PHYSICAL REVIEW B **73**, 037302 (2006)

Reply to "Comment on 'Spin splitting in modulation-doped $\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{N/GaN}$ heterostructures' "

Ikai Lo,* J. K. Tsai, W. J. Yao, P. C. Ho, Li-Wei Tu, and T. C. Chang
Department of Physics, Center for Nanoscience and Nanotechnology, National Sun Yat-Sen University, Kaohsiung, Taiwan, Republic of China

S. Elhamri and W. C. Mitchel
Air Force Research Laboratory, MLPS, Wright-Patterson Air Force Base, Dayton, Ohio 45433, USA

K. Y. Hsieh, J. H. Huang, and H. L. Huang
Institute of Materials Science and Engineering, National Sun Yat-Sen University, Kaohsiung, Taiwan, Republic of China

Wen-Chung Tsai
Advanced Epitaxy Technology Inc., 119 Kuangfu N. Road, Hsinchu Industrial park, Taiwan, Republic of China

(Received 10 November 2005; revised manuscript received 14 December 2005; published 25 January 2006)

In this Reply, we reexamine the beating Shubnikov–de Haas oscillations by a nonlinear curve-fitting technique. The results do not support the arguments of Tang *et al.* [Phys. Rev. B **73**, 037301 (2006)], and it is unlikely that the beating Shubnikov–de Haas oscillations we observed in $\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{N/GaN}$ heterostructures originate from magnetointersubband scattering.

DOI: 10.1103/PhysRevB.73.037302 PACS number(s): 73.21.−b, 71.18.+y, 72.20.Fr, 71.70.Ch

Tang *et al.* commented on our paper[1] that the beating Shubnikov–de Haas (SdH) oscillations we observed in $\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{N/GaN}$ heterostructures might originate from magnetointersubband scattering (MIS) instead of zero-field spin splitting. To support their argument, they pointed out that (i) a second-subband population with SdH oscillation frequency 16.7 T might exist in sample 3, (ii) the theoretical calculation in wurtzite GaN was still not available, (iii) the phase difference between SdH and MIS oscillations was equal to π, and (iv) the amplitude of the beating pattern induced by the MIS effect was determined by $A_{\rm MIS} \sim \sin \nu\pi$, where $\nu=(E_F-E_2)/\hbar\omega_c$.

In order to examine the accurate phases for the individual SdH oscillations, we applied the nonlinear curve-fitting technique to the original data in Fig. 3 of Ref. 1. After the removal of the background noise signal (nonoscillating signal), the oscillatory resistivity $\rho_{\rm osc}(B)$ was fitted to the superposition of two independent cosine functions,[2]

$$\rho_{\rm osc}(B)=\sum_{i=1}^{2}\rho_i\frac{(\mu_iB)^2}{1+(\mu_iB)^2}\exp\left(\frac{-\pi}{\mu_iB}\right)\frac{1/X_iB}{\sinh(1/X_iB)}\times\cos(2\pi f_i/B+\phi_i), \qquad (1)$$

where ρ_i is a constant proportional to the zero-field resistivity, $\mu_i=\omega_c\tau_i/B$, τ_i is the quantum lifetime of the carrier, and $\omega_c=eB/m^*$, $X_i=\hbar e/2\pi^2k_BTm^*$, and f_i and ϕ_i are SdH frequency and phase of the ith subband. It is noted that $f_i=n_ih/2e$ is for the spin-degenerated ith subband and n_i is the carrier concentration of that subband, but $f_i=n_ih/e$ is for the spin-splitting subband due to the lift of spin degeneracy. The fitting results are shown in Fig. 1: the upper black lines are the experimental data and the lower red (gray) lines are the fitting data for each set of different illuminated times. The fitting parameters are shown in Table I.

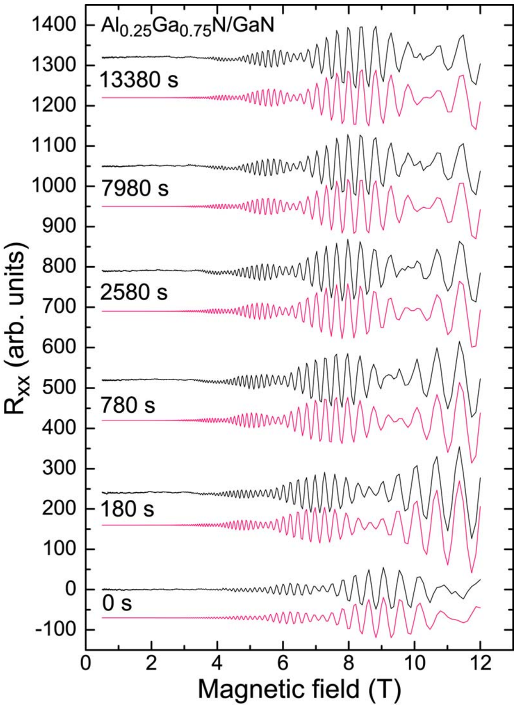

FIG. 1. (Color online) The nonlinear curve fitting to the SdH data of Fig. 3 in Ref. 1. The upper black lines denote the experimental data and the lower red (gray) lines denote the fitting results for each set.

TABLE I. The parameters of curve fitting results for SdH oscillations in Fig. 1.

Time (s)	ρ_1	ρ_2	μ_1	μ_2	X_1	X_2	f_1(T)	f_2(T)	ϕ_1	ϕ_2
0	91	263	1.07	0.31	0.045	0.039	178.8	159.3	82	165
180	118	172	0.82	0.59	0.058	0.044	179.0	164.3	104	173
780	126	184	0.80	0.79	0.061	0.041	179.6	164.5	124	157
2580	129	229	0.94	0.65	0.056	0.040	180.3	164.8	125	146
7980	142	240	0.80	0.56	0.054	0.042	181.0	164.7	120	148
13380	149	246	0.79	0.56	0.053	0.041	181.2	164.7	117	147

The two fitting frequencies for zero illumination time (178.8 and 159.3 T) give the carrier concentrations of spin up and spin down, 4.34 and 3.86×10^{12} cm^{-2}, which is in agreement with the low-temperature carrier concentration determined by Hall measurement, 8.96×10^{12} cm^{-2} (the sum of spin-up and spin-down carrier concentrations).[1] This is our reply to comment (i) of Tang *et al.* In addition, none of the phase difference $(\phi_1 - \phi_2)$ in Table I is equal to π. This is our reply to comment (iii) of Tang *et al.* Based on the theory of magnetointersubband resonant scattering,[3,4] where Sander *et al.*[5] derived their equations, the magnetointersubband resonant scattering produces a series of resistivity oscillations with a frequency of $(f_1 - f_2)$ against inverse magnetic field $(1/B)$. In the equation, A_1 and A_2 (the fundamental SdH terms) are the first-order terms of $(1/g_0)$, but B_{12} (the MIS term) is the second-order term of $(1/g_0^2)$; see Eq. (2) of Ref. 3. The second-order term (MIS) is always small as compared with the first-order terms (SdH)—i.e., $B_{12} \ll A_1, A_2$. However, in Table I, it is shown that the difference of fast Fourier transform (FFT) amplitudes for the two SdH oscillations (in Ref. 1) is mainly due to the presence of carriers having different "mobility" (μ_i). Here, the "mobility" is defined as $\mu_i = e\tau_i / m^*$. In the Comment of Tang *et al.*, the argument that the amplitude of the beating pattern (node) induced by the MIS effect was determined by $A_{\mathrm{MIS}} \sim \sin \nu\pi$, where $\nu = (E_F - E_2)/\hbar\omega_c$, is incorrect. It should be $\nu = (E_{02} - E_{01})/\hbar\omega_c = (f_1 - f_2)/B$; i.e., see Eq. (7) in Ref. 3. The beat frequency $(f_{\mathrm{beat}} = f_1 - f_2)$ should be determined by the slope of Laudau plot (Landau levels versus $1/B$), the inset in Fig. 5 of Ref. 1, instead of the y-axis intersection of the plot of Das *et al.* (δ versus B) in Fig. 4 of Ref. 6. The spin splitting determined by the the plot of Das *et al.* is $\delta_0 = 2.31$ meV. The spin splitting determined by the beat frequency of the Landau plot $(f_{\mathrm{beat}} = 0.925$ T) is $\delta_0 = 1.16$ meV. The linear fittings of the two plots are shown in Fig. 2, which are used the same data points (sample *A* of Ref. 6). The results can support our argument.[1,6] This is our reply to comment (iv) of Tang *et al.*

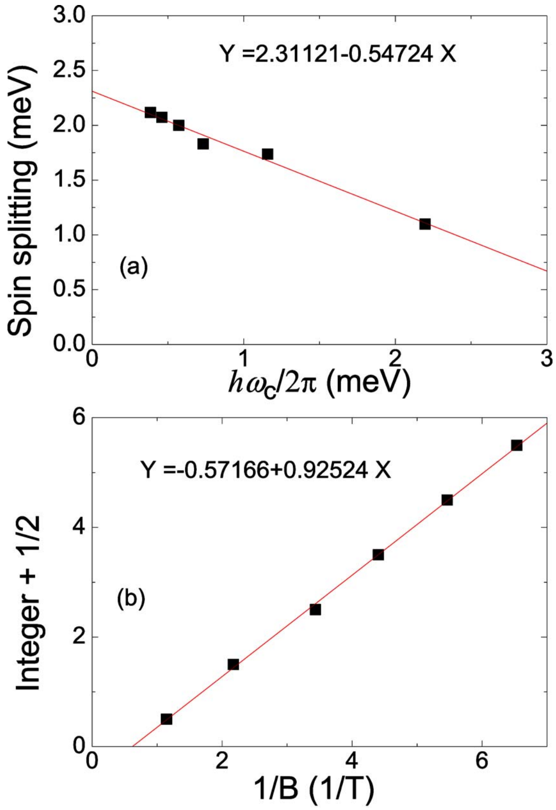

FIG. 2. (Color online) (a) The spin splitting determined by the y-axis intersection of the plot of Das *et al.* (δ versus B) in Fig. 4 of Ref. 6 is $\delta_0 = 2.31$ meV. (b) The spin splitting determined by the beat frequency of the Landau plot $(f_{\mathrm{beat}} = 0.925$ T) is $\delta_0 = 1.16$ meV.

A new mechanism (Δ_{C1}-Δ_{C3} coupling) was recently proposed to describe the large spin splitting in wurtzite GaN, which is originated from the band-folding effect and intrinsic wurtzite structure inversion asymmetry.[7] The band-folding effect generates two conduction bands (Δ_{C1} and Δ_{C3}), in which the p-wave probability has tremendous change when k_z approaches the anticrossing zone. The Δ_{C1}-Δ_{C3} coupling can produce a spin-splitting energy much larger than traditional Rashba or Dresselhaus effects. This is our reply to comment (ii) of Tang *et al.*

In conclusion, we have shown that the nonlinear curve-fitting results do not support the arguments of Tang *et al.* and it is unlikely that the beating Shubnikov–de Haas oscillations observed in $\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{N}/\mathrm{GaN}$ heterostructures originate from magnetointersubband scattering.

This project is supported in part by NSC Core Facilities Laboratory in Kaohsiung-Pintung area, Taiwan (ROC).

*Electronic address: ikailo@mail.phys.nsysu.edu.tw

[1] Ikai Lo, J. K. Tsai, W. J. Yao, P. C. Ho, L.-W. Tu, T. C. Chang, S. Elhamri, W. C. Mitchel, K. Y. Hsieh, J. H. Huang, H. L. Huang, and W.-C. Tsai, Phys. Rev. B **65**, 161306(R) (2002).

[2] J. Singh, *Physics of Semiconductors and Their Heterostructures* (McGraw-Hill, New York, 1993), Chap. 17, p. 693.

[3] D. R. Leadley, R. Fletcher, R. J. Nicholas, F. Tao, C. T. Foxon, and J. J. Harris, Phys. Rev. B **46**, 12439 (1992).

[4] M. E. Raikh and T. V. Shahbazyan, Phys. Rev. B **49**, 5531 (1994).

[5] T. H. Sander, S. N. Holmes, J. J. Harris, D. K. Maude, and J. C. Portal, Phys. Rev. B **58**, 13856 (1998).

[6] B. Das, D. C. Miller, S. Datta, R. Reifenberger, W. P. Hong, P. K. Bhattacharya, J. Singh, and M. Jaffe, Phys. Rev. B **39**, 1411 (1989).

[7] Ikai Lo, W. T. Wang, M. H. Gau, S. F. Tsai, and J. C. Chiang, cond-mat/0510831, Phys. Rev. B (to be published).