Magnetoresistance of Two-Dimensional Fermions in a Random Magnetic Field

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We perform a semiclassical calculation of the magnetoresistance of spinless two-dimensional fermions in a long-range correlated random magnetic field. In the regime relevant for the problem of the half filled Landau level the perturbative Born approximation fails and we develop a new method of solving the Boltzmann equation beyond the relaxation time approximation. In absence of interactions, electron density modulations, in-plane fields, and Fermi surface anisotropy, we obtain a quadratic negative magnetoresistance in the weak field limit. [S0031-9007(96)01015-0]

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The problem of two-dimensional transport in spatially random static magnetic fields (RMF) attracted a lot of attention over the last few years. There exists a number of experimental realizations of two-dimensional (2D) RMF created by randomly pinned flux vortices in a type-II superconducting gate, grains of a type-I superconductor, or a demagnetized permanent magnet placed nearby the 2D electron gas [1]. A formally similar problem arises in the contexts of the gauge theory of high T_c cuprates [2] and the composite fermion theory of the half filled Landau level (HFLL) [3,4].

Various analytical and numerical results obtained so far seem to indicate that in the case of a continuum system with unbound spectrum all states are localized [5], while in the lattice case there might be an extended state at the center of the band [6]. The effective low-energy description in terms of the unitary σ model constructed in [7] suggests that the RMF problem belongs to the unitary random ensemble. Compared to the conventional problem of potential scattering corresponding to the orthogonal case, the effects of quantum interference in the RMF are suppressed as a result of broken time-reversal symmetry. In particular, the logarithmic temperature dependent weak localization corrections to the conductivity appear only in the next order in the metallicity parameter $k_F l \gg 1$: $\delta_{\rm wl}\sigma = -(1/\pi^2 k_F l)\ln(\tau_\varphi/\tau_{\rm tr})$ [where the temperature dependence comes from the inelastic phase breaking time $\tau_\varphi(T)$ in the regime $\tau_\varphi(T) \gg \tau_{\rm tr} = l/v_F$]. It suggests a larger localization length $L_{\rm loc} \sim l\exp[(\pi^2/4)(k_F l)^2]$ than in the orthogonal case.

It was shown in [8] that quantum fluctuations of particle's positions in the direction transverse to their classical trajectories are strongly reduced by the RMF. And so is the contribution of quantum backscattering which leads to a negative magnetoresistance (MR) in the presence of a weak uniform external magnetic field B.

Therefore, in the RMF problem one might expect that the semiclassical transport theory is applicable in a wider range of length scales and temperatures than in the case of time-reversal invariant random potential scattering. Yet, a semiclassical treatment of the RMF scattering beyond the relaxation time approximation may lead to a nontrivial MR. This classical contribution due to the bending of particle's trajectories dominates over the suppressed effects of weak localization, at least, at not very low temperatures.

The semiclassical approach to the RMF problem was undertaken in a number of publications. In [9] the Subnikov–de Haas (SdH) oscillations of $\rho_{xx}(B)$ in the strong field limit $\Omega_c = B/m \gg 1/\tau_{\rm tr}$ were studied by summing over classical cyclotron orbits. However, to find $\rho_{xx}(B)$ in the weak field limit before the onset of the SdH oscillations one has to develop a semiclassical analysis in terms of particle-hole pairs rather than single fermions.

The authors of [10] used the linear Boltzmann equation where the RMF played a role of a random driving force instead of including it into the collision integral. The condition which makes it possible is $k_F\xi \gg 1$, where ξ is a correlation length (scale of a typical spatial variation) of the RMF. However, the analysis performed in [10] was restricted onto the case relevant for the experiments [1] where, first, strong potential scattering provides a large bare value of ρ_{xx}, and, second, the spatial correlation of the RMF has a finite range $\sim\xi^{-1}$.

In the present Letter we address the case when ρ_{xx} is solely due to the scattering by the RMF described by the (not necessarily short-range) correlator $\langle b_q b_{-q}\rangle = f(q)$ independent of the external field B. We will not restrict our consideration onto the lowest order of the perturbation theory and be able to discuss the case $f(q) \sim e^{-2q\xi}$ relevant for the problem of HFLL where the spatial RMF correlations decay as r^{-3} and the lowest order result simply diverges.

We start with the Boltzmann equation for the distribution function $g(t,\vec r,\phi,\epsilon) = g_0(\epsilon) + \delta g(t,\vec r,\phi,\epsilon)$ which reads as

$$\left[i\frac{\partial}{\partial t} + i\vec v_\phi\cdot\vec\nabla + i\left(\Omega_c + \frac{b(\vec r)}{m}\right)\frac{\partial}{\partial\phi}\right]\times \delta g(t,\vec r,\phi,\epsilon) = ie\vec v_\phi\cdot\vec E\,\frac{\partial g_0}{\partial\epsilon}, \tag{1}$$

where $\vec v_\phi = v_F\vec n_\phi$ is a vector of Fermi velocity normal to the circular Fermi surface parametrized by the angular variable ϕ, $\vec{E}$ stands for an infinitesimal external electric field, and $g_0 = \theta(\varepsilon_F - \epsilon)$ is the unperturbed Fermi-Dirac distribution. By contrast to the case of ordinary potential scattering (which can also be treated at $k_F\xi \gg 1$ as a random electric field) Eq. (1) implies a trivial dependence of δg on ϵ, namely, $\delta g(t,\vec{r},\phi,\epsilon) = -\delta g(t,\vec{r},\phi)\delta(\varepsilon_F - \epsilon)$.

We note, in passing, that one can use Eq. (1) even in the presence of long-range retarded gauge interactions between fermions which spoil the Fermi liquid coherence. Even though there are no well-defined fermionic quasiparticles, the kinetic equation in terms of local displacements of the fluctuating Fermi surface can still be derived [11], thereby making a close contact with the idea of bosonization of 2D non-Fermi-liquids [12].

The perturbed distribution function can be found in terms of the (retarded) Green function of the Boltzmann operator for a given RMF configuration

$$\delta g(t,\vec{r},\phi,\epsilon) = ie \oint d\phi' \int d\vec{r}' \times G(t;\vec{r},\vec{r}';\phi,\phi')\vec{v}_{\phi'}\vec{E}\delta(\epsilon_F - \epsilon) \quad (2)$$

and then averaged over all possible configurations. As usual, the averaging over disorder restores the translational invariance in both $\vec{r}$ and ϕ spaces: $\langle\langle G(t;\vec{r},\vec{r}';\phi,\phi')\rangle\rangle = \mathcal{G}(t;\vec{r} - \vec{r}';\phi - \phi')$. Then the calculation of the frequency-dependent conductivity amounts to computing the p-wave harmonics of the $\vec{q} = 0$ component of the Fourier transform of

$$\mathcal{G}(\omega;\vec{q};\phi - \phi') = \sum_l e^{il(\phi-\phi')+iR_c\vec{q}\times(\vec{n}_\phi - \vec{n}_{\phi'})}\,\frac{\mathcal{G}_l(\omega)}{2\pi},$$

where $R_c = v_F/\Omega_c$ is the cyclotron radius of fermions with density n_e,

$$\sigma_{xx}(\omega) = \frac{ie^2n_e}{2m}[\mathcal{G}_1(\omega) + \mathcal{G}_{-1}(\omega)],$$

$$\sigma_{xy}(\omega) = \frac{e^2n_e}{2m}[\mathcal{G}_1(\omega) - \mathcal{G}_{-1}(\omega)].$$

In the absence of the RMF the bare Green function is given by its harmonics $G_l^{(0)}(\omega) = 1/\omega - l\Omega_c + i\delta$.

In our case of no potential disorder ($\delta = 0^+$) the lowest order (bosonic) self-energy correction to $\mathcal{G}_l^{-1}(\omega) = \omega - l\Omega_c + \Sigma_l(\omega)$ found in [10] makes it only possible to study the B-dependent correction to the bare ρ_{xx} at high frequencies or magnetic fields. To proceed with a more complete account of the effects of the RMF we first solve the equation for the Green function

$$\left[i\frac{\partial}{\partial t} + i\vec{v}_\phi\vec{\nabla} + i\left(\Omega_c + \frac{b(\vec{r})}{m}\right)\frac{\partial}{\partial\phi}\right]G(t - t';\vec{r},\vec{r}';\phi,\phi') = \delta(t - t')\delta(\vec{r} - \vec{r}')\delta(\phi - \phi') \quad (3)$$

for an arbitrary RMF configuration by Fourier transforming with respect to $t - t'$ and $\vec{r} - \vec{r}'$. The solution of the resulting equation

$$\left[\omega - \vec{v}_\phi\vec{q} + i\vec{v}_\phi\vec{\nabla} + i\left(\Omega_c + \frac{b(\vec{r})}{m}\right)\frac{\partial}{\partial\phi}\right]G(\omega;\vec{q},\vec{r};\phi,\phi') = \delta(\phi - \phi') \quad (4)$$

can be searched in the form

$$G(\vec{r},\vec{q};\phi,\phi') = -i\int_0^\infty d\tau \times \oint d\phi''\, e^{i\tau G_0^{-1}(\vec{q};\phi,\phi'')}e^{i\Psi(\tau;\vec{r};\phi'',\phi')}. \quad (5)$$

Here Ψ can be viewed as an "eikonal" phase of a particle-hole pair propagating in the RMF. Substituting (5) into (4) and integrating by parts we obtain the equation for Ψ,

$$\left(i\frac{\partial}{\partial\tau} + i\vec{v}_\phi\vec{\nabla}\right)\Psi = -e^{-i\Psi}e^{-i\tau G_0^{-1}}\frac{b(\vec{r})}{m}\frac{\partial}{\partial\phi}e^{i\tau G_0^{-1}}e^{i\Psi}. \quad (6)$$

Provided the RMF correlation length is large enough ($k_F\xi \gg 1$) one can linearize Eq. (6) (we will comment on this point below) and then end up with an explicit solution

$$\Psi(\tau;\vec{r};\phi,\phi') = \frac{1}{m}\int_0^\tau d\tau'\, b(\vec{r} - \vec{R}_{\tau'})\frac{\partial}{\partial\phi}\delta(\phi - \phi'), \quad (7)$$

where $\vec{R}_\tau = R_c e^{\Omega_c\tau(\partial/\partial\phi)}\vec{n}_\phi$ is a classical trajectory corresponding to the Larmor precession along the cyclotron orbit. Now taking the Gaussian average of the exponent $e^{i\Psi}$ in (7) over different realizations of the RFM we obtain the averaged Green function

$$\mathcal{G}(\omega,\vec{0};\phi - \phi') = \frac{1}{i}\int_0^\infty d\tau\, e^{i\tau[\omega + i\Omega_c(\partial/\partial\phi)]}\exp\left[\frac{1}{2}\int_0^\tau d\tau_1\int_0^\tau d\tau_2\, F\left(2R_c\sin\frac{\Omega_c(\tau_1 - \tau_2)}{2}\right)\frac{\partial^2}{\partial\phi^2}\right]\delta(\phi - \phi'), \quad (8)$$

where $F(r)$ is a Fourier transform of the RMF correlator $f(q)$. The use of Eq. (8) yields the conductivity

$$\sigma_{xx}(\omega) = \frac{e^2n_e}{m}\int_0^\infty d\tau\cos(\Omega_c + \omega)\tau\exp\left[-\int_0^\tau d\tau'(\tau - \tau')F\left(2R_c\sin\frac{\Omega_c\tau'}{2}\right)\right] + (\omega \to -\omega). \quad (9)$$

The formula for $\sigma_{xy}(\omega)$ differs from (9) by the additional factor $\tan(\Omega_c \pm \omega)\tau$ in the integrand.

The exponential factor in the integrand in (9) is controlled by the amplitude of the RMF correlator $F(r)$. To check the validity of (9) in the perturbative regime we expand the exponent up to the first order in $F(r)$ and perform the τ integration first [it has to be done with an infinitesimal exponent $e^{-\delta\tau}$ inserted into the integrand which specifies the retarded nature of the Green function (8)]. Then one can readily obtain

$$\sigma_{xx}(\omega) = 1/\tau_{\rm tr}(\Omega_c - \omega)^{-2} + (\omega \to -\omega),$$

where the RMF scattering rate is given by the expression

$$1/\tau_{\rm tr}(B) = \int_0^\infty d\tau \cos\Omega_c\tau F\left(2R_c \sin\frac{\Omega_c\tau}{2}\right), \qquad (10)$$

which is nothing but the result of the first Born approximation (BA) [10]. One can also obtain the formula (10) by estimating the $\sim\tau$ term in the exponent in (9) in the $\tau \to \infty$ limit which would be equivalent to the relaxation time approximation.

In what follows we will concentrate on the case $f(q) = 8\pi^2\alpha n_e e^{-2q\xi}$ relevant for the problem of HFLL where the dimensionless coupling constant $\alpha = \frac{1}{2}\Phi^2(n_i/n_e)$ is proportional to the density n_i of ionized Coulomb impurities separated by the spacer of the width ξ from the 2D electron gas and the amount of gauge flux quanta Φ attached to every electron in the case of a filling factor ν with an even denominator Φ [3]. In the picture of composite fermions (CF) each charged impurity also becomes a source of a gauge magnetic flux [3,4]. It is believed that the scattering by randomly distributed fluxes provides the main mechanism of the CF momentum relaxation while the potential scattering is negligible.

At $B = 0$ Eq. (10) gives the CF elastic transport rate $1/\tau_{\rm tr}^{\rm CF} = (v_F/2\xi)\alpha$ which coincides with the BA result found in [3]. Although this estimate certainly holds for $\alpha \ll 1$, it is no longer valid at large α when the CF mean free path (MFP) $l_{\rm CF} = 2\xi\alpha^{-1}$ gets shorter than ξ. The case of HFLL ($\nu = 1/2$) appears to be marginal ($\alpha = 2$), and l determined this way just equals ξ. Since α is proportional to Φ^2, the situation becomes even worse for compressible states at fractions with higher even denominators.

This observation signals about a failure of the BA at large coupling α when the matrix element $M_{\vec{p},\vec{p}'} \sim \sqrt{\alpha}\,(k_F/m\xi)$ describing a single event of CF scattering by a typical magnetic impurity satisfies neither of the two conditions $M_{\vec{p},\vec{p}'} \ll 1/m\xi^2$ nor $(k_F\xi)/m\xi^2$ required for the validity of the BA [13]. This is the semiclassical regime of a small ($\Delta\phi \sim 1/k_F\xi \ll 1$) angle scattering, which, however, cannot be treated in the lowest order of the perturbation theory.

Nevertheless, if the parameter $k_F\xi$ is large enough, so that

$$k_F\xi \gg \sqrt{\alpha}, \qquad (11)$$

one can resort to the so-called eikonal approach [13] which was essentially implemented by the above solution of the Boltzmann equation. In fact, it is the condition (11) which allows one to solve Eq. (6) in the first order in $b(\vec{r})$.

The use of (9) then leads to

$$\sigma_{xx} = \frac{e^2}{2h}(k_F l) = \frac{e^2}{h}(k_F\xi)e^\alpha K_1(\alpha), \qquad (12)$$

where $K_1(x)$ is the modified Bessel function of the second kind. At small α Eq. (12) reproduces the above BA result while at strong coupling it predicts the MFP to be $l \approx \xi\sqrt{2\pi/\alpha}$.

Thus the condition (11) actually provides that $k_F l \gg 1$ which is necessary to verify the very use of the kinetic equation (1) and to obtain the metallic value of the conductivity.

In the intermediate coupling regime relevant for the case of HFLL the condition (11) is fairly well met ($\alpha = 2$ and $k_F\xi \approx 15$) and Eq. (12) gives the CF conductivity $\sigma_{xx}^{\rm CF} = (e^2/h)(k_F\xi)e^2K_1(2)$ which is 2.06 times greater than the BA result. It is worthwhile mentioning that the experimentally measured resistivity at $\nu = 1/2$ is about 3 times smaller than the BA estimate [3].

The strong coupling behavior of the MFP ($l \sim \xi/\sqrt{\alpha}$ as opposed to $l \sim \xi/\alpha$ at weak coupling) can already be seen in the self-consistent BA which in the case of a finite-range potential scattering is applicable at $\xi \ll B^{-1/2}$. Adapting this method to the RMF problem one can achieve a self-consistent improvement of the lowest order result by inserting a factor $e^{-\tau/\tau_{\rm tr}}$ into the integrand in (10) and solving the resulting nonlinear equation for $\tau_{\rm tr}$.

For the case of HFLL this equation reads as

$$1/\tau_{\rm tr} = \frac{2\xi\alpha}{v_F}\int_0^\infty d\tau\, e^{-\tau/\tau_{\rm tr}}\left[\tau^2 + \left(\frac{2\xi}{v_F}\right)^2\right]^{-3/2} \qquad (13)$$

and gives the MFP $l(\alpha)$ in agreement with the above eikonal calculation. In terms of the conventional diagrammatics Eq. (13) corresponds to the sum of uncrossed "rainbow" diagrams for the bosonic self-energy. By contrast, the eikonal result (8) includes contributions of the relevant crossed diagrams in all orders of the perturbation theory.

In [10] the MR of the RMF problem was identified as the B dependence of $\tau_{\rm tr}$. It is obvious, however, that in the absence of other mechanisms of momentum relaxation one cannot use Eq. (10) in order to obtain the MR at $\Omega_c\tau_{\rm tr} < 1$.

Moreover, in the HFLL case the long-range character of $F(r) \sim r^{-3}$ leads to a logarithmic divergency of the second derivative of $\rho_{xx}(B)$ at $B \to 0$, which is, of course, an artifact of the above expansion. In the framework of the more accurate self-consistent BA one obtains that in the weak field limit $\tau_{\rm tr}(B)$ is an increasing function of B which implies a negative MR. Our self-consistent analysis shows that the decreasing behavior of $\tau_{\rm tr}(B)$ (positive MR) in the case of $f(q) \sim e^{-(1/2)q^2\xi^2}$ and small α reported in [10], in fact, holds only for large enough fields, $\Omega_c > 1/\tau_{\rm tr}(1/\ln 1/\alpha^2)$.

The more complete eikonal treatment of the RMF problem confirms this prediction. Expanding (9) up to the second order in Ω_c we obtain the MR defined as $\Delta\rho_{xx}(B)/\rho_{xx} = -[\Delta\sigma_{xx}(B)/\sigma_{xx}] - [\sigma_{xy}(B)/\sigma_{xx}]^2$ in the form

$$\frac{\Delta\rho_{xx}(B)}{\rho_{xx}} = \frac{\Omega_c^2}{2\langle 1\rangle}\left(\langle\tau^2\rangle - 2\frac{\langle\tau\rangle^2}{\langle 1\rangle} + \frac{1}{12}\left\langle\int_0^\tau d\tau'\,(3\tau'^2\tau - 4\tau'^3)F(\tau')\right\rangle\right), \tag{14}$$

where $\langle\tau^n\rangle = \int_0^\infty d\tau\,\tau^n \exp[-\int_0^\tau d\tau'(\tau - \tau')F(\tau')]$.

Notice that in the relaxation time approximation ($\langle\tau^n\rangle = \int_0^\infty d\tau\,\tau^n e^{-\tau/\tau_{\rm tr}}$) expression (14) contains only two terms proportional to $\langle\tau^2\rangle$ and $\langle\tau\rangle^2$ which exactly cancel out and result in zero MR. Since the exponential factor $\int_0^\tau d\tau'(\tau - \tau')F(\tau')$ appearing in all our calculations instead of $\tau/\tau_{\rm tr}$ behaves as $\sim\tau^2$ at small τ, the combined effect of these terms on the MR is negative and can dominate over the remaining (strictly positive) contribution.

The MR remains negative for all couplings in a wide class of realistic RMF correlation functions $f(q)$ including the HFLL case at $\alpha = 2$ when the Eq. (14) yields

$$\frac{\Delta\rho_{xx}(B)}{\rho_{xx}} = -0.06(\Omega_c\tau_{\rm tr})^2, \tag{15}$$

while in the strong coupling limit of large α the MR becomes $\Delta\rho_{xx}(B)/\rho_{xx} = (\Omega_c\tau_{\rm tr})^2(\pi/4 - 1)$.

Equation (9) also suggests that the MR may change sign at $\Omega_c\tau_{\rm tr} \sim 1$ and remain positive up to $\Omega_c\tau_{\rm tr} \approx 8$ although our semiclassical analysis becomes insufficient at such high fields.

In the existing artificial realizations of the RMF [1] the random field depends on the applied external one. Namely, the RMF correlation function $f(q)$ appropriate for the references (a) and (c) from [1] is proportional to B while the one relevant for the reference (b) is $\sim B^2$. In either case this trivial B dependence necessarily leads to a positive MR and does not allow one to explore the MR (14) which is the only one present in the case of $f(q)$ independent of B.

The quadratic positive MR was also reported in a numerical simulation of the lattice version of the RMF modeled as an assembly of uncorrelated random fluxes [14]. Again, the anisotropy of the lattice fermion dispersion and the obvious lack of the large parameter $k_F\xi$ in the short-range case preclude us from making a direct comparison with our results.

In the case of HFLL one may also think of the broad minimum of $\rho_{xx}(B)$ at $\nu = 1/2$ suggesting a positive MR of CF as resulting from the interference between the RMF scattering of CF and their residual gauge interactions. It was recently shown [15] that this interference can explain the strong nonuniversal $\ln T$ correction to $\rho_{xx}(T)$ observed at $\nu = 1/2$ and $3/2$ [16].

Indeed, it turns out that by contrast to the case of ordinary Coulomb interacting spin-polarized electrons [17] the analysis of the interference correction to the CF conductivity at finite $B_{\rm eff} = B - 2\pi\Phi n_e$ reveals a positive MR, which appears to be greater than the negative RMF contribution (15). The details will be presented elsewhere [18].

To summarize, we develop a new method of solving the semiclassical Boltzmann equation for the RMF problem beyond the relaxation time approximation provided $k_F l \gg 1$. The obtained solution is used to calculate the RMF transport time, mean free path, and zero-field conductivity which all appear to be greater than the corresponding results of the Born approximation. In particular, we propose a new estimate of the semiclassical conductivity at HFLL which is about twice the value found in [3] and agrees better with the experimental data. In the absence of interactions, Fermi surface and fermion dispersion anisotropy, periodic electron density modulations, and in-plane fields (all these factors are known to yield positive contributions to the MR) we obtain the negative quadratic MR in the weak field limit. It remains to be understood, however, whether or not the semiclassically found negative MR is related to the quantum localization phenomena in the RMF.

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[1] (a) A.K. Geim *et al.*, Phys. Rev. B **49**, 5749 (1994); (b) A. Smith *et al.*, Phys. Rev. B **50**, 14 726 (1994); (c) P.B. Mancoff *et al.*, Phys. Rev. B **51**, 13 269 (1995).

[2] L.B. Ioffe and A.I. Larkin, Phys. Rev. B **39**, 8988 (1989); N. Nagaosa and P.A. Lee, Phys. Rev. Lett. **64**, 2450 (1990); Phys. Rev. B **45**, 966 (1992).

[3] B.I. Halperin, P.A. Lee, and N. Read, Phys. Rev. B **47**, 7312 (1993).

[4] V. Kalmeyer and S.-C. Zhang, Phys. Rev. B **46**, 9889 (1992).

[5] T. Sugiyama and N. Nagaosa, Phys. Rev. Lett. **70**, 1980 (1993); D.K.K. Lee and J.T. Chalker, Phys. Rev. Lett. **72**, 1510 (1994).

[6] J. Miller and J. Wang, Phys. Rev. Lett. **76**, 1461 (1996).

[7] A.G. Aronov, A.D. Mirlin, and P. Wolfle, Phys. Rev. B **49**, 16 609 (1994).

[8] B.L. Altshuler and L.B. Ioffe, Phys. Rev. Lett. **69**, 2979 (1992); D.V. Khveshchenko and S.V. Meshkov, Phys. Rev. B **47**, 12 051 (1993).

[9] A.D. Mirlin, E. Altshuler, and P. Wolfle, Ann. Phys. **5**, 281 (1996).

[10] P. Hedegard and A. Smith, Phys. Rev. B **51**, 10 869 (1995).

[11] Y.B. Kim, P.A. Lee, and X.-G. Wen, Phys. Rev. B **52**, 17 275 (1995).

[12] F.D.M. Haldane, *Varenna Lectures*, edited by R. Schrieffer and P.A. Broglia (North-Holland, New York, 1994), Vol. 121.

[13] L.D. Landau and E.M. Lifshitz, *Quantum Mechanics* (Pergamon, New York, 1977).

[14] V. Kalmeyer, D. Wei, D. Arovas, and S.C. Zhang, Phys. Rev. B **48**, 11 095 (1993).

[15] D.V. Khveshchenko, Phys. Rev. Lett. **77**, 362 (1996).

[16] L.P. Rokhinson *et al.*, Phys. Rev. B **52**, R11 588 (1995); W. Kang *et al.*, Phys. Rev. Lett. **75**, 4106 (1995); R. Haug *et al.* (unpublished).

[17] P.A. Lee and T.V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).

[18] D.V. Khveshchenko, Report No. cond-mat/9605051.