Guided Beams in Concave Metallic Waveguides

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Abstract—A new waveguiding technique employs a single concave metallic strip as the guiding medium. With appropriate curvature of the strip, the beam modes may be described in terms of Hermite-Gaussian functions parallel to the strip and Airy functions in the perpendicular direction. Such devices would be especially useful in frequency or power regimes where appropriate dielectric waveguides are not available. Experiments have been performed with concave waveguides formed to circular and helical geometries.

I. Introduction

One of the long-standing problems in many laser applications involves the low-loss transmission of a laser beam from its source to a distant detector or target. The atmosphere is often an unsuitable medium, and in recent years, dielectric optical fibers have been gaining in effectiveness and popularity as transmission media. The principal advantages of fibers for transmitting laser beams are their small size, low cost, and good mode control. There remain, however, certain application areas where the properties of a metallic waveguide are essential. For example, in the infrared and far infrared portions of the spectrum there are broad wavelength regions where low-loss fiber materials are not readily available, although steady progress is being made. Similarly in the far ultraviolet and soft X-ray regimes, transparent materials are virtually unknown, while grazing reflectivities from metals can be quite high. For such wavelengths and for extremely high-power applications, metallic waveguides would be the only choice.

There have been extensive theoretical studies of propagation in single-mode or low-order-mode metallic waveguides, and curved and twisted guides have also been considered [1]. More recently, curved dielectric waveguides for optical frequencies have been studied in detail [2]-[4], and flexible metallic waveguides of rectangular cross section have been developed for transmitting infrared wavelengths [5]-[8]. Lately it has been shown that effective light guiding is possible using a single curved metallic surface rather than a fully enclosed waveguide [9]. In that work, the surface of the waveguiding strip was flat in the transverse direction. The purpose of the present study is to investigate the characteristics of a waveguiding strip which is curved in the transverse direction. The advantage of this configuration is that it eliminates diffraction losses and results in low-loss Airy-Hermite-Gaussian modes. These modes are obtained in Section III as approximate solutions of the wave equation, and experimental results are presented in Section IV.

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II. The Waveguide Equation

The electromagnetic modes of a waveguide can be found as solutions of the wave equations

$$\nabla \times \nabla \times \overline{E} - \omega^2 \mu\epsilon \overline{E} = (\nabla\mu/\mu) \times \nabla \times \overline{E} \tag{1}$$

$$\nabla \times \nabla \times \overline{H} - \omega^2 \mu\epsilon \overline{H} = (\nabla\epsilon/\epsilon) \times \nabla \times \overline{H} \tag{2}$$

where $\overline{E}$ and $\overline{H}$ are the complex vector amplitudes of the time harmonic electric and magnetic fields and μ and ϵ are, respectively, the permeability and permittivity of the component media. To be specific, we emphasize solutions of (2), and (1) can be solved in an identical fashion. When (2) is applied to the cylindrical components of a vector expressed in cylindrical coordinates, the resulting equation is [10]

$$\begin{aligned}\overline{i}_r\nabla^2 H_r + \overline{i}_\phi\nabla^2 H_\phi + \overline{i}_z\nabla^2 H_z + k^2\overline{H} &= (1/r^2)\{\overline{i}_r[H_r + 2(\partial H_\phi/\partial\phi)] + \overline{i}_\phi[H_\phi - 2(\partial H_r/\partial\phi)]\} \\ &\quad - \nabla[(\nabla\mu/\mu)\cdot\overline{H}] - (\nabla\epsilon/\epsilon)\times\nabla\times\overline{H}\end{aligned} \tag{3}$$

where $k = \omega(\mu\epsilon)^{1/2}$ is a propagation constant. The permittivity and permeability in the component media are independent of space, so the last terms in (3) vanish. For the moment we are interested in field distributions which are dominated by the radial component H_r. At large radii the coupling terms on the right-hand side of (3) also vanish and H_r is governed by the familar scalar wave equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H_r}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 H_r}{\partial\phi^2} + \frac{\partial^2 H_r}{\partial z^2} + k^2 H_r = 0. \tag{4}$$

For the metallic strip waveguide, the region outside of the guide is essentially free space and k may be replaced by $k_0 = \omega(\mu_0\epsilon_0)^{1/2}$.

The most general beam modes of interest here propagate helically about the z axis as shown schematically in Fig. 1. Thus a useful substitution is

$$H_r(r,\phi,z) = A(r,\phi,z)\exp[-ik_0(r_0\phi\cos\theta + z\sin\theta)] \tag{5}$$

where $A(r, \phi, z)$ is a slowly varying amplitude, r_0 is the radial distance to the metal strip, and θ is the propagation angle with respect to the $r - \phi$ plane. When (5) is substituted into (4), one obtains

$$\begin{aligned}&\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r} + \frac{1}{r^2}\left(\frac{\partial^2 A}{\partial\phi^2} - 2ik_0 r_0\cos\theta\frac{\partial A}{\partial\phi}\right) \\ &+ \left(\frac{\partial^2 A}{\partial z^2} - 2ik_0\sin\theta\frac{\partial A}{\partial z}\right) \\ &+ k_0^2\left[1 - \left(\sin^2\theta + \frac{r_0^2}{r^2}\cos^2\theta\right)\right]A = 0.\end{aligned} \tag{6}$$

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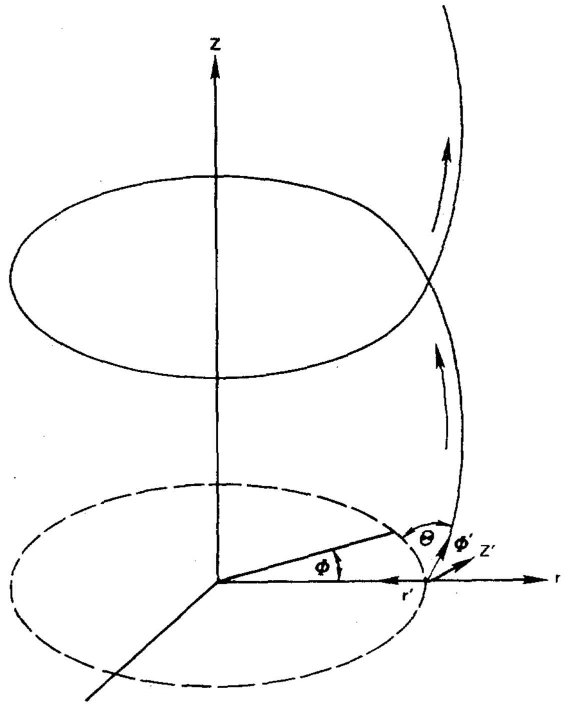

Fig. 1. Schematic respresentation of a helically propagating waveguide mode showing some of the coordinates used in the analysis.

To evaluate the amplitude A, it is helpful to transform the ϕ and z coordinates to a system rotated by the angle θ. The appropriate transformation is

$$\phi' = \phi \cos\theta + (z/r)\sin\theta \tag{7}$$

$$z' = -r\phi \sin\theta + z\cos\theta. \tag{8}$$

With these substitutions (6) becomes

$$\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r} + \frac{1}{r^2}\frac{\partial^2 A}{\partial \phi'^2} + \frac{\partial^2 A}{\partial z'^2} - \frac{2ik_0 r_0}{r^2}\cos\theta\left(\cos\theta\frac{\partial A}{\partial\phi'} - r\sin\theta\frac{\partial A}{\partial z'}\right) - 2ik_0\sin\theta\left(\frac{\sin\theta}{r}\frac{\partial A}{\partial\phi'} + \cos\theta\frac{\partial A}{\partial z'}\right) + k_0^2\left[1 - \left(\sin^2\theta + \frac{r_0^2}{r^2}\cos^2\theta\right)\right]A = 0. \tag{9}$$

At this point, it is helpful to make some approximations. After some analytical experimentation, one concludes that for dimensions much larger than the wavelength ($r_0 \gg \lambda$), the radial coordinate r can be approximate by r_0 except in the last term, and the second term may be neglected entirely. Thus one obtains

$$\frac{\partial^2 A}{\partial r^2} + \frac{1}{r_0^2}\frac{\partial^2 A}{\partial\phi'^2} + \frac{\partial^2 A}{\partial z'^2} - \frac{2ik_0}{r_0}\frac{\partial A}{\partial\phi'} + k_0^2\left[1 - \left(\sin^2\theta + \frac{r_0^2}{r^2}\cos^2\theta\right)\right]A = 0. \tag{10}$$

We can also make the standard paraxial approximation assuming that A varies so slowly in the direction of propagation that its second derivative with respect to ϕ' can be neglected. Thus we have

$$\frac{\partial^2 A}{\partial r^2} + \frac{\partial^2 A}{\partial z'^2} - \frac{2ik_0}{r_0}\frac{\partial A}{\partial\phi'} + k_0^2\left[1 - \left(\sin^2\theta + \frac{r_0^2}{r^2}\cos^2\theta\right)\right]A = 0. \tag{11}$$

The boundary conditions must be applied across a curved boundary, and it is helpful to transform the r coordinate using the substitution

$$r = r_0 - r' - z'^2/2R_0 \tag{12}$$

where R_0 is the radius of curvature across the waveguiding strip, and r' measures distance inward from the strip. Thus (11) becomes

$$\frac{\partial^2 A}{\partial r'^2} + \frac{\partial^2 A}{\partial z'^2} - \frac{2ik_0}{r_0}\frac{\partial A}{\partial\phi'} + k_0^2\left\{1 - \left[\sin^2\theta + \frac{r_0^2\cos^2\theta}{(r_0 - r' - z'^2/2R_0)^2}\right]\right\}A = 0. \tag{13}$$

The last term in (13) may be expanded into a binomial series, and to lowest order the result is

$$\frac{\partial^2 A}{\partial r'^2} + \frac{\partial^2 A}{\partial z'^2} - \frac{2ik_0}{r_0}\frac{\partial A}{\partial\phi'} - \frac{2k_0^2(r' + z'^2/2R_0)}{r_0}\cos^2\theta A = 0. \tag{14}$$

This equation is the principal result of this section and it is solved exactly in the following paragraphs.

III. Derivation of the Waveguide Modes

A. The Gaussian Factor

Equation (14) is in some respects similar to the equations of conventional beam optics, and the initial portion of the solution proceeds along fairly standard lines [10], [11]. As a first step we factor out a form corresponding to an off-axis Gaussian beam by means of the substitution

$$A(r', \phi', z') = B(r', \phi', z')\exp\{-i[Q(\phi')z'^2/2 + S(\phi')z']\}. \tag{15}$$

By setting equal to zero the sum of terms in z' and z'^2, (14) separates into the set

$$Q^2 + \frac{k_0}{r_0}\frac{dQ}{d\phi'} + \frac{k_0^2}{r_0 R_0}\cos^2\theta = 0 \tag{16}$$

$$QS + \frac{k_0}{r_0}\frac{dS}{d\phi'} = 0 \tag{17}$$

$$\frac{\partial^2 B}{\partial r'^2} + \frac{\partial^2 B}{\partial z'^2} - 2i(S + Qz')\frac{\partial B}{\partial z'} - S^2 B - iQB - \frac{2ik_0}{r_0}\frac{\partial B}{\partial\phi'} - \frac{2k_0^2 r'}{r_0}\cos^2\theta B = 0. \tag{18}$$

In these equations, Q is the usual beam parameter

$$Q(\phi') = \frac{k_0}{R(\phi')} - i\frac{2}{w^2(\phi')} \tag{19}$$

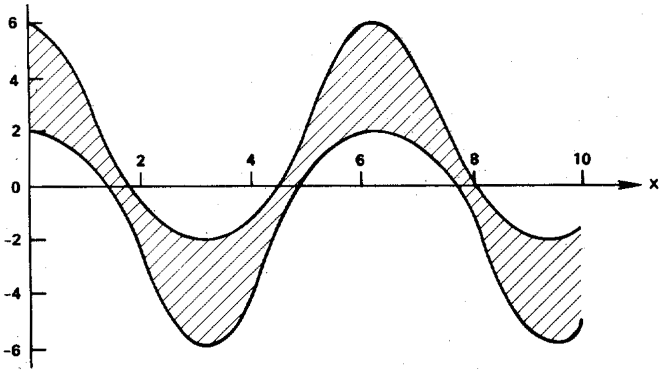

Fig. 2. Normalized plot of the beam displacement and spot size as a function of the distance $x = (\cos\theta/r_0R_0)^{1/2} r_0\phi'$ in a concave metal strip waveguide.

where R and w are the radius of curvature of the phase fronts and the $1/e$ amplitude spot size in the z' direction, respectively. The ratio $d_a = -S_i/Q_i$ is the displacement in the z' direction of the amplitude center of the beam, and the ratio $d_p = -S_r/Q_r$ is the displacement in the z' direction of the phase center of the beam. The subscripts i and r denote the imaginary and real parts of the parameters Q and S, respectively.

The solution of equations like (16) has been given previously [10], [11], and for this case the result is

$$\frac{Q(r_0\phi')}{k_0} = \frac{-(\cos\theta/r_0R_0)^{1/2}\sin[(\cos\theta/r_0R_0)^{1/2}r_0\phi'] + [Q(0)/k_0]\cos[(\cos\theta/r_0R_0)^{1/2}r_0\phi']}{\cos[(\cos\theta/r_0R_0)^{1/2}r_0\phi'] + [Q(0)/k_0](r_0R_0/\cos\theta)^{1/2}\sin[(\cos\theta/r_0R_0)^{1/2}r_0\phi']}. \tag{20}$$

Equations (16) and (17) may also be combined to yield a formula for the location of the beam's amplitude center

$$\frac{1}{r_0^2}\frac{d^2 d_a}{d\phi'^2} = -\frac{\cos\theta}{r_0R_0} d_a. \tag{21}$$

The integral of (21) is the oscillatory function

$$d_a(r_0\phi') = d_a(0)\cos[(\cos\theta/r_0R_0)^{1/2}r_0\phi'] + d_a'(0)(r_0R_0/\cos\theta)^{1/2}\sin[(\cos\theta/r_0R_0)^{1/2}r_0\phi'] \tag{22}$$

where $d_a'(0)$ is the input slope with respect to the $z' = 0$ surface. Thus the beam center propagates along a ray-like trajectory, while the spot size oscillates periodically according to the real part of (20).

A typical plot of (20) and (22) is given in Fig. 2. This example shows clearly the sinusoidal oscillation of the beam center displacement and the periodic pinching of the beam waist. It is always true that the period of the beam center oscillation is twice as large as the period of the pinching. However, the relative phase of these phenomena depends on the initial conditions.

B. The Hermite Polynomial Factor

In solving (18) it is helpful now to introduce the new variable [12]

$$\zeta = a(\phi')z' + b(\phi') \tag{23}$$

and the result is

$$\frac{\partial^2 B}{\partial r'^2} + a^2\frac{\partial^2 B}{\partial \zeta^2} - 2\left[iaS + i(\zeta - b)Q + \frac{ik_0}{r_0}\frac{(\zeta - b)}{a}\frac{da}{d\phi'} + \frac{ik_0}{r_0}\frac{db}{d\phi'}\right]\frac{\partial B}{\partial \zeta} - S^2B - iQB - \frac{2ik_0}{r_0}\frac{\partial B}{\partial \phi'} - \frac{2k_0^2 r'}{r_0}\cos^2\theta B = 0. \tag{24}$$

This substitution makes it more convenient to express the off-axis Hermite polynomial functions. For simplicity, we have retained the original notation for the unchanged variables r' and ϕ'.

The substitution

$$B(r', \phi', \zeta) = C(\phi', \zeta)D(r', \phi') \tag{25}$$

makes possible the arbitrary separation

$$a^2\frac{\partial^2 C}{\partial \zeta^2} - 2\left[iaS + i(\zeta - b)Q + \frac{ik_0}{r_0}\frac{(\zeta - b)}{a}\frac{da}{d\phi'} + \frac{ik_0}{r_0}\frac{db}{d\phi'}\right]\frac{\partial C}{\partial \zeta} - \frac{2ik_0}{r_0}\frac{\partial C}{\partial \phi'} - 2mQ_iC = 0 \tag{26}$$

$$\frac{\partial^2 D}{\partial r'^2} - S^2D - iQD - \frac{2ik_0}{r_0}\frac{\partial D}{\partial \phi'} - \frac{2k_0^2 r'}{r_0}\cos^2\theta D + 2mQ_iD = 0. \tag{27}$$

The purpose of this substitution is to split apart the r' and ζ dependences. This particular separation was chosen because it leads to a set of Hermite polynomial functions of real argument. An alternate set of complex Hermite polynomials can be derived in a similar manner [12]. Neglecting (27) for the moment, the requirement that $a(\phi')$, $b(\phi')$, and $C(\phi', \zeta)$ all be real, separates (26) into the two equations

$$a^2\frac{\partial^2 C}{\partial \zeta^2} - 2[-aS_i + i(\zeta - b)Q_i]\frac{\partial C}{\partial \zeta} - 2mQ_iC = 0 \tag{28}$$

$$\left[aS_r + (\zeta - b)Q_r + \frac{k_0}{r_0}\frac{(\zeta - b)}{a}\frac{da}{d\phi'} + \frac{k_0}{r_0}\frac{db}{d\phi'}\right]\frac{\partial C}{\partial \zeta} + \frac{k_0}{r_0}\frac{\partial C}{\partial \phi'} = 0. \tag{29}$$

If one imposes the conditions

$$a^2 = -Q_i \tag{30}$$

$$aS_i = bQ_i \tag{31}$$

(24) reduces to

$$\frac{\partial^2 C}{\partial \zeta^2} - 2\zeta\frac{\partial C}{\partial \zeta} + 2mC = 0. \tag{32}$$

If C is assumed to be independent of ϕ', (32) is just the ordinary Hermite differential equation. The significance of (30) and (31) and the definition of ζ in (23) is that the Hermite polynomial functions are displaced from the waveguide axis by the same amount as the Gaussian factor.

With C independent of ϕ' the bracketed quantity in (29) must vanish. This is only possible if a and b satisfy the additional constraints

$$Q_r + \frac{k_0}{r_0 a}\frac{da}{d\phi'} = 0 \tag{33}$$

$$S_r + \frac{k_0}{r_0 a}\frac{db}{d\phi'} = 0. \tag{34}$$

Equations (30), (31), (33), and (34) may be readily shown to be consistent with (16) and (17), so the validity of the Hermite polynomial factorization is confirmed.

Some simple checks can be performed on the Hermite-Gaussian modes that have been found so far. For this purpose we consider first a steady-state nonpinching mode. Thus (16) has the solution

$$Q = \left(-\frac{k_0^2 \cos^2\theta}{r_0 R_0}\right)^{1/2} \tag{35}$$

and with (19) the spot size is

$$w = \left[\frac{2(r_0 R_0)^{1/2}}{k_0 \cos\theta}\right]^{1/2} \tag{36}$$

If θ is equal to zero, this result corresponds to a beam propagating around the equator of a spheroid, and for a sphere ($r_0 = R_0$), the spot size is simply $w = (2r_0/k_0)^{1/2}$. In a previous analysis of beam propagation around a sphere the spot size was obtained in the form $w = (2/n)^{1/2} r_0$, where n is the azimuthal mode order [13]. But these results are equivalent since $n = k_0 r_0$. The higher order Hermite-Gaussian modes obtained in both analyses are also identical.

Another way to check these results is by considering off-axis ($z' \neq 0$) beam modes. From (22) it follows that the oscillation period of such modes is

$$\phi_0' = 2\pi (R_0/r_0 \cos\theta)^{1/2}. \tag{37}$$

For the case of a sphere ($\theta = 0$, $R_0 = r_0$), (37) is simply $\phi_0' = 2\pi$. This is a reasonable result, since a beam propagating near the equator of a sphere would clearly have to return to its initial displacement after an angular distance of 2π.

C. The Airy Function Factor

It only remains now to break down the function $D(r', \phi')$ governed by (27). Unfortunately, standard tabulated functions are not sufficient to describe the solutions of this equation which characterize a beam "bouncing along" with its r' distribution depending on its ϕ' position. We can, however, obtain a useful set of radial eigenmodes in which the r' distribution is independent of ϕ'. For this purpose the appropriate substitution is

$$D(r', \phi') = F(r') \exp[-iP(\phi')] \tag{38}$$

with the resulting separation

$$\frac{d^2F}{dr'^2} + \left(\alpha - \frac{2k_0^2 r'}{r_0}\cos^2\theta\right)F = 0 \tag{39}$$

$$\frac{1}{r_0}\frac{dP}{d\phi'} = -\frac{iQ_r}{2k_0} + \frac{(m+\frac{1}{2})Q_i}{k_0} - \frac{S^2+\alpha}{2k_0} \tag{40}$$

where α is a separation constant.

Equation (39) can be reduced by the change of variables

$$\rho = \left(\frac{2k_0^2\cos^2\theta}{r_0}\right)^{-2/3}\left(\frac{2k_0^2\cos^2\theta}{r_0}r' - \alpha\right) \tag{41}$$

and the result is

$$\frac{d^2F}{d\rho^2} - \rho F = 0. \tag{42}$$

This is the standard form of Airy's equation and the solutions are well known [14]. Equation (40) governs the amplitude and phase of the modes, and the integral of this equation is also known [10]. Thus the waveguide equation has been completely solved and the resulting field distributions may be described in terms of Airy-Hermite-Gaussian modes.

To apply these results to a practical waveguide, one should, in principle, solve (42) for the various waveguide regions and apply appropriate boundary conditions. For the case of interest, one would derive the field distribution above the metal waveguiding strip and within the metal and apply the boundary conditions at the curved surface. However, the effective dielectric constant of typical metals is very large, and it is sufficient here to require that the normal component of the magnetic field vanish at the metal surface. Thus the separation constant α must be chosen in such a way that a zero of the Airy function occurs at $r' = 0$. From (41) it follows that the α value for the nth mode must be given by

$$\alpha_n = -\left(\frac{2k_0^2\cos^2\theta}{r_0}\right)^{2/3}\rho_n \tag{43}$$

where ρ_n is the nth zero of the Airy function. The coordinate ρ is now related to r' by

$$\rho - \rho_n = \left(\frac{2k_0^2\cos^2\theta}{r_0}\right)^{1/3} r' \equiv r''. \tag{44}$$

The Airy function solutions of (42) are plotted in Fig. 3 as functions of the coordinate r'' defined in (44) for several low order modes. It is evident from this figure that, with increasing mode order, the fields extend farther away from the waveguide surface. An alternate set of modes can be obtained from (1) with the requirement that the electric field be dominated by its radial component. The solution would be the same in all respects, except for changes in the boundary conditions. It should be noted that these results apply also to acoustic modes, and the Airy function radial dependence has been obtained long ago with reference to whispering gallery modes [15]-[17]. The attenuation of the waveguides modes can be calculated using standard techniques [18], and the loss for a circular metal strip has been given [9].

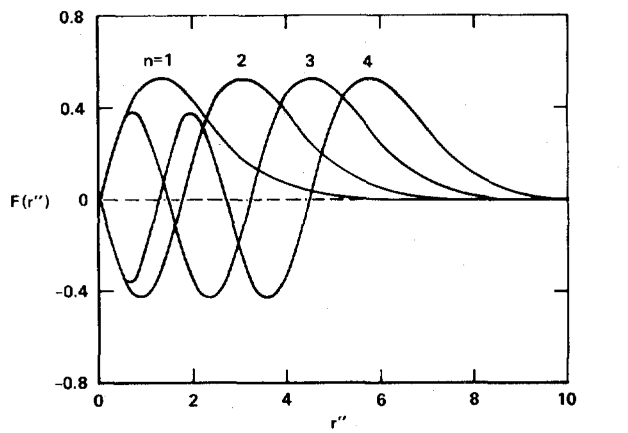

Fig. 3. Low order Airy function radial modes of a metal strip waveguide.

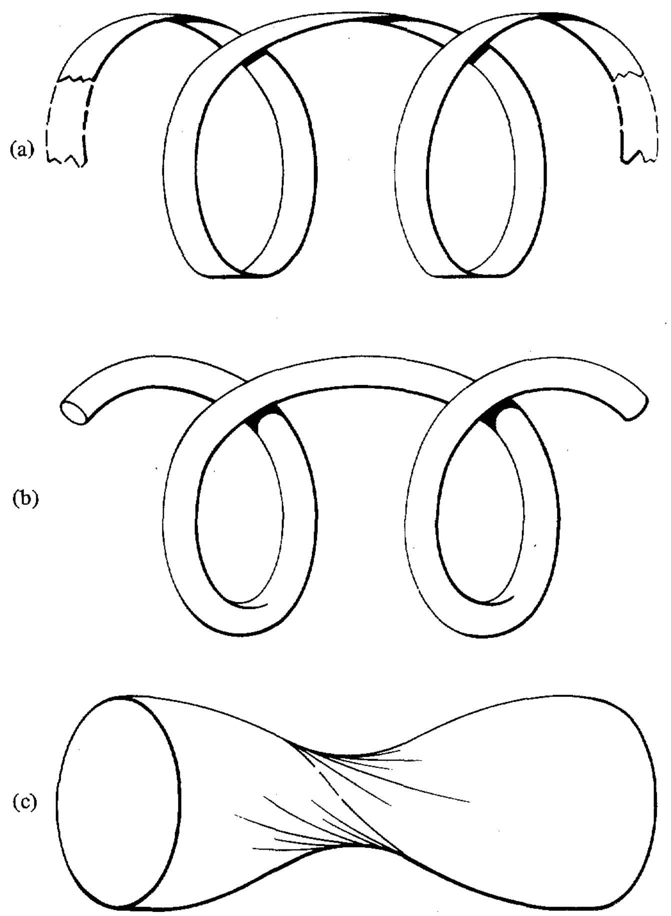

Fig. 4. Possible waveguides configurations for propagating Airy-Hermite-Gaussian modes: (a) helical metal strip; (b) helical metal tube of circular cross section; (c) twisted metal tube of elliptical cross section.

IV. Experiment

From the discussions in the previous sections it follows that one should be able to guide a light beam for long distances by means of a simple concave metallic strip. Some possible realizations of this waveguide method are shown in Fig. 4. For applications requiring vacuum techniques, it might be desirable to use a helical tube of circular cross section or a twisted tube of elliptical cross section instead of an open metal strip. The open strip configuration of Fig. 4(a) is, of course, much easier to polish. Several experiments have been performed in order to test the theoretical predictions. We find that satisfactory waveguides can be made by sawing the side from a metal tube and polishing the concave inner surface. The circular or helical shape can be readily formed. Alternatively, some types of weather stripping are concave and with polishing make excellent waveguides.

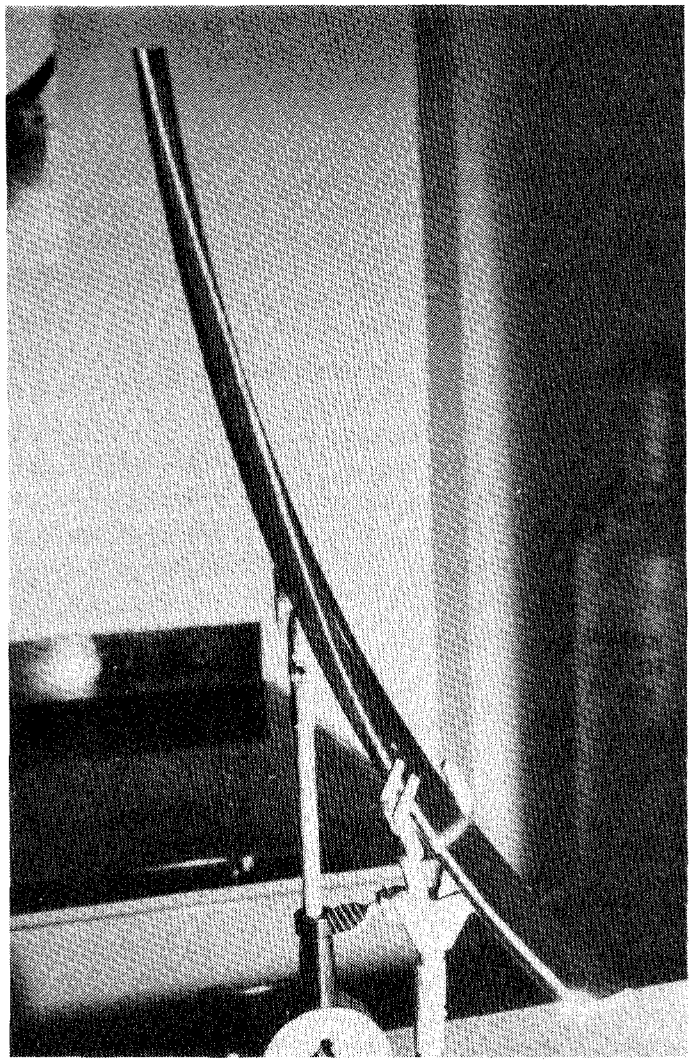

Fig. 5. A 92 cm long concave metal strip showing light scattered from a propagating 6328 Å helium-neon laser beam. Periodic displacement and pinching of the beam may be seen.

A typical waveguide strip is shown in Fig. 5. In this example, the strip is 92 cm in length and formed from aluminum. The principal radii of curvative are $r_0 = 36$ cm and $R_0 = 3.25$ cm. This photograph used a time exposure and shows the actual beam from a 6328 Å helium neon laser as it propagates along the guide. The beam shows up because of scattering imperfections on the waveguide surface.

The oscillating trajectory of the beam center and pinching of the beam waist are conspicuous features of Fig. 5. From (37) the oscillation period of the beam center should be

$$r_0 \phi_0' = 2\pi (r_0 R_0)^{1/2} = 68.0 \text{ cm} \tag{45}$$

and this result is confirmed by the experiment to within our experimental accuracy. The pinching data also agree with the theory. Our uncertainty in these measurements is less than about 2 percent, and the most uncertain measurement involves determining the concaveness R_0.

In principle, one could also test to see whether the experimental radial field variations are in agreement with the theoretical Airy functions. From Fig. 3 or suitable tables, the $1/e$ radius of the fundamental Airy function mode occurs at the radius $r'' = 3.00$. Thus, for the previously described experiment the mode thickness is

$$\Delta r' = 3.00(r_0/2k_0^2)^{1/3} = 3.67 \times 10^{-5} \text{ m}. \tag{46}$$

Our present setup does not select radial modes, so this result has not been confirmed.

Experiments have also been performed with helical waveguides. One device consists of an aluminum strip 365 cm in length which has been coiled to a helix of radius $r_0 = 14.7$ cm. The concaveness of the strip is 3.7 cm. The resulting coil is highly flexible, and when compressed the theoretical oscillation period is

$$r_0 \phi_0' = 2\pi(r_0 R_0)^{1/2} = 46.3 \text{ cm}. \tag{47}$$

When the coil is extended to a helix angle of $\theta = 27.1°$ (keeping r_0 constant), the oscillation period is

$$r_0 \phi_0' = 2\pi(r_0 R_0/\cos\theta)^{1/2} = 49.1 \text{ cm}. \tag{48}$$

These results are in agreement with the corresponding experimental measurements.

V. Conclusion

A beam guiding technique has been developed in which the waveguiding medium consists of a single metallic strip. The mode properties of this device have been expressed in terms of oscillating Airy-Hermite-Gaussian functions. Our experimental results have confirmed the validity of the theoretical formalism for waveguides of almost 4 m length. An input beam which is displaced from the waveguide axis propagates with periodic variations of the beam center and width. The approximations of the analysis (paraxial approximation, large r_0) have produced no discernible effects, and any possible errors would have still less significance for on-axis eigenmode propagation.

Besides the flexibility of the strip waveguides, another advantage is ease of fabrication. Polishing, in particular, is straightforward and can be done before the waveguide is bent. However, if the inside of a metal tube can be polished, then the other designs shown in Fig. 4 may also be of interest. The main applications of these waveguides would be for wavelength and power regimes where low-loss dielectric materials are not available.

Note Added in Proof: Related studies by M. E. Marhic *et al.* have recently been published in *Appl. Phys. Lett.*, vol. 33, pp. 609-611, Oct. 1, 1978; and *Appl. Phys. Lett.*, vol. 33, pp. 874-876, Nov. 15, 1978.

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