Optics Communications 282 (2009) 1998–2008

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Generation of TE- and TH-polarized Bessel beams using one-dimensional photonic crystal

V.N. Belyi, N.S. Kazak, S.N. Kurilkina *, N.A. Khilo

B.I. Stepanov Institute of Physics of NAS Belarus, Laboratory of Optical Diagnostics, 68 Independence Ave., Minsk 220072, Belarus

ARTICLE INFO

Article history:
Received 24 November 2008
Received in revised form 5 February 2009
Accepted 6 February 2009

Keywords:
Bessel light beams
TH- and TE-polarized modes
One-dimensional photonic structure

ABSTRACT

Based on the matrix method, a theory of propagation of TE- and TH-polarized Bessel light beams (BLBs) in a one-dimensional photonic crystal (1DPC) is developed. The transmission through a 1DPC (with and without a defect impurity) of a quasi-circularly-polarized incident Bessel beam generated by an axicon from a circularly-polarized Gaussian beam has been calculated and analyzed. Also a solution of the problem on the transmission of BLBs through crystalline plate (layer of a uniaxial crystal with the orientation of the optical axis orthogonally to its interfaces) and reflection from it has been presented.

Based on this, a new method of formation of TE- and TH-polarized Bessel light beams has been proposed. It has been shown that it is possible to control this process by changing the cone angle of an incident Bessel light beam. The effect of generation of a coherent superposition of two Bessel beams with different cone angles in the case of a high birefringence of defect layer has been predicted theoretically.
© 2009 Elsevier B.V. All rights reserved.

1. Introduction

Bessel light beams (BLBs) suggested in 1987 [1] have attracted considerable interest due to their properties of propagation invariance and self-reconstruction, which have found applications in different fields of science and technology, for example, for optical trapping and manipulation of microparticles and atoms [2–6]. More investigated from both theoretical and experimental viewpoints (see, for example, [1,7–11]) are scalar BLBs. Recently, attention has been concentrated on the vector Bessel light beams of order m, which are rotating wave solutions of the homogeneous Maxwell equations in the circular cylindrical coordinates and are associated with the dependence on the azimuthal angle [12–14].

One of important directions of optics of Bessel beams is the investigation of their polarization properties and elaboration of methods of forming the beams having a definite polarization [13,15–17]. In particular, the polarization and space-energetic characteristics of BLBs [18], the specific azimuthal component of energy flux [19], properties of influence of anisotropy and gyrotropy on vector Bessel beams [20,21] are of great scientific interest.

If the component of the electric (magnetic) field along the axis of beam is absent, it is called TE (TH) mode. Some dynamic and propagation invariance properties of these beams are analyzed in papers [13,22,23]. The lowest-order TH and TE BLBs are radially (ρ) and azimuthally (φ)-polarized, respectively. Their experimental realizations in free space have been reported recently [24]. Note, that the beams with TE or TH polarization have variety of applications. It is established that their focusing allows one to obtain a higher axis concentration of the electrical and correspondingly magnetic fields in comparison with the case of linear or circular polarization [25]. Consequently, such beams are promising in photolithography, confocal microscopy, devices of optical record-reading of information. TH and TE BLBs with large cone angle have a high value of the transverse gradient of intensity and possibilities of reconstruction of the spatial field configuration. Hence, they show good promise for the microparticles keeping and management of their movement [26–29], and also for the transportation of laser radiation energy in the open space and in hollow light guides. TH and TE beams are optimal for the use in different schemes of probing the cylindrical objects, for example, in profilometry [30,31]. Moreover, in the last years the outlook of their use for laser cutting has been established [32].

By now several methods of producing TE- and TH-polarized light fields have been proposed. The simplest method is the use of the polarization property of the conical surface when light is incident at the Brewster's angle [33]. But for the complete separation of TE- and TH-polarized component there is necessity of the repeated passage of light through the conical surface that makes the scheme more complex. TH-polarized field can be obtained by the method of the mode converter due to the formation of a superposition of two orthogonally polarized Hermit–Gaussian modes of the first-order [34]. A number of intra-cavity methods of obtaining TH- and TE-polarized modes for traditional (for example, Gaussian) beams are known (see, for example, [35–38]).

* Corresponding author. Tel.: +375 17 284 04 39; fax: +375 17 284 08 79.
E-mail address: s.kurilkina@ifanbel.bas-net.by (S.N. Kurilkina).

0030-4018/$ - see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.optcom.2009.02.020

Recently the experimental generation has been demonstrated of the lowest- and first-order Bessel light beams with TE and TH polarization in free space by means of a Mach–Zender interferometer [39]. But the measured optical efficiency for the output vector BLB was about several percents. Unfortunately, all mentioned methods do not permit one to achieve a high efficiency.

One of the effective methods of transformation of polarization of vector BLB is based on the use of anisotropic uniaxial and biaxial crystals [40,41]. By using this method, the transformers have been developed, which convert the Bessel zero-order function into the first- and second-order one. It is also shown that gyrotropic crystals [42] allow one to transform mutually TH and TE polarization types [43].

Anisotropic crystals in a combination with one-dimensional photonic crystal (1DPC) allow one also to solve the problem of the direct formation of TH- and TE-polarized Bessel beams. 1DPCs, exhibiting electromagnetic stop bands for the photon propagation over a wide band of frequencies, are widely used while creating a polarizer for traditional (for example, Gaussian) beams [44–48]. Recently the possibility is shown of generating Bessel light beams with the help of a resonant point source, located on the side of one-dimensional photonic crystal with a defect inclusion [49]. These modes that are coupled with the localized modes supported by the one-dimensional photonic crystal are selectively transmitted. This is used to produce a single narrow band of transmission, which, combined with the circular symmetry of the system, yields a propagating Bessel beam.

In this report (Section 2) the problem will be solved of developing an exact, analytical expressions for vector Bessel light beams in a finite, N-period, one-dimensional photonic crystal, in terms of the complex transmission coefficient of a unit cell of the structure. As an example, we shall consider a 1DPC made from periodic, multilayered dielectric stacks. Particular attention will be given to Bessel light beams in a one-dimensional photonic crystal having a defect impurity – a crystalline layer. Results of numerical simulations, illustrated the properties of BLB transmission through one-dimensional photonic crystal (with and without the defect crystalline layer), will be presented in Section 3.

In Section 4 a new method of formation of TE- and TH-polarized Bessel beams will be suggested. It is based on the transformation of a circular-polarized zero-order J_0 Bessel beam into superposition of the TH and TE Bessel modes mutually splitting in the space. The initial J_0 BLB may be obtained from the basic laser mode by the usual method, for example, by an axicon [50].

2. Description of transformation of a vector Bessel light beam in one-dimensional photonic crystal

Let us consider the transformation of vector BLBs in a one-dimensional photonic crystal containing at its center a defect in the form of a plate of a uniaxial crystal. It is assumed that 1DPC is surrounded by a dielectric with the refractive index n_0 (in particular case, by air). In front of the entrance face of this structure an axicon is placed which is illuminated by a circularly-polarized a well-collimated Gaussian – or single-ringed Laguerre–Gaussian beam with azimuthal index m (LG_0^m). This beam, when passed through the axicon, is transformed into a circularly-polarized J_0 – or J_m vector BLB. The origin of the coordinated system $z = 0$ is assumed to be chosen at the interface between the dielectric n_0 and the first elementary cell of the photonic crystal (Fig. 1).

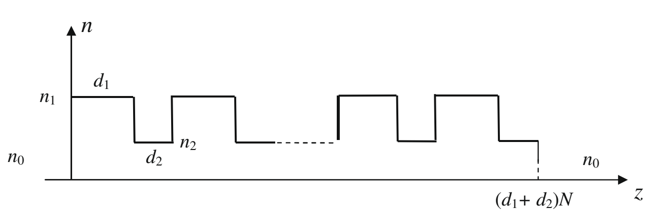

Fig. 1. Example: one-dimensional photonic crystal with a N-period stack composed of two-layer unit cells of thickness d_1 and d_2 and refractive indices n_1 and n_2, respectively.

2.1. Transmittance and reflectance of the vector Bessel light beam at the boundary of two dielectrics

At first, the peculiarity of the reflection and transmission of BLBs on the boundary "air-dielectric" will be analyzed. Let a normally incident arbitrary polarized vector Bessel beam falls from the air on the boundary of an isotropic nonmagnetic material with the refractive index n. Let the direction of propagation coincide with z-axis of coordinate system. It is known (see, for example, [1,22]) that the electromagnetic field propagating in a medium can be represented as a superposition of TE ($E_z = 0$) and TH ($H_z = 0$) modes:

$$\vec{E}(R) = A^{TE}\vec{E}_{TE}(R) + A^{TH}\vec{E}_{TH}(R),$$
$$\vec{E}_{TE,TH}(R) = \vec{E}^{(TE,TH)}(\rho,\varphi)\exp i(k_z z + im\varphi),$$
$$\vec{H}_{TE,TH}(R) = \vec{H}^{(TE,TH)}(\rho,\varphi)\exp i(k_z z + im\varphi), \tag{1}$$

where $R = (\rho, \varphi, z)$ are the cylindrical coordinates; m is the integer; k_z is the z-projection of the wave vector. It follows from the solution of Maxwell's equations that the components of the vector amplitudes $\vec{E}^{(TE,TH)}(\rho,\varphi)$, $\vec{H}^{(TE,TH)}(\rho,\varphi)$ are represented as:

$$E_\rho^{TE} = \frac{im}{q\rho}J_m(q\rho);\quad E_\varphi^{TE} = -J'_m(q\rho);\quad E_z^{TE} = 0;$$
$$E_\rho^{TH} = i\cos\gamma J'_m(q\rho);\quad E_\varphi^{TH} = -\cos\gamma\frac{m}{q\rho}J_m(q\rho);\quad E_z^{TH} = \sin\gamma J_m(q\rho).$$

$$H_\rho^{TE} = n\cos\gamma J'_m(q\rho);\quad H_\varphi^{TE} = in\cos\gamma\frac{m}{q\rho}J_m(q\rho);\quad H_z^{TE} = -in\sin\gamma J_m(q\rho);$$
$$H_\rho^{TH} = n\frac{m}{q\rho}J_m(q\rho);\quad H_\varphi^{TH} = inJ'_m(q\rho);\quad H_z^{TH} = 0. \tag{2}$$

Here $J_m(q\rho), J'_m(q\rho) = \partial J_m(q\rho)/\partial(q\rho)$ are the m-order Bessel functions and their derivatives, respectively, $q = \sqrt{(\omega/c)^2n^2 - k_z^2} = (\omega/c)n\sin\gamma$ is the parameter of conicity (transversal component of wave vector, which is unchanged during the crossing of the boundary of two media), ω is the frequency of the incident Bessel beam, c is the light velocity in vacuum, γ is the half-cone angle of BLB in the medium.

For excitation of fields describing by Eq. (2), which are characterized by the homogeneous azimuthal intensity distribution, the incident Bessel beam must be circularly-polarized. In the common case of vector BLBs this condition is satisfied when the beams are formed by the axicon from circularly-polarized Laguerre–Gaussian beam (LG_0^m). Then the transverse component $\vec{E}^{\pm}_{\perp i}(R)$ of the electrical vector of the incident Bessel beam has the following form [51]:

$$\vec{E}^{\pm}_{\perp i}(R) = iA\left[(\vec{e}_\rho \pm i\cos\gamma_i\vec{e}_\varphi)\frac{m}{q\rho}J_m(q\rho) \pm (\cos\gamma_i\vec{e}_\rho \pm i\vec{e}_\varphi)J'_m(q\rho)\right] \times \exp\left[i\left(\frac{\omega}{c}\cos\gamma_i z + m\varphi\right)\right]. \tag{3}$$

Here $\vec{e}_{\rho,\varphi}$ are the unit vectors of the cylindrical coordinate system, A is the constant amplitude factor, signs "+" and "−" corresponds to the right and left circular polarization of Laguerre–Gaussian beam falling to axicon, respectively, γ_i is the half-cone angle of the incident light Bessel beam formed by the axicon. For definition below we shall suppose that the field, formed behind the axicon, is right-polarized. Then, comparing Eqs. (2) and (3), we obtain:

$$\vec{E}^{+}_{\perp i}(R) = A\left[\vec{E}^{TE}_{i}(\rho,\varphi) + \vec{E}^{TH}_{i}(\rho,\varphi)\right]\exp\left[i\left(\frac{\omega}{c}\cos\gamma_i z + m\varphi\right)\right]. \quad (4)$$

Here the components of vectors $\vec{E}^{TE,TH}_{i}(\rho,\varphi)$ of the incident Bessel beam are determined by Eq. (2), where it should be assumed $n = 1$ for an air. It follows from Eq. (2) that for the field (4) z-projection of the energy flux is the sum of two equal components corresponding to TE and TH modes. Then, while comparing Eqs. (4) and (1), one can see that for the incident Bessel beam (3) the conditions $A^{TE} = A^{TH} = A$ are fulfilled.

Taking into account the relations:

$$\frac{m}{q\rho}J_m(q\rho) = \frac{1}{2}[J_{m-1}(q\rho) + J_{m+1}(q\rho)];$$
$$J'_m(q\rho) = \frac{1}{2}[J_{m-1}(q\rho) - J_{m+1}(q\rho)],$$

we can represent vectors $\vec{E}^{TE,TH}_{i}(\rho,\varphi)$ for incident field (4) in the form:

$$\vec{E}^{TE}_{i}(\rho,\varphi) = \frac{i}{\sqrt{2}}[J_{m-1}(q\rho)\vec{e}_{+} + J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_{-}]\exp(-i\varphi),$$
$$\vec{E}^{TH}_{i}(\rho,\varphi) = \frac{i}{\sqrt{2}}\cos\gamma_i[J_{m-1}(q\rho)\vec{e}_{+} - J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_{-}]\exp(-i\varphi).$$

Then Eq. (4) can be rewritten:

$$\begin{aligned}\vec{E}^{+}_{\perp i}(R) = \frac{iA}{\sqrt{2}}\exp i[(m-1)\varphi + \frac{\omega}{c}\cos\gamma_i z] \times \{[J_{m-1}(q\rho)\vec{e}_{+} \\ + J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_{-}] + \cos\gamma_i[J_{m-1}(q\rho)\vec{e}_{+} \\ - J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_{-}]\},\end{aligned} \quad (5)$$

where $\vec{e}_{\pm} = (\vec{e}_1 \pm i\vec{e}_2)/\sqrt{2}$ are the unit circular vectors.

In similar manner, we can represent the transverse component $\vec{E}_{\perp t}(R)$ of the field (1) refracted into the medium having refractive index n in the form:

$$\begin{aligned}\vec{E}_{\perp t}(R) &= [A^{TE}_t\vec{E}^{TE}_t(\rho,\varphi) + A^{TH}_t\vec{E}^{TH}_t(\rho,\varphi)]\exp[i(k_z z + m\varphi)] \\ &= \frac{i}{\sqrt{2}}\exp i[(m-1)\varphi + k_z z] \times \{A^{TE}_t[J_{m-1}(q\rho)\vec{e}_{+} \\ &\quad + J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_{-}] + A^{TH}_t\cos\gamma_t[J_{m-1}(q\rho)\vec{e}_{+} \\ &\quad - J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_{-}]\}.\end{aligned} \quad (6)$$

Here the components of vectors $\vec{E}^{TE,TH}_{t}(\rho,\varphi)$ are determined by Eq. (2); $A^{TE,TH}_t$ are amplitude factors; index "t" indicates the refracted BLB; $\cos\gamma_t = (1 - \sin^2\gamma_i/n^2)^{1/2}$, $k_z = (\omega/c)n\cos\gamma_t$. Let us further introduce amplitude transmission coefficients $t^{TE,TH} = A^{TE,TH}_t/A$ for TE and TH BLBs, respectively. Finally, the electric vector of the refracted Bessel beam can be represented in the form:

$$\begin{aligned}\vec{E}_{\perp t}(R) = \frac{iA}{\sqrt{2}}\exp i[(m-1)\varphi + k_z z] \times \{t^{TE}[J_{m-1}(q\rho)\vec{e}_{+} \\ + J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_{-}] + t^{TH}\cos\gamma_t[J_{m-1}(q\rho)\vec{e}_{+} \\ - J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_{-}]\}.\end{aligned} \quad (7)$$

In similar manner, for the transversal component of reflected field (it is indicated by index "r") we obtain:

$$\begin{aligned}\vec{E}_{\perp r}(R) = \frac{iA}{\sqrt{2}}\exp i[(m-1)\varphi - \frac{\omega}{c}\cos\gamma_i z] \times \{r^{TE}[J_{m-1}(q\rho)\vec{e}_{+} \\ + J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_{-}] - r^{TH}\cos\gamma_i[J_{m-1}(q\rho)\vec{e}_{+}] \\ - J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_{-}]\}.\end{aligned} \quad (8)$$

Here $r^{TE,TH} = A^{TE,TH}_r/A$ are the amplitude reflection coefficients.

2.2. Boundary problem for Bessel light beam (the case of the boundary of two dielectrics)

There is need to determine the transmission and reflection coefficients for TE and TH vector BLB of m-order, which enter into Eqs. (7) and (8). For this aim, let us consider the normal incident BLB of an arbitrary polarization on the interface of two isotropic media with the refractive indices n_i and n_j. The incident, reflected and refracted waves are to satisfy the boundary conditions of continuity of the tangential components of the electric and magnetic fields.

$$\begin{aligned}E^i_\rho + E^r_\rho = E^t_\rho; \quad H^i_\rho + H^r_\rho = H^t_\rho; \\ E^i_\varphi + E^r_\varphi = E^t_\varphi; \quad H^i_\varphi + H^r_\varphi = H^t_\varphi.\end{aligned} \quad (9)$$

Here *i, r and t* designate the incident, reflected and refracted optical fields, respectively. Substituting the field components from (2) into (13) and equating the coefficients at the same Bessel functions, the system of scalar equations for the amplitudes $A^{TE,TH}_i$, $A^{TE,TH}_r$ and $A^{TE,TH}_t$ of the incident, reflected and refracted waves of TH- and TE-polarization, can be written in the form:

$$\cos\gamma_i(A^{TH}_i - A^{TH}_r) = \cos\gamma_j A^{TH}_t, \quad n_i(A^{TH}_i + A^{TH}_r) = n_j A^{TH}_t,$$

$$A^{TE}_i + A^{TE}_r = A^{TE}_t, \quad n_i\cos\gamma_i(A^{TE}_i - A^{TE}_r) = n_j\cos\gamma_j A^{TE}_t, \quad (10)$$

where γ_i is the half-cone angle of BLB in the medium with the refractive index n_i.

From Eq. (10) the expression for amplitude coefficients of transmission $t^{TE,TH}_{ij} = A^{TE,TH}_t/A^{TE,TH}_i$ and reflection $r^{TE,TH}_{ij} = A^{TE,TH}_r/A^{TE,TH}_i$ can be obtained:

$$t^{TE}_{ij} = \frac{2n_i\cos\gamma_i}{n_i\cos\gamma_i + n_j\cos\gamma_j}, \quad r^{TE}_{ij} = \frac{n_i\cos\gamma_i - n_j\cos\gamma_j}{n_i\cos\gamma_i + n_j\cos\gamma_j}, \quad (11)$$

$$t^{TH}_{ij} = \frac{2n_i\cos\gamma_i}{n_i\cos\gamma_j + n_j\cos\gamma_i}, \quad r^{TH}_{ij} = \frac{n_j\cos\gamma_i - n_i\cos\gamma_j}{n_j\cos\gamma_i + n_i\cos\gamma_j}, \quad (12)$$

where the refractive indices n_i and n_j are related by the law $n_i\sin\gamma_i = n_j\sin\gamma_j$. From Eqs. (11) and (12), it is seen that the coefficients of reflection and refraction of Bessel TE- and TH-beams of the m-order coincide with ones for plane waves polarized, correspondingly, transversely and parallel to the plane of incidence.

It should be noted that coefficients $t^{TE,TH}$, $r^{TE,TH}$ entering into Eqs. (7) and (8) are determined by the Eqs. (11) and (12), in which it is necessary to perform the following substitutions: $n_i \to 1$, $n_j \to n$, and $\gamma_j \to \gamma_t$.

2.3. Description of BLB propagation in one-dimensional photonic crystal

Let us consider now the propagation of the BLBs in a one-dimensional photonic crystal without a defect layer. Its special case is a multilayer stack containing *N-periods* of dielectric materials. Each period has layers with a high n_1 and a low n_2 refractive indices (Fig. 1). Such a structure exhibits an electromagnetic stop band for the photon propagation over a wide range of frequencies. We will consider a normally incident circularly-polarized Bessel beam (3) that impinges from the medium with the refractive index n_0 on such a structure. As BLB falls on 1DPC from the medium with the n_0 refractive index, in Eq. (3) it is necessary to make a replacement $(\omega/c)\cos\gamma_i \to k_{oz} = [(\omega/c)^2 n_0^2 - q^2]^{1/2}$. As mentioned above, the origin of the used coordinate system, for which the z-axis is collinear to the direction of periodicity, is situated at the interface between dielectric n_0 and the first elementary cell of the photonic crystal.

2.3.1. Transformation of BLB into a unit cell of one-dimensional photonic crystal

First of all we will consider the properties of transformation of Bessel beam into a unit cell with a thickness $d = d_1 + d_2$ (Fig. 1) which is surrounded by a medium having refractive index n_2. We shall present electric vector $\vec{E}$ (TE or TH BLB) in the unit cell as a superposition of vectors corresponding to forward and backward propagating Bessel beams, labeled plus and minus, respectively [52]:

$$\vec{E} = \begin{pmatrix} E^+ \\ E^- \end{pmatrix}. \tag{13}$$

As a consequence of periodicity of the structure,

$$\vec{E}(0) = M\vec{E}(d), \tag{14}$$

where M is the transfer matrix. Taking into account the boundary conditions which can be represented in the form:

$$\vec{E}(0) = \begin{pmatrix} 1 \\ r_{cell} \end{pmatrix}, \quad \vec{E}(d) = \begin{pmatrix} t_{cell} \\ 0 \end{pmatrix}, \tag{15}$$

where r_{cell} and t_{cell} are the complex coefficients of reflection and transmission of the unit cell, respectively, and the time-reversal symmetry of the transfer matrix, that is the consequence of the assumptions of energy conservation, one can obtain:

$$M = \begin{pmatrix} 1/t_{cell} & r^*_{cell}/t^*_{cell} \\ r_{cell}/t_{cell} & 1/t^*_{cell} \end{pmatrix}. \tag{16}$$

Here the asterisk $(*)$ denotes the complex conjugation. It follows from (16), that the general form of the transfer matrix for Bessel beams is similar to the same obtained in [52] for the case of a plane wave propagating in a one-dimensional photonic crystal.

In analogy to the case of plane wave, the inverse transfer matrix, describing the transformation of BLB propagating left to right, is represented as a product of matrices of only two types:

$$M^{-1} = \Pi(p_2)\Delta_{12}\Pi(p_1)\Delta_{21}. \tag{17}$$

Here Δ_{ij} is a discontinuity matrix describing the transformation of BLB on left to right across the $n_i \to n_j$ interface; the propagation matrix $\Pi(p_i)$ is describing the phase change of Bessel beam in a layer with the refractive index n_i.

$$\Pi(p_i) = \begin{pmatrix} e^{ip_i} & 0 \\ 0 & e^{-ip_i} \end{pmatrix}, \tag{18}$$

where $p_i = \frac{\omega}{c} n_i d_i \cos\gamma_i$, γ_i is the half-cone angle of BLB propagating in the medium with the refractive index n_i. From Eq. (18) it follows that $\Pi(p_i)\Pi(-p_i) = I$, where I is the unit matrix.

Let $z = z_0$ be at the $n_i \to n_j$ interface. Representing the boundary conditions in the form:

$$\vec{E}(z_0 - 0) = \begin{pmatrix} 1 \\ r_{ij} \end{pmatrix}, \quad \vec{E}(z_0 + 0) = \begin{pmatrix} t_{ij} \\ 0 \end{pmatrix}, \tag{19}$$

where $\vec{E}(z_0 - 0) = \Delta_{ij}\vec{E}(z_0 + 0)$, and taking into account the time-reversal symmetry of Δ_{ij}, that is the consequence of the law of energy conservation, one can obtain:

$$\Delta_{ij} = \begin{pmatrix} a^+_{ij} & a^-_{ij} \\ a^-_{ij} & a^+_{ij} \end{pmatrix}, \qquad a^+_{ij} = 1/t_{ij}, \quad a^-_{ij} = r_{ij}/t_{ij}, \tag{20}$$

where coefficients r_{ij} and t_{ij} in Eq. (20) are determined by Eqs. (11) and (12) for TE and TH Bessel beams, respectively. As is evident from Eq. (20), the Δ_{ij} matrices have a number of useful properties: commutativity ($\Delta_{ij}\Delta_{kl} = \Delta_{kl}\Delta_{ij}$), transitivity ($\Delta_{ij}\Delta_{jk} = \Delta_{ik}$) and identity ($\Delta_{ij}\Delta_{ji} = I$).

Using the main properties of matrices Δ_{ij} and $\Pi(p_i)$, and relation

$$(AB)^{-1} = B^{-1}A^{-1}, \tag{21}$$

where A and B are arbitrary matrices, we can express the transfer matrix M in the following form:

$$M = \Delta_{12}\Pi(-p_1)\Delta_{21}\Pi(-p_2). \tag{22}$$

After substituting Eqs. (18) and (20) into Eq. (22), it is easy to find the elements of matrix M and then, using Eq. (16), to determine transmission t_{cell} and reflection r_{cell} coefficients for the unit cell:

$$t_{cell} = \frac{T\exp[i(p_1 + p_2)]}{1 - R\exp(2ip_1)}, \quad r_{cell} = r_{12}\frac{1 - \exp(2ip_1)}{1 - R\exp(2ip_1)}. \tag{23}$$

The quantities T and R in Eq. (23) are

$$T = t_{12}t_{21}, \quad R = |r_{12}r_{21}|, \tag{24}$$

where coefficients t_{ij} and r_{ij} are determined by expressions (11) and (12). Finally, one can obtain the quantities T and R for TE BLBs:

$$T = \frac{4n_1n_2\cos\gamma_1\cos\gamma_2}{(n_1\cos\gamma_1 + n_2\cos\gamma_2)^2}; \quad R = \left|\frac{n_1\cos\gamma_1 - n_2\cos\gamma_2}{n_1\cos\gamma_1 + n_2\cos\gamma_2}\right|^2. \tag{25}$$

For TH BLB modes, the transmission and reflection coefficients of the unit cell are also determined by Eqs. (23)–(25), but in Eq. (25) we should replace $n_i \to 1/n_i$.

2.3.2. Propagation of BLB in infinite one-dimensional photonic crystal

Let us consider now the BLBs propagating in an infinite periodical medium. It is known that properties of the light waves transformation in such a medium are described by the Bloch functions changing only the phase by the value $\mu_b = \exp(\pm i\beta)$ from cell to cell [53]. As a consequence, μ_b is an eigenvalue of the transfer matrix M. On the other hand, it follows from (16) that the eigenvalues μ of the matrix M are determined from the equation:

$$\mu^2 - 2\mu\mathrm{Re}(1/t_{cell}) + 1 = 0. \tag{26}$$

Inserting μ_b into Eq. (26), and equating the real and imaginary terms, we arrive at the very important relation

$$\cos\beta = \frac{1}{2}SpM = \mathrm{Re}(1/t_{cell}). \tag{27}$$

According to Eq. (27), the frequency region of the pass band is determined by the condition $|\cos\beta| < 1$, and, as a consequence, $|(1/2)SpM| < 1$, or $\mathrm{Re}(1/t_{cell}) < 1$. If $|(1/2)SpM| > 1$, or $\mathrm{Re}(1/t_{cell}) > 1$, the value β is imaginary and forbidden bands for BLBs are observed. It follows from Eq. (27), with allowance for account Eq. (23), that for the case of small half-cone angles and at the fulfilled condition $n_1d_1 = n_2d_2 = \lambda_0/4$, where λ_0 is the wavelength corresponding to the midgap frequency, the width of the band gap is determined approximately by the expression $\Delta\lambda \approx 4\lambda_0|n_2 - n_1|/(\pi(n_2 + n_1))$.

At that, owing to difference of t_{cell} for TE and TH modes, the edges of band gaps for these BLBs are not coincided.

2.3.3. Propagation of BLB in finite one-dimensional photonic crystal

From the Cayley–Hamilton theorem [54] it follows:

$$M^N = \frac{1}{\sin\beta}[M\sin N\beta - I\sin(N-1)\beta]. \tag{28}$$

Here N is the number of unit cells. Then, representing M^N by means of the transmission t_N and reflection r_N coefficients, as in the case of Eq. (16),

$$M^N = \begin{pmatrix} 1/t_N & r^*_N/t^*_N \\ r_N/t_N & 1/t^*_N \end{pmatrix}, \tag{29}$$

and using Eq. (28), one can obtain the expressions for a periodical medium having N unit cells:

$$\begin{aligned} \frac{1}{t_N} &= \frac{1}{t_{cell}\sin\beta}(\sin N\beta - t_{cell}\sin(N-1)\beta), \\ \frac{r_N}{t_N} &= \frac{r_{cell}}{t_{cell}}\frac{\sin N\beta}{\sin\beta}, \end{aligned} \tag{30}$$

where the quantity β is determined by Eq. (27), and the quantities r_{cell} and t_{cell} are determined by Eq. (23).

Taking into account the form of the transfer matrix M for the unit cell (see (22)) and the property of transitivity $\Delta_{02}\Delta_{21} = \Delta_{01}$ and commutativity $\Delta_{ij}\Delta_{kl} = \Delta_{kl}\Delta_{ij}$ of Δ_{ij} matrix, in the case of a finite one-dimensional photonic crystal, containing N unit cells and surrounded by a dielectric with the refractive index n_0, the transfer matrix M' relating light fields at the entrance and exit of the structure can be represented as

$$M' = \Delta_{20}M^N\Delta_{02}, \tag{31}$$

where matrix Δ_{ij} is determined by Eqs. (20), (11), and (12), and M is given by expression (22).

Using Eq. (31), taking into account Eqs. (20) and (30) and writing down the matrix M' in the form similar to Eq. (16):

$$M' = \begin{pmatrix} 1/t & r^*/t^* \\ r/t & 1/t^* \end{pmatrix}, \tag{32}$$

it is possible to calculate the transmission t and reflection r coefficients for TE and TH Bessel modes propagating in a finite periodical medium:

$$\begin{aligned} \frac{1}{t} &= \frac{1}{T_{02}}\left\{\frac{1}{t_N} - 2i\,\mathrm{Im}\left(\frac{r_N}{t_N}\right)r_{02} - \frac{R_{02}}{t_N^*}\right\}, \\ \frac{r}{t} &= \frac{1}{T_{02}}\left\{\frac{r_N}{t_N} - \frac{r_N^*}{t_N^*}R_{02} - 2ir_{02}\,\mathrm{Im}\left(\frac{1}{t_N}\right)\right\}. \end{aligned} \tag{33}$$

In Eq. (33) the quantities T_{02}, R_{02} and r_{02} for TE Bessel beams are expressed as:

$$\begin{aligned} T_{02}^{TE} &= t_{02}t_{20} = \frac{4n_0n_2\cos\gamma_0\cos\gamma_2}{(n_0\cos\gamma_0 + n_2\cos\gamma_2)^2}; \\ r_{02}^{TE} &= \frac{n_0\cos\gamma_0 - n_2\cos\gamma_2}{n_0\cos\gamma_0 + n_2\cos\gamma_2}; \quad R_{02}^{TE} = |r_{02}^{TE}|^2. \end{aligned} \tag{34}$$

In turn, for TH Bessel light beams in Eq. (34) there should be done the following replacement $n_i \to 1/n_i$.

2.4. Transmission and reflection of a vector Bessel light beam on the boundary of dielectric – uniaxial crystal

Since in the investigated photonic structure the layer of anisotropic crystal serves as a symmetrically situated defect, first of all it is necessary to investigate the problem of propagation of Bessel beams through the crystalline plate surrounded by a dielectric medium.

Let a normally incident arbitrary polarized vector Bessel beam impinges from an isotropic medium with the refractive index n_2 on the entrance of a plate made of an uniaxial crystal with the principal refractive indices n_o and n_e (see Fig. 2). Here the direction of propagation coincides with the z-axis of the coordinate system and optical axis of the crystal.

From Maxwell's equations for a uniaxial crystal in the cylinder coordinate system, one can obtain the formula for the radial (ρ) and azimuthal (φ) components of the electric and magnetic fields [51]:

$$\begin{aligned} E_\rho &= \frac{i\omega}{cq}\left(i\frac{m}{q\rho}H_z + \frac{n(\gamma)\cos\gamma}{q}\frac{\partial E_z}{\partial\rho}\right); \\ E_\varphi &= \frac{i\omega}{cq}\left(i\frac{mn(\gamma)\cos\gamma}{q\rho}E_z - \frac{\partial H_z}{\partial\rho}\right); \\ H_\rho &= -\frac{i\omega}{cq}\left(i\frac{m\varepsilon_o}{q\rho}E_z - \frac{n(\gamma)\cos\gamma}{q}\frac{\partial H_z}{\partial\rho}\right); \\ H_\varphi &= \frac{i\omega}{cq}\left(\frac{\varepsilon_o}{q}\frac{\partial E_z}{\partial\rho} + i\frac{mn(\gamma)\cos\gamma}{q\rho}H_z\right). \end{aligned} \tag{35}$$

In Eq. (35) $n(\gamma)$ is the refractive index, γ is the half-cone angle, $\varepsilon_o = n_o^2$, $q = \sqrt{(\omega/c)^2n_o^2 - k_z^2}$ is the parameter of conicity (transversal

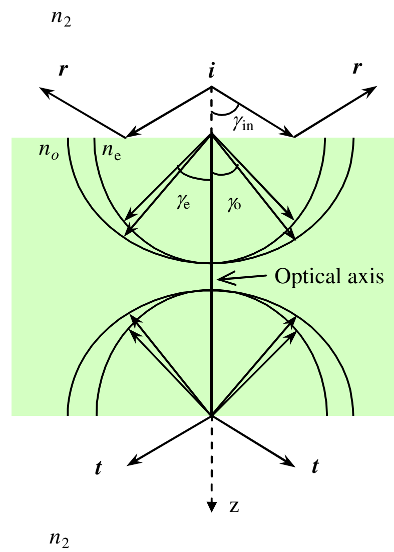

Fig. 2. Transmission of Bessel beam through the layer of uniaxial crystal. i, r and t designate input, reflected and transmitted Bessel beams.

component of wave vector, which is unchanged during the crossing of the boundary of two media). Substitution of the expression (35) for the transverse components of the electric and magnetic fields in the Maxwell's equations allows one to get the differential equation of the second-order for the longitudinal components H_z and E_z:

$$\frac{\partial^2 H_z}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial H_z}{\partial\rho} - \frac{m^2}{\rho^2}H_z = -q^2H_z; \quad \frac{\partial^2 E_z}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial E_z}{\partial\rho} - \frac{m^2}{\rho^2}E_z = \frac{\varepsilon_e}{\varepsilon_o}q^2E_z; \tag{36}$$

where $\varepsilon_e = n_e^2$. Two independent solutions of the Eqs. (36) are

$$\text{a)}\,H_{ez} = 0, \quad E_{ez} = J_m(q\rho)\exp[i(k_{ez}z + m\varphi)], \tag{37}$$

$$\text{b)}\,E_{oz} = 0, \quad H_{oz} = J_m(q\rho)\exp[i(k_{oz}z + m\varphi)], \tag{38}$$

The transverse components k_{oz} and k_{ez} of the wave vectors in Eqs. (37) and (38) are given by the expressions $k_{o,z} = (\omega/c)n_o\cos\gamma_o$, $k_{e,z} = (\omega/c)n_e(\gamma_e)\cos\gamma_e$ and are linked to the radial component q by the relations $q^2 + k_{oz}^2 = (\omega/c)^2n_o^2$, $q^2 + k_{ez}^2 = (\omega/c)^2n_e^2(\gamma_e)$. Here

$$n_e^2(\gamma_e) = \frac{\varepsilon_o\varepsilon_e}{\varepsilon_o\sin^2(\gamma_e) + \varepsilon_e\cos^2(\gamma_e)}. \tag{39}$$

Thus, the fields described by Eqs. (37) and (38) correspond to the ordinary (o) and extraordinary (e) plane waves in the theory of uniaxial crystals [53]. That is why they may be designated as o- and e-type Bessel beams, respectively.

From the Eqs. (35), (37), and (38) one can find all the components for e- and o-type Bessel beams (the phase multiplier $\exp[i(k_{o,ez}z + m\varphi)]$ is omitted):

$$\begin{aligned} &E_\rho^o = \frac{im}{q\rho}J_m(q\rho); \quad E_\varphi^o = -J'_m(q\rho); \quad E_z^o = 0; \\ &E_\rho^e = i\cos\gamma_e J'_m(q\rho); \quad E_\varphi^e = -\cos\gamma_e\frac{m}{q\rho}J_m(q\rho); \\ &E_z^e = \sin\gamma_e J_m(q\rho). \end{aligned} \tag{40a}$$

$$\begin{aligned} &H_\rho^o = n_o\cos\gamma_o J'_m(q\rho); \quad H_\varphi^o = in_o\cos\gamma_o\frac{m}{q\rho}J_m(q\rho); \\ &H_z^o = -in_o\sin\gamma_o J_m(q\rho); \quad H_\rho^e = \frac{\varepsilon_o}{n_e(\gamma_e)}\frac{m}{q\rho}J_m(q\rho); \\ &H_\varphi^e = i\frac{\varepsilon_o}{n_e(\gamma_e)}J'_m(q\rho); \quad H_z^e = 0. \end{aligned} \tag{40b}$$

For the azimuthal index $m = 0$, it follows from Eq. (40) that a zero-order Bessel beam of o-type has only the φ-component and BLB of the e-type is described by the ρ-component. Such a structure of Bessel beams coincides completely with the polarization of ordinary and extraordinary plane waves in an uniaxial crystal. For the o- and e-type of BLBs with $m \neq 0$ there appear also components that are orthogonal to the ones mentioned above.

The analysis of Eq. (40) leads to the conclusion that the ordinary and extraordinary Bessel beams are analogous to TE and TH modes of Bessel beams in an isotropic medium for which the components of the electrical and magnetic fields are determined by the expressions (2). The difference is in that for the isotropic medium the phase velocities of the above-mentioned modes are degenerate.

2.5. Transmission of Bessel light beams through crystalline plate and reflection from it

Let us consider the normal incidence of arbitrary polarized BLBs onto the interface between an isotropic medium (the refractive index n_2) and uniaxial crystal having the principal refractive indices n_o and $n_e(\gamma_e)$. The incident BLB can be represented as a superposition of TE and TH modes. Let us analyze the transformation of these modes at the boundary. The incident, reflected and refracted waves must satisfy the conditions (see Eqs. (9)) of continuity of the tangent components of the electrical and magnetic fields. Substituting the components of the fields from Eq. (40) into Eq. (9) and equating the coefficients at the same Bessel functions, we obtain that at the boundary there occurs only two kind of the transformation, namely, TE mode into o-type BLB and TH mode into e-type BLB. In the case of the TE BLB → o-type BLB transformation, the amplitudes of incident A_i^{TE}, reflected A_{rc}^{TE} and transmitted into crystal A_{tc}^{o} Bessel beams obey the equations:

$$A_i^{TE} + A_{rc}^{TE} = A_{tc}^{o}, \quad n_2 \cos\gamma_2 (A_i^{TE} - A_{rc}^{TE}) = n_o \cos\gamma_o A_{tc}^{o}. \tag{41}$$

Here γ_2 is the half-cone angle of BLB in the medium with the refractive index n_2 and γ_o is the half-cone angle of o-type BLB.

In the case of the TH BLB → e-type BLB transformation, the amplitudes of incident A_i^{TH}, reflected A_{rc}^{TH} and transmitted into crystal A_{tc}^{e} beams obey the equations:

$$n_2 (A_i^{TH} + A_{rc}^{TH}) = \frac{\varepsilon_o}{n_e(\gamma_e)} A_{tc}^{e}, \quad \cos\gamma_2 (A_i^{TE} - A_{rc}^{TE}) = \cos\gamma_e A_{tc}^{e}, \tag{42}$$

where γ_e is the half-cone angle of e-type BLB.

Let us determine the transmission coefficient $t_{ic}^{o} = A_{tc}^{o}/A_i^{TE}$ and reflection $r_{ic}^{TE} = A_{rc}^{TE}/A_i^{TE}$ one for o-type Bessel beams, as well as the transmission coefficient $t_{ic}^{e} = A_{tc}^{e}/A_i^{TH}$ and reflection one $r_{ic}^{TH} = A_{rc}^{TH}/A_i^{TH}$ for e-type Bessel beam. These coefficients can be easily determined from the solutions of Eqs. (41) and (42):

$$t_{ic}^{o} = \frac{2 n_2 \cos\gamma_2}{n_2 \cos\gamma_2 + n_o \cos\gamma_o}, \quad r_{ic}^{TE} = \frac{n_2 \cos\gamma_2 - n_o \cos\gamma_o}{n_2 \cos\gamma_2 + n_o \cos\gamma_o}, \tag{43}$$

$$t_{ic}^{e} = \frac{2 n_2 n_e(\gamma_e) \cos\gamma_2}{n_2 n_e(\gamma_e) \cos\gamma_e + n_o^2 \cos\gamma_2}, \quad r_{ic}^{TH} = \frac{n_o^2 \cos\gamma_2 - n_2 n_e(\gamma_e) \cos\gamma_e}{n_2 n_e(\gamma_e) \cos\gamma_e + n_o^2 \cos\gamma_2}, \tag{44}$$

where, as is evident from Eq. (39):

$$n_e(\gamma_e) \cos\gamma_e = \sqrt{n_o^2 - n_o^2 n_2^2 \sin^2\gamma_2 / n_e^2}. \tag{45}$$

Similarly, we can obtain the amplitude coefficients of the transformation (transmission t_{ci}^{TH}, t_{ci}^{TE} and reflection r_{ci}^{e}, r_{ci}^{o}) of the e-type BLB → TH BLB and the o-type BLB → TE BLB at the boundary between the uniaxial crystal and isotropic medium with the refractive index n_2:

$$t_{ci}^{TE} = \frac{2 n_o \cos\gamma_o}{n_o \cos\gamma_o + n_2 \cos\gamma_2}, \quad r_{ci}^{o} = \frac{n_o \cos\gamma_o - n_2 \cos\gamma_2}{n_o \cos\gamma_o + n_2 \cos\gamma_2}. \tag{46}$$

$$t_{ci}^{TH} = \frac{2 n_o^2 \cos\gamma_e}{n_2 n_e(\gamma_e) \cos\gamma_e + n_o^2 \cos\gamma_2}, \quad r_{ci}^{e} = -\frac{n_o^2 \cos\gamma_2 - n_2 n_e(\gamma_e) \cos\gamma_e}{n_2 n_e(\gamma_e) \cos\gamma_e + n_o^2 \cos\gamma_2}. \tag{47}$$

The transfer matrix, describing the transformation of BLB propagating through the crystalline plate, is represented as a product of matrices of two types:

$$M_D = \Delta_{ci} \Pi(-p_{o,e}) \Delta_{ic}. \tag{48}$$

Here the matrices $\Delta_{ic,ci}$ describe the transformation of BLB at the boundary between the isotropic medium and uniaxial crystal and between the crystal and isotropic medium, respectively, and are determined by Eq. (20), where it should be replaced: for Δ_{ic} matrix $a_{ij}^{+} \to 1/t_{ic}^{o,e}$; $a_{ij}^{-} \to r_{ic}^{TE,TH}/t_{ic}^{o,e}$, and for Δ_{ci} matrix $a_{ij}^{+} \to 1/t_{ci}^{TE,TH}$; $a_{ij}^{-} \to r_{ci}^{o,e}/t_{ci}^{TE,TH}$.

Matrix $\Pi(p_{o,e})$ describes the phase change of Bessel beam in a crystalline layer and is determined by Eq. (18), where $p_o = D(\omega/c) n_o \cos\gamma_o$; $p_e = D(\omega/c) n_e(\gamma_e) \cos\gamma_e$, D is the crystal thickness. The transfer matrix can be expressed in terms of the transmission t_D and reflection r_D coefficient of the crystal in the following form:

$$M_D = \begin{pmatrix} 1/t_D & r_D^*/t_D^* \\ r_D/t_D & 1/t_D^* \end{pmatrix}, \tag{49}$$

where in the case of incidence of TE and TH modes of BLBs on the crystalline plate, the coefficients t_D and r_D are given by the expressions:

$$t_D = \frac{t_{ic}^{o,e} t_{ci}^{TE,TH} \exp[i p_{o,e}]}{1 + r_{ic}^{TE,TH} r_{ci}^{o,e} \exp[2 i p_{o,e}]}, \quad r_D = \frac{r_{ci}^{o,e} + r_{ic}^{TE,TH} \exp[2 i p_{o,e}]}{1 + r_{ic}^{TE,TH} r_{ci}^{o,e} \exp[2 i p_{o,e}]}. \tag{50}$$

2.6. Propagation of BLB in a finite one-dimensional photonic crystal having a defect impurity – a layer of uniaxial crystal

Let us consider the peculiarities of BLB transformation in a stratified periodic medium with a defect inclusion in the form of a layer of a uniaxial crystal (Fig. 3) which is surrounded by a dielectric having refractive index n_0.

As is seen from Fig. 3, in this case one-dimensional photonic crystal is divided into two sub-structures, each of which has N unit cells. Then the total transfer matrix M_S is determined by the product of transfer matrices for the first (located to the left from the defect) M^N and the second (located to the right from the defect) $\widetilde{M}^N$ sub-structures and defect inclusion M_D: $M_S = \Delta_{20} M^N M_D \widetilde{M}^N \Delta_{02}$. Note that in the last expression transfer matrices are different: M^N is determined by Eqs. (23), (24), (25), (27), and (28), whereas

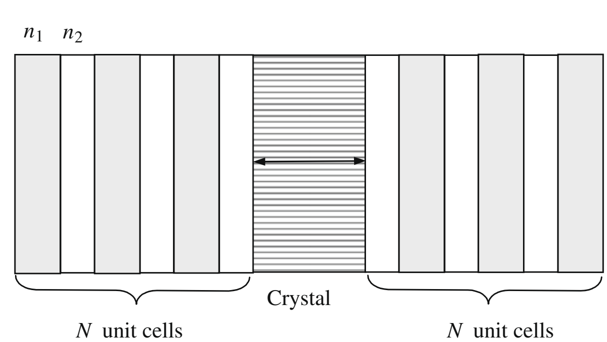

Fig. 3. Layered-periodic medium with the defect inclusion of uniaxial crystal. The orientation of the optical crystal axis is shown by arrow.

$\widetilde{M}^N$ is obtained by multiplying the transfer matrix $\widetilde{M}$ for the unit cell of the second sub-structure:

$$\widetilde{M} = \Pi(-p_2)\Delta_{12}\Pi(-p_1)\Delta_{21}. \quad (51)$$

From Eq. (51) it follows that for the unit cell of the second sub-structure:

$$\tilde{t}_{cell} = \frac{T\exp[i(p_1+p_2)]}{1-R\exp(2ip_1)},$$
$$\tilde{r}_{cell} = -r_{12}\exp[2i(p_1+p_2)]\frac{1-\exp(-2ip_1)}{1-R\exp(2ip_1)}, \quad (52)$$

where the coefficients T and R are determined by Eqs. (24) and (25).

By representing M_S in the form similar to Eq. (16), one can calculate its elements M_{S11}, M_{S21} and, as a consequence, the transmission t_S and reflection r_S coefficients for the periodic medium including the defect and surrounded by a dielectric with the refractive index n_0. One can show that these coefficients can be represented in the form of Eq. (33), where the following replacements should be made:

$$\frac{1}{t_S}\to\frac{1}{t};\quad \frac{r_S}{t_S}\to\frac{r}{t};$$
$$\frac{1}{t_N}\to\frac{1}{t'} = \frac{1}{t_N\tilde{t}_N t_D} + \frac{r_N^* r_D}{t_N^*\tilde{t}_N t_D} + \frac{\tilde{r}_N r_D^*}{t_N\tilde{t}_N t_D^*} + \frac{r_N^*\tilde{r}_N}{t_N^*\tilde{t}_N t_D^*};$$
$$\frac{r_N}{t_N}\to\frac{r'}{t'} = \frac{r_N}{t_N\tilde{t}_N t_D} + \frac{r_D}{t_N^*\tilde{t}_N t_D} + \frac{r_N\tilde{r}_N r_D^*}{t_N\tilde{t}_N t_D^*} + \frac{\tilde{r}_N}{t_N^*\tilde{t}_N t_D^*}. \quad (53)$$

Here t_D and r_D are the amplitude transmission and reflection coefficients for the crystalline layer, respectively, determined by the relations (50).The quantities $\tilde{r}_N,\ \tilde{t}_N$ can be find from Eqs. (30) and (27), where it should be replaced $r_{cell}\to\tilde{r}_{cell}$ and $t_{cell}\to\tilde{t}_{cell}$, respectively.

The fields, transmitted through the structure surrounded by an air (the most frequently realizable case) and reflected from it, are determined by equations, similar to (7), (8):

$$\vec{E}_{\perp t}(R) = \frac{iA}{\sqrt{2}}\exp i[(m-1)\varphi + \frac{\omega}{c}\cos\gamma_i z] \times \{t_s^{TE}[J_{m-1}(q\rho)\vec{e}_+ + J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_-] + t_s^{TH}\cos\gamma_i[J_{m-1}(q\rho)\vec{e}_+ - J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_-]\}, \quad (54)$$

$$\vec{E}_{\perp t}(R) = \frac{iA}{\sqrt{2}}\exp i[(m-1)\varphi - \frac{\omega}{c}\cos\gamma_i z] \times \{r_s^{TE}[J_{m-1}(q\rho)\vec{e}_+ + J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_-] - r_s^{TH}\cos\gamma_i[J_{m-1}(q\rho)\vec{e}_+ - J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_-]\}, \quad (55)$$

Eqs. (54) and (55), with allowance for Eqs. (23), (24), (25), (27), (30), (34), (46), (47), (50), and (52), give the complete solution of the problem of determining the reflection and transmission coefficients for TE and TH Bessel modes in a one-dimensional photonic crystal having an anisotropic defect impurity.

As it follows from obtained expressions, the spectral dependences of transmission and reflection are different for TE and TH Bessel beams. Furthermore, the defect levels caused by the existence of a defect insert, appear in the band gap. Owing to anisotropy of the crystalline layer, the defect levels for TE and TH modes of BLBs are different in their location. So, one can realize a condition when we will observe TH (TE)-polarized BLB at the output of the structure and TE (TH)-polarized BLB at its input.

3. Numerical calculation of transmittance of circularly-polarized BLB through the one-dimensional photonic crystal

Using the above-derived formula (54), (55), (23), (24), (25), (27), (30), (34), (46), (47), (50), and (52), we will firstly consider the properties of Bessel light beams transformation in a perfect stratified periodic medium (in the absence of a defect layer), containing alternate layers of dielectrics of ZrO_2 (n_1 = 2.0 [55]; d_1 = 72 nm) and SiO_2 (n_2 = 1.45 [56]; d_2 = 100 nm). Note that both these materials are transparent in the wide spectral region: 0.25 ÷ 7 μm [57]. The structure is surrounded by the air. It follows from Eq. (27) that, as a consequence of difference of t_{cell}, the width of the band gap for TE mode $\Delta\lambda^{TE}$ exceeds one for TH modes $\Delta\lambda^{TH}$. Therefore, the edges of the photon band gap for both modes do not coincide. Then, if the frequency of incident Bessel light beam is near band gap edges, it is possible to realize the condition when TH BLB is transmitted through the medium and TE BLB is reflected from it. At small half-cone angles (within 20°) the difference of the band gap edges, corresponding to the case of TE and TH Bessel modes, is little, and incident radiation with a high-stable wavelength and a great number of the unit cells of stratified medium are necessary to split the incident Bessel light beam into transmitted TH BLB and reflected TE-polarized Bessel beam. It follows from the numerical simulation that at a larger half-cone angle ($\gamma_i \geqslant 20°$) the extinction ratio $|t^{TH}|^2/|t^{TE}|^2$ achieves large values in a wider spectral region. It is shown in Fig. 4a and b that for γ_i = 20° and N = 20, in order to achieve both a high transmission of TH modes ($|t^{TH}|^2$ > 90%) and essential (>10) extinction ratio, the incident wavelength instability $\Delta\lambda$ must not exceed 0.6 nm. Note that the value of $\Delta\lambda$ depends on the half-cone angle of incident BLB. In particular, as it follows from Fig. 4c and d, for γ_i = 25° and N = 20, we have $\Delta\lambda$ = 1.7 nm.

As N increases, the transmission maxima become sharper (Fig. 4e and f). Owing to this, it is possible to achieve simultaneously a high transmission of TH modes ($|t^{TH}|^2$ > 90%) and extreme extinction ratio (for example, more than 900) in a spectral region about 1 nm (see Fig. 4e and f). Note that for selecting only transmitted TH Bessel mode from incident field, one can use BLBs with a higher wavelength instability. For example, it follows from Fig. 4e and f, that the extinction ratio greater than 100 is achieved for the width of the spectral line of incident Bessel beams ~5 nm.

Moreover, the transmission and reflection of the perfect layered structure for TE and TH modes depend on the half-cone angle of incident BLB (Fig. 5). Then, the splitting of the incident Bessel beam into TE- and TH-polarized BLBs can be realized also in the case of incident beams with half-cone angles lying in a certain interval. It follows from Fig. 5a and b, that if N = 20, it is possible to achieve both a high transmission of TH modes ($|t^{TH}|^2$ > 90%) and high (>10) extinction ratio for an incident BLB having half-cone angle dispersion ~1°. As the number of the unit cells of the one-dimensional photonic crystal increases, the extinction ratio is significantly enhanced. For example, as is seen from Fig. 5c and d, based on perfect 1DPC with 30 unit cells, it is possible to realize an optical element, providing the splitting of an incident BLB having a half-cone angle dispersion of about 0.5°, on transmitted TH and reflected TE BLB modes with both high transmission of TH modes and extreme (>600) extinction ratio.

Thus, the numerical calculations demonstrate the prospects of application of one-dimensional photonic crystals for generation of TH and TH Bessel modes. Relying on 1DPC, it is feasible to design and fabricate optical elements, which produce transmitted TH-polarized Bessel beams and reflected TE-polarized Bessel beams. From numerical simulation it follows that these elements are characterized by a high efficiency (it is possible to achieve the extinction ratios higher than 100) and can be used for incident light beams with a relatively wide spectral line and half-cone angle lying in a certain interval.

Now we will consider the features of the Bessel light beams transformation in a stratified periodic medium containing a defect insert – a layer of a uniaxial crystal, the optical axis of which is perpendicular to the input interface of 1DPC (Fig. 3). The calculation for the periodic medium consisting of alternate layers of dielectrics of ZrO_2 (n_1 = 2.0; d_1 = 79 nm) and SiO_2 (n_2 = 1.45; d_2 = 109 nm) has been performed. It is a structure, the first band gap of which is

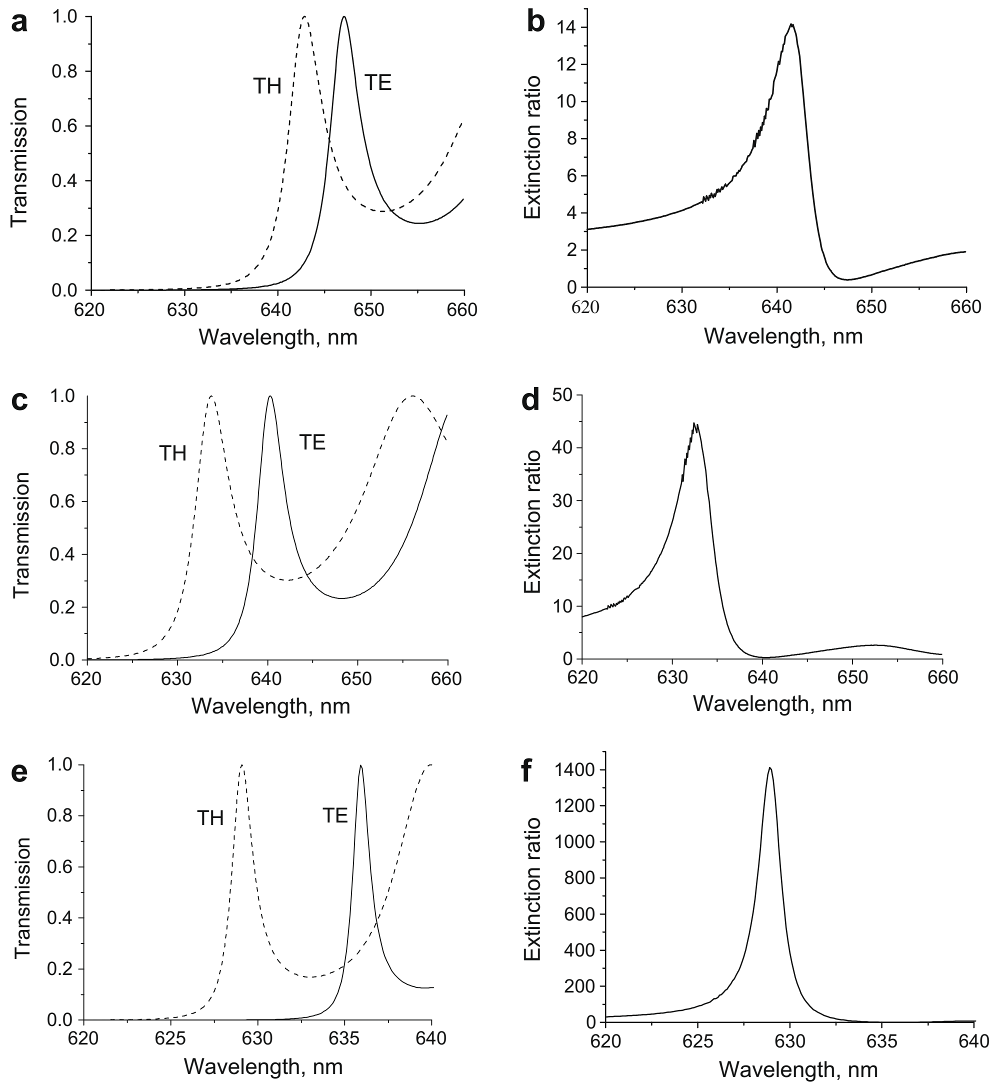

Fig. 4. Spectral dependence of the transmission $\widetilde{T} = |t|^2$ and extinction ratio of perfect 1DPC $[ZrO_2/SiO_2]^N$, containing $N = 20$ (a–d) or $N = 30$ (e, f) unit cells, for incident BLB having half-cone angle 20° (a, b) or 25° (c–f).

centered at $\lambda_0 = 4n_1d_1 = 4n_2d_2 = 632.8$ nm. At the center of the structure a layer of $LiNbO_3$ is placed. The TE and TH defect modes appearing as a consequence of a crystalline defect will be introduced into the band gap. At certain wavelength λ of the incident BLB the location of TE and TH defect modes depends on the half-cone angle γ_i.

In Fig. 6 the dependences of the transmission coefficient of the layered structure $[ZrO_2/SiO_2]^5/LiNbO_3/[ZrO_2/SiO_2]^5$ on the half-cone angle of the incident BLB at small (a) and large (b) angles are shown. It is evident that for certain intervals of the half-cone angles, a spatial separation of TE and TH BLBs is possible. It follows from Fig. 6a and b that one can achieve both a high transmission of TE or TH modes and a high (more 30) extinction ratio for an incident BLB having half-cone angle dispersion about 0.007°. As the half-cone angle γ_i increases, the angle width of the separated maxima decreases, and the extinction ratio is significantly enhanced (up to 100 and higher). It follows from Fig. 6c and d that at the variation of the layer thickness as compared with $\lambda/4$ the widening of the transmission lines for both modes takes place. Thus, the fulfillment of the condition $d_1 = \lambda/(4n_1)$; $d_2 = \lambda/(4n_2)$ provides the better separation of TE and TH modes.

It is seen (Fig. 6) that while increasing the half-cone angle, the frequency of the maxima of the angle spectrum of transmission for both modes becomes higher. Thus, the oscillation frequency for TH mode appears to be higher, as well. This effect is observed also, when the thicknesses of the ZrO_2 and SiO_2 layers are not quarter-wave ones (Fig. 6c and d).

When anisotropy of the defect layer characterized by birefringence $\Delta n = n_o - n_e$ is large, an essential increase in the relative frequency of transmission oscillation for TE and TH modes is observed. Then, the situation is realized, in which the interval between neighbor maxima of TE modes more than one maximum for TH modes are located. This case is shown in Fig. 7, where the situation has been analyzed, in which the birefringence of a defect insert in a one-dimensional photonic crystal is 0.4, 0.6 and 0.8, respectively. As is known, such a birefringence is, in principle, achievable in nematic liquid crystals [58,59]. It is seen that as anisotropy increases, the angle width of the maxima decreases and the extinction ratio are enhanced. For the cases, illustrated in Fig. 7a and b, the extinction ratio achieves an extreme value of 770.

The revealed effect can have an interesting application, because it allows one to form the field with a two-annular angular spectrum from an incident BLB which is a superposition of plane waves with wave vectors lying on the surface of a cone and in its small neighboring. In the spatial domain the field with a two-annular

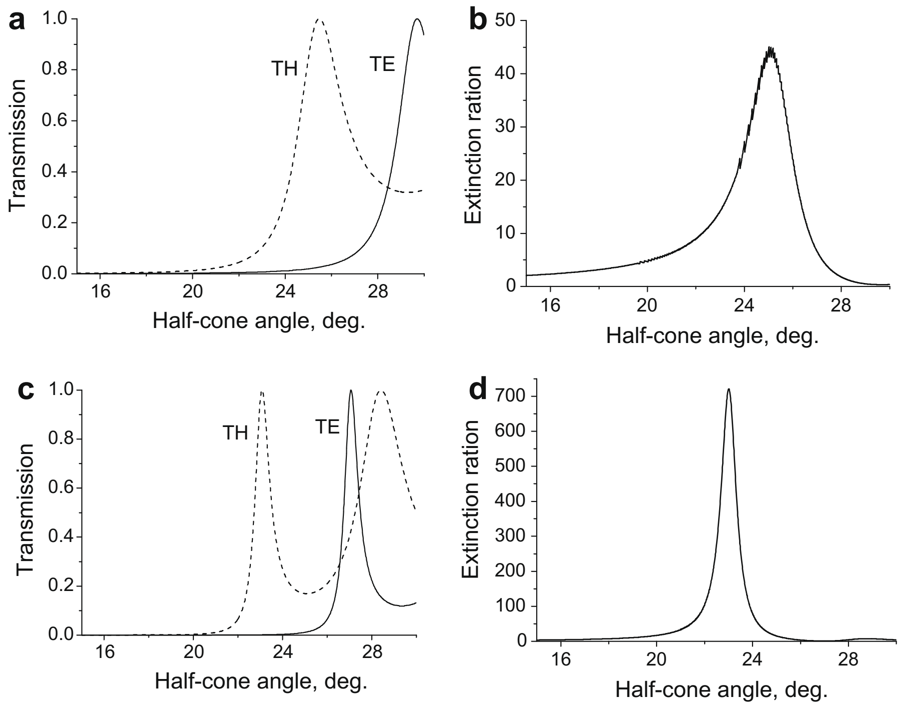

Fig. 5. Angle dependence of transmission $\widetilde{T} = |t|^2$ and extinction ratio of perfect 1DPC $[ZrO_2/SiO_2]^N$, containing $N = 20$ (a, b) or $N = 30$ (c, d) unit cells, for incident BLB having wavelength $\lambda = 0.6328$ μm.

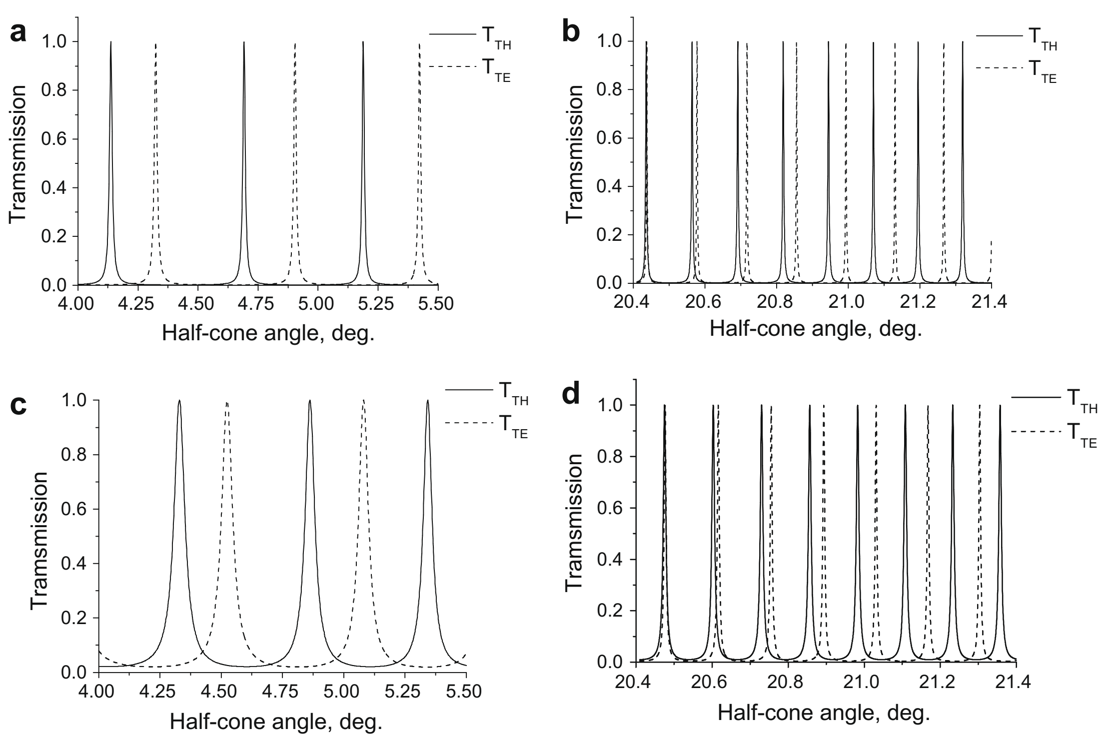

Fig. 6. Dependences of transmission $\widetilde{T} = |t_S|^2$ of $[ZrO_2/SiO_2]^5/LiNbO_3/[ZrO_2/SiO_2]^5$ structure on half-cone angle of incident BLB having the wavelength $\lambda = 0.6328$ μm. Parameters: $d_{1,2} = \lambda/4n_{1,2}$ (a, b); $d_{1,2} = 1.12\lambda/4n_{1,2}$ (c, d). The principle refractive indices of $LiNbO_3$ are $n_o = 2.2878$ and $n_e = 2.1890$.

angular spectrum is represented as a coherent superposition of two BLBs that differ in the cone angles.

Thus, it is shown that for both small and large half-cone angles γ the angle regimes exist, where TE and TH modes are divided with a high extinction ratio. The possible scheme for formation and spatial separation of a circularly-polarized incident Bessel beam of zero-order into the TE- and TH-polarized BLBs is presented in Fig. 8.

Here the incident circularly-polarized Gaussian beam is transformed by the axicon A_1 into the zero-order BLB. The beam transmitted through the photonic crystal has the TH polarization and can be transformed by the axicon A_2 into the Gaussian beam. The

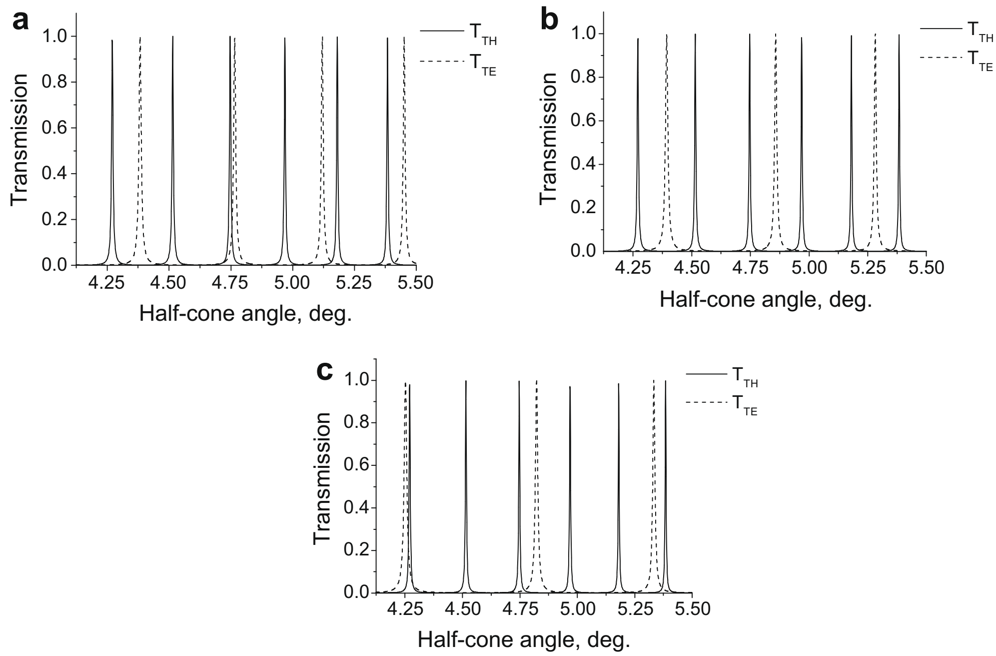

Fig. 7. Dependences of transmission $\widetilde{T} = t_S|^2$ of $[ZrO_2/SiO_2]^5$/anisotropy layer/$[ZrO_2/SiO_2]^5$ structure on the half-cone angle of incident BLB having wavelength λ = 0.6328 µm. Parameters: $d_{1,2} = \lambda/4n_{1,2}$. The principle refractive indices of model anisotropy layer are n_o = 1.85 (a), 2.05 (b), 2.25 (c); n_e = 1.45.

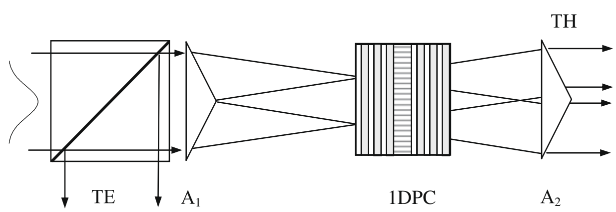

Fig. 8. Optical scheme for formation of TE- and TH-polarized BLBs. $A_{1,2}$ are axicons; 1DPC is a one-dimensional photonic crystal (perfect of having a crystalline defect layer).

reflected beam has the TE polarization. It can be converted back into the Gaussian beam by the axicon A_1 and removed out of the optical system by a splitting cube. Two beams will have an equal power, owing to the circular polarization of the incident field.

4. Conclusion

In this paper, on the basis of a matrix method, a theory of propagation of TE- and TH-polarized Bessel light beams in one-dimensional photonic crystals has been developed. The cases of perfect photonic structures, as well as one containing defect layer, have been examined. Defect inclusion is a layer of a uniaxial crystal with the optical axis, oriented along the direction of periodicity. The problem of the TE- and TH-polarized BLB transmission through the crystal plate has been solved. Analytical expressions have been derived for the reflection and transmission coefficients of TE and TH Bessel modes propagating through a symmetric (relatively anisotropic inclusion) photonic structure.

Based on the obtained expressions, the transmission through the perfect photonic structure of quasi-circularly-polarized incident Bessel beam generated by an axicon from a circularly-polarized Gaussian beam has been calculated and analyzed. From numerical simulation it follows that both spectral and angular width of the band gap of such a structure is different for TE- and TH-polarized modes. It is shown that on the basis of this effect, observed in 1DPC, it is possible to create optical elements, which produce transmitted TH-polarized Bessel beams and reflected TE-polarized Bessel beams. It has been proved that these elements are characterized by a high efficiency (it is possible to achieve the extinction ratios higher than 100) and can be used for incident beams with an essential wavelength instability and half-cone angle dispersion.

The possibility of generation of TH- and TE-polarized BLBs with the use of defect modes in the band gap, appearing as a consequence of anisotropic layer in the structure, has been studied. For this aim the difference of transmission dependences for TH and TH Bessel beams on the half-cone angle of incident circularly-polarized Bessel beam is proposed to use. It is shown that the efficient splitting of TH and TE modes is achieved even at a little number of unit cells (about 10). As the anisotropy of inclusion increases, the extinction ratio is essentially enhanced and the angular width of transmission maxima of the modes decreases. The effect of a considerable increase in the relative frequency of oscillation of transmission function for TH- and TE-polarized Bessel modes has been established, as the defect layer birefringence enhances. Thus, this situation can be realized when in the interval between neighbor TE mode maxima more than one TH mode maxima are located. This effect has been proposed to use for field formation with a two-annular angle spectrum from an incident BLB which is a superposition of plane waves with wave vectors lying on the surface of a cone and in its small neighboring. In the spatial domain a field having a two-annular angle spectrum allows one to form a coherent superposition of two BLBs with different half-cone angles.

Thus, using both perfect photonic structures and 1DPC with a defect layer, efficient and compact polarized elements for zero- and higher-order Bessel light beams can be elaborated. It is important to point out that during experiment some factors can modify the results of theoretical predictions made above, including absorption effects, variations in the thicknesses of the layers, non-normal incidence of Bessel light beam onto the structure, misalignment of optical axis of the crystalline defect from the direction of propagation of BLB. However, the numerical estimations show that the impact of the most of these factors is small enough for photonic crystals containing several tens of the unit cells. For example, the account of optical losses in the layers ≈0.5 cm^{-1} (as for ZrO_2 and SiO_2 thin films in visible light [57]) causes small (<0.5%) changes of spectral dependences of transmission for TE and TH Bessel modes and extinction ratio. The deviation of the direction of BLB propagation from the normal one (up to 1°) causes the widening the transmission maxima of TE and TH modes and, hence, the decrease of extinction ratio (up to 5%). The misalignment of optical axis of the crystalline defect from the direction of propagation of BLB up to 1° (that is possible during manufacturing) causes negligible (<0.01%) changes of spectral dependences of transmission for TE and TH modes. The variations in the thicknesses of the layers may have the appreciable influence on the conditions (wavelength and the value of half-cone angle of incident Bessel beam) of generation of TH- and TE-polarized BLBs, but only in the case if the thicknesses of both layers in every unit cell are increasing or decreasing simultaneously. However, it should be noted that when in every unit cell the thickness of the first layer is greater (smaller) and the thickness of the second one is smaller (greater) than the values of thicknesses of $d_{1,2} = \lambda_0/4n_{1,2}$ up to quantity p% ($p \leqslant 5$%), the position of transmission maxima, as well as the value of the cone angle of the incident BLB, at which the generation of TE- and TH-polarized BLBs is observed, does not practically change.

It is important to point out that the proposed method allows one not only to generate Bessel beams with TE and TH polarizations but to form their various superpositions as well. It is achieved by changing the half-cone angle of an incident BLB, which results in the transformation of the TE and TH transmission spectra, and, as a result, the control of the ellipticity of the reflected and refracted Bessel beams.

References

[1] J. Durnin, J.J. Miceli, J.H. Eberly, Phys. Rev. Lett. 58 (1987) 1499.
[2] H. Little, C. Brown, V. Garcéz, W. Sibbett, K. Dholakia, Opt. Express 12 (2004) 2560.
[3] G. Milne, K. Dholakia, D. McGloin, K. Volke-Sepulveda, P. Zemanek, Opt. Express 15 (2007) 13972.
[4] V. Garcés-Chavéz, D. Roskey, M.D. Summers, H. Melville, D. McGloin, E.M. Wright, K. Dholakia, Appl. Phys. Lett. 8 (2004) 4001.
[5] L. Paterson, E. Papagiakoumou, G. Milne, V. Garcés-Chavéz, T. Briscoe, W. Sibbett, L. Dholakia, A. Riches, J. Biomed. Opt. 12 (2007) 054017.
[6] T. Čizmár, V. Garcés-Chavéz, K. Dholakia, P. Zemánek, Appl. Phys. Lett. 86 (2005). 101-1-174.
[7] P. Sprangle, B. Hafizi, Phys. Rev. Lett. 66 (1991) 837.
[8] Z. Bouchal, J. Wagner, M. Chlup, Opt. Commun. 151 (1998) 207.
[9] Y. Lin et al., Appl. Opt. 31 (1992) 2708.
[10] D. McGloin, K. Dholakia, Contemp. Phys. 46 (2005) 15.
[11] A.V. Novitsky, L.M. Barkovsky, J. Opt. A: Pure Appl. Opt. 7 (2005) 550.
[12] J. Turunen, A.T. Friberg, Pure Appl. Opt. 2 (1993) 51.
[13] R. Horak, Z. Bouchal, J. Bajer, Opt. Commun. 133 (1997) 315.
[14] S.R. Mishra, Opt. Commun. 85 (1991) 159.
[15] D.G. Hall, Opt. Lett. 21 (1996) 9.
[16] S.S. Girgel, S.N. Kurilkina, Proc. SPIE 4358 (2001) 258.
[17] P.L. Greene, D.G. Hall, J. Opt. Soc. Am. A 15 (1998) 3020.
[18] Z. Bouchal, M. Olivik, J. Mod. Opt. 42 (1995) 1555.
[19] J. Arlt, J. Mod. Opt. 50 (2003) 1573.
[20] A. Ciattoni, C. Palma, Opt. Commun. 224 (2003) 175.
[21] P.I. Ropot, R.Yu. Vasil'ev, J. Appl. Spectrosc. 73 (2006) 551.
[22] R. Jauregui, S. Hacyan, Phys. Rev. A 71 (2005) 033411.
[23] S. Hacyan, R. Jauregui, J. Phys. B 39 (2006) 1669.
[24] A. Niv, G. Biener, V. Kleiner, E. Hasman, Opt. Lett. 29 (2004) 238.
[25] R. Dorn, S. Quabis, G. Leuchs, Phys. Rev. Lett. 91 (2003) 233901.
[26] J. Arlt, V. Garces-Chavez, W. Sibbett, K. Dholakia, Opt. Commun. 197 (2001) 239.
[27] L. Novotny, M.R. Beversluis, K.S. Youngworth, T.G. Brown, Phys. Rev. Lett. 86 (2001) 5251.
[28] K.T. Gahagan, G.A. Swartzlander, JOSA B 16 (1999) 533.
[29] T. Kuga, Y. Torii, N. Shiokawa, T. Hirano, Y. Shimizu, H. Sasada, Phys. Rev. Lett. 78 (1997) 4713.
[30] N. Lindlein, R. Schreiner, S. Brinkmann, T. Dresel, J. Schwider, Appl. Opt. 36 (1997) 2791.
[31] N. Khilo, V. Belyi, N. Kazak, A. Mashchenko, P. Ropot, Proc. SPIE 6616 (2007) 66162L-01.
[32] A.V. Nesterov, V.G. Niziev, J. Phys. D: Appl. Phys. 33 (2000) 1817.
[33] F.P. Schafer, Appl. Phys. B 39 (1986) 1.
[34] S.C. Tidwell, B.P. Ford, W.D. Kimura, Appl. Opt. 29 (1990) 2234.
[35] J.J. Wynne, IEEE J. Quantum Electron. 10 (1972) 125.
[36] M.E. Maric, E. Garmire, Appl. Phys. Lett. 38 (1981) 743.
[37] D.J. Armstrong, M.C. Philips, A.V. Smith, Appl. Opt. 42 (2003) 3550.
[38] A.V. Nesterov et al., Appl. Phys. 32 (1999) 2871.
[39] A. Flores-Pérez, J. Hernández-Hernández, R. Jáurequi, K. Volke-Sepulveda, Opt. Lett. 31 (2006) 1732.
[40] V.N. Belyi, N.S. Kazak, N.A. Khilo, Quantum Electron. 30 (2000) 753.
[41] T.A. King, W. Hogervorst, N.S. Kazak, N.A. Khilo, A.A. Ryzhevich, Opt. Commun. 187 (2001) 407.
[42] F.I. Fedorov, Theory of Gyrotropy, Minsk, Nauka i tehnika, 1976.
[43] E.S. Petrova, Proc. SPIE 4358 (2001) 265.
[44] R.S. Tyan, A.A. Savekar, H.P. Chou, C.C. Cheng, A. Scherer, P.C. Sun, F. Xu, Y. Fainman, J. Opt. Soc. Am. A 14 (1997) 1627.
[45] I. Hodgkinson, Q.H. Wu, Appl. Phys. Lett. 74 (1999) 1794.
[46] L. Li, J.A. Dobrowolski, Appl. Opt. 35 (1996) 2221.
[47] P.W. Baumeister, Appl. Opt. 36 (1997) 3610.
[48] L. Li, J.A. Dobrowolski, Appl. Opt. 39 (2000) 2754.
[49] W.B. Williams, J.B. Pendy, J. Opt. Soc. Am. 22 (2005) 992.
[50] M. Lei, B. Yao, Opt. Commun. 239 (2004) 367.
[51] N.A. Khilo, E.S. Petrova, A.A. Ryzhevich, Quantum Electron. 31 (2001) 85.
[52] J.M. Bendickson, J.P. Dowling, Phys. Rev. E 53 (1996) 4107.
[53] A. Yariv, P. Yeh, Optical Waves in Crystals, Wiley, New York, 1984.
[54] F.R. Gantmaher, Theory of Matrixes, Nauka, Moscow, 1967.
[55] H. Tagechi, Y. Takahashi, C. Matsumoto, Jpn. J. Appl. Phys. 39 (1980) 1417.
[56] H.K. Pulker, Appl. Opt. 12 (1979) 1969.
[57] R.E. Hummel, K.H. Guenther (Eds.), Handbook of Optical Properties, Thin Films for Optical Coatings, CRC-Press, 1995.
[58] L.M. Blinov, Sov. Phys. Usp. 17 (1975) 658.
[59] L.M. Blinov, V.G. Chigrinov, Electrooptic Effects in Liquid Crystal Materials: Partially or Dered Systems, Springer, 1996.