmz}\overline{v^y h^m_{\boldsymbol{k}}Z_{\boldsymbol{k}}},$$

$$M^j = \frac{i}{2\Gamma}Q^m\overline{(\nabla^m_{\boldsymbol{k}}n^j_{\boldsymbol{k}})h^3_{\boldsymbol{k}}Z_{\boldsymbol{k}}}, \tag{12}$$

where $Z_{\boldsymbol{k}}=(\Gamma^2+h^2_{\boldsymbol{k}})^{-1}$ and $\boldsymbol{n}_{\boldsymbol{k}} \equiv \boldsymbol{h}_{\boldsymbol{k}}/h_{\boldsymbol{k}}$. The overbar in (12) denotes the average over directions of $\boldsymbol{k}$ which has the magnitude $k=k_F$. In Eq. (12) ϵ^{lmz} is the antisymmetric tensor with $\epsilon^{xyz}=1$, and all doubly repeated superscripts should be summed over x, y, and z.

$D^{ij}(\Omega,\boldsymbol{Q})$ satisfies the spin diffusion equation.[13,30] For $Qv_F \ll h_{k_F}$ we can neglect in this equation the diffusion and spin precession terms which are proportional to the gradient of the spin propagator. We then have

$$-i\Omega D^{mj}(\Omega,\boldsymbol{Q}) = 2\Gamma\delta^{mj} - \Gamma^{ml}D^{lj}(\Omega,\boldsymbol{Q}), \tag{13}$$

where Γ^{ml} is the spin relaxation matrix element. At low frequency the relaxation term dominates and so $D^{mj}(\Omega,\boldsymbol{Q})$ is simply given by the inverse of Γ^{ml}, and

$$\Gamma^{ml} = 2\Gamma\overline{[\delta^{ml}h^2_{\boldsymbol{k}} - h^m_{\boldsymbol{k}}h^l_{\boldsymbol{k}}]Z_{\boldsymbol{k}}}. \tag{14}$$

This equation differs by a factor $\Gamma^2 Z_{\boldsymbol{k}}$ from the standard definition of the spin relaxation matrix, for example, in Ref. 28.

This factor is not unity because we consider the situation that the spin splitting $\Delta=2h_{\boldsymbol{k}}$ can be comparable to the electron elastic scattering rate 2Γ.

Let us first consider the case of Rashba SOI (3). We then set $Q^y=0$ and $E=-iQ^xV$ to calculate Ψ, Ψ^y, and M^y from Eq. (12). In this case both the spin relaxation matrix and the spin diffusion-relaxation propagator are diagonal. Substituting the so calculated Ψ, Ψ^y, M^y, and D^{yy} into Eqs. (6) and (8), the currents J_1 and J_2 are obtained as

$$J_1=-J_2=E\frac{e}{8\pi}\frac{\Delta^2}{4\Gamma^2+\Delta^2}\frac{\Omega}{\Omega+iDQ^2}, \tag{15}$$

where $\Delta=2\alpha k_F$. Hence, the total current J_1+J_2 vanishes even for small impurity scattering rate $\Gamma \ll \Delta$, in agreement with the existing results.[12–16] We should mention that in deriving this result for $\Omega \ll \Gamma^{yy}$, in the denominator $(-i\Omega+\Gamma^{yy})$ of the spin diffusion-relaxation propagator the frequency term has been removed. If we retain Ω, J_1, and J_2 will cancel each other not exactly, but the accuracy[13] is up to Ω/Γ^{yy}. As was pointed out by Mishchenko *et al.*,[13] near the sample boundaries J_2 can also differ from J_1 because of the rapid spatial variation of the spin diffusion propagator. We have ignored this effect by neglecting the gradient terms in the diffusion equation (13). If necessary, in our approach we can consider the boundary problem by substituting into Eq. (8) the complete solution $D^{mj}(\Omega,\boldsymbol{Q})$ of the spin diffusion equation.[30] Our main goal is, however, to show that the spin current is not zero in the bulk of the sample when the Dresselhaus SOI is taken into account. In this case the total spin accumulation near the sample edge will be determined by a direct inflow of the spin polarization from the bulk.

Let us assume that the SOI contains only the Dresselhaus interaction (2), which has terms both linear and cubic in $\boldsymbol{k}$. When the cubic interaction is ignored, there is no spin-Hall effect because the linear Dresselhaus SOI can be obtained from the Rashba SOI via a unitary transformation of the spin operators.[16] For the complete Dresselhaus interaction (2), following Eqs. (6), (8), and (12)–(14), the calculation of the spin-Hall current is straightforward. We obtain the total spin current $J=J_1+J_2$ as

$$J=E\sigma_{sH}\frac{\Omega}{\Omega+iDQ^2}, \tag{16}$$

where σ_{sH} is the DC spin-Hall conductivity at $Q\to 0$. The calculated $\sigma_{sH}/(e/16\pi)$ is plotted in Fig. 1 as a function of a/k_F, for three values of $\Gamma^2/\beta^2 k_F^6=10^{-4}$, 10^{-3}, and 10^{-1}. The ratio a/k_F is a measure of relative strength of the linear to cubic terms in Eq. (2). As expected, the σ_{sH} vanishes for large a. It is important to notice the singular behavior at small Γ of σ_{sH} in the vicinity of $a/k_F=1/\sqrt{2}$ and $a/k_F=0$. The singularities appear because at these points the spin-orbit splitting $2h_k$ vanishes for certain $\boldsymbol{k}$ directions. As a result, in such angular integrals $Z_{\boldsymbol{k}}^{-1}\to\infty$ when the elastic scattering rate $\Gamma\to 0$. It is also interesting to notice that in the range $0<a/k_F<1/\sqrt{2}$, as $\Gamma\to 0$ the spin Hall conductivity has a plateau shape with the universal value of $\sigma_{sH}=3e/8\pi$. This plateau and the sharp change of sign at $a/k_F=1/\sqrt{2}$ can be useful in device applications.

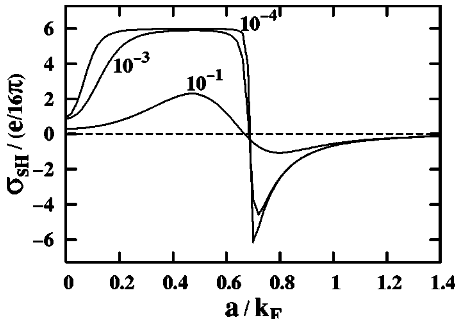

FIG. 1. Spin Hall conductivity as a function of a/k_F for $\Gamma^2/\beta^2 k_F^6=10^{-4}$, 10^{-3}, and 10^{-1}.

We would like to elaborate the nonanalytic behavior of Eq. (16) when both Ω and Q approach zero, a consequence of the diffusion denominator in J. When $Q\to 0$ first, Eq. (16) gives the dc flow of the spin Hall current induced by the spatially homogeneous electric field. At the opposite regime $DQ^2 \gg \Omega$, we neglect the Ω in the denominator and rewrite Eq. (16) in a coordinate independent form as

$$J_l=\frac{i\Omega\sigma_{sH}}{DQ^2}\epsilon^{ljz}E^j. \tag{17}$$

Here J_l is the z-polarized spin current flowing along the l axis. To arrive at Eq. (17) we have assumed that DQ^2 is much less than the spin relaxation rate. Otherwise, the term DQ^2 should be added to Eq. (13).

Equation (17) yields the hydrodynamics of the spin Hall current flow. Since $\boldsymbol{E}=-i\boldsymbol{Q}V$ is a longitudinal field, we have

$$\nabla\cdot\boldsymbol{J}=0, \qquad (\nabla\times\boldsymbol{J})_z=\frac{\sigma_{sH}}{D}\frac{\partial V}{\partial t}. \tag{18}$$

The first equation indicates that the spin current is conserving. The second equation tells us that in each spatial point the flux is perpendicular to the local electric field, similar to the spin Hall effect in a homogeneous field. In the field of spherically symmetric potential a circular vortex flow of the spin current is thus induced around a central charge. The physics of this effect is similar to the screening of scalar potential by electric charges. To clarify this analogy, let us introduce the conjugate current $\tilde{J}_x=J_y$ and $\tilde{J}_y=-J_x$, as well as the vortex "charge" density ρ defined by the continuity equation

$$e\,\nabla\tilde{\boldsymbol{J}}=\frac{\partial\rho}{\partial t}. \tag{19}$$

We can then rewrite the second equation in Eq. (18) as

$$\rho=e\frac{\sigma_{sH}}{D}V, \tag{20}$$

which has the same form as the equation for the electrostatic screening of the scalar potential V, with $e\sigma_{sH}/D$ playing the role of the inverse screening length.

It should be noted that because of the above-mentioned close relationship between the spin Hall effect and the accumulation of in-plane spin polarization, the latter will also appear as a screening cloud around the external charge. The in-plane polarization, in its turn, can give rise to a z-polarized component via the spin precession term of the diffusion equation.[30] This precession is proportional to $v_F Q/\Gamma$, which is small in the diffusion approximation and was neglected in Eq. (18). Consequently, the spin Hall current turns out to be conserved, as one can expect in the absence of the relaxation of z polarization. On the other hand, in the near vicinity of the vortex core, the precession term becomes more important because of the larger gradient of the electric field. Hence, the accumulation of the z polarized spin density will be expected in the region of the core. The detailed analysis of this phenomenon is outside the scope of the present paper. It is worthwhile to notice that the core has a macroscopic size about $\hbar v_F/\Delta$, which is of the order microns. Therefore, the spin accumulation in the vortex core can be observed by, for example, the method of Faraday rotation.[31]

In conclusion, within the quasiclassical perturbation theory we have shown that, in agreement with existing results, impurity scattering reduces the dc spin Hall current to zero if the SOI is due to the Rashba interaction. On the other hand, the spin Hall current remains finite for the Dresselhaus SOI. Nevertheless, this current becomes zero if it is induced by a *spatially* varying dc electric field. The field must be time dependent in order to produce a finite effect. In this case the spin-current flow in the field of a scalar potential has the form of a vortex. The physics of this phenomenon is formally equivalent to the screening of external electric potential by electrons.

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