Models of dynamical supersymmetry breaking and quintessence

P. Binétruy
LPTHE, Université Paris-XI, Bâtiment 211, F-91405 Orsay Cedex, France

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We study several models of relevance for the dynamical breaking of supersymmetry which could provide a scalar component with an equation of state $p=w\rho, -1<w<0$. Such models would provide a natural explanation for recent data on the cosmological parameters. [S0556-2821(99)02710-1]

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INTRODUCTION

There are increasing indications that the energy density of matter in the Universe is smaller than the critical density [1]. If one sticks to the inflation prediction of $\Omega_T=1$, then the natural question is the origin of the extra component providing the missing energy density. An obvious candidate is a cosmological constant, whose equation of state is $p=-\rho$. This presents particle physics with the unpleasant task of explaining why the energy of the vacuum should be of order $(0.003\ \mathrm{eV})^4$, a task possibly even harder than the one of explaining why the cosmological constant is zero. In particular, it seems to require new interactions with a typical scale much lower than the electroweak scale, long range interactions that would have remained undetected.

It has recently been proposed to consider instead a dynamical time-dependent and spatially inhomogeneous component, with an equation of state $p=w\rho$, $-1<w<0$. Such a component has been named ''quintessence'' by Caldwell, Dave, and Steinhardt [2]. Indeed, present cosmological data seem to prefer [3], in the context of cold dark matter models, a value for w of the order of -0.6. Several candidates have been proposed for this component: tangled cosmic strings [4] and pseudo Goldstone bosons [5]. Of particular relevance to some issues at stake in the search for a unified theory of fundamental interactions is a scalar field with a scalar potential decreasing to zero for infinite field values [2,6].

It has been noted that such a behavior appears naturally in models of dynamical supersymmetry breaking (DSB) [6]. Typically, the scalar potential of supersymmetric models has many flat directions, i.e., directions in field space where the scalar potential vanishes. Once supersymmetry is broken dynamically, the degeneracy corresponding to the flat direction is lifted but generally the flat direction is restored at infinite values of the scalar field.[1] We are thus precisely in the situation of a potential smoothly decreasing to zero at infinity. This is usually considered as a drawback of spontaneous supersymmetry breaking models from the point of view of cosmology: in the standard approach, the potential has a stable ground state, where the potential is fine tuned to zero (in order to account for a vanishing cosmological constant), but the initial conditions and the subsequent cosmological evolution may lead to a situation where the field misses the ground state and evolves to infinite values.

Dynamical supersymmetry breaking is often favored because it can more easily account for large mass scale hierarchies such as M_W/M_P through some powers of Λ/M_P where Λ is the dynamical scale of breaking. It is thus a natural question to ask whether the corresponding models may account for quintessence. Indeed, in this case, there is a fundamental reason why the scalar potential vanishes at infinity: this is related to the old result that global supersymmetry yields a vanishing ground state energy. And there may be reasons as to why once it dominates, the contribution of the scalar field to the energy density is very small (again through powers of Λ/M_P).

In the following, we will discuss two models of dynamical supersymmetry breaking which may be considered as representative of semirealistic models for high energy physics. One is based on gaugino condensation coupled to the dynamics of a dilaton field; the other uses the condensation of N_f flavors in a SU(N_c) gauge theory.

MODELS WITH A DILATON

We start with a class of models, reminiscent of many superstring models, where supersymmetry is broken through gaugino condensation [7] along the flat direction corresponding to the dilaton field. Indeed, in many superstring models, the dilaton field s does not appear in the superpotential and thus corresponds to a flat direction in the scalar potential. It couples to the gauge fields in a model-independent way:

$$\mathcal{L}=-\frac{1}{4}sF^{\mu\nu}F_{\mu\nu}, \tag{1}$$

where $F_{\mu\nu}$ is the field strength corresponding to a generic gauge symmetry group G and, throughout this article, s is expressed in Planck mass units. Thus the vacuum expectation value $\langle s\rangle$ can be interpreted as the inverse of the gauge coupling $1/g^2$ at the string scale. Indeed, it is directly related to the inverse of the string coupling constant (see below). The interaction corresponding to the gauge group G becomes strong at a scale

$$\Lambda=M_Pe^{-1/2bg^2}=M_Pe^{-s/2b_0}, \tag{2}$$

[1] In some cases, the field value may be interpreted as the inverse coupling constant associated with the dynamics responsible for supersymmetry breaking. An infinite field value means a vanishing gauge coupling and thus restoration of supersymmetry.

where b_0 is the one-loop beta function coefficient of the gauge group G. The corresponding gaugino fields are expected to condense,

$$\langle\overline{\lambda}\lambda\rangle=\Lambda^3=M_P^3 e^{-3s/2b_0}, \tag{3}$$

and they lead to a potential energy, quadratic in the gaugino condensates, that scales like e^{-3s/b_0}. In the limit of infinite s, that is, of vanishing gauge coupling, the dynamics is inoperative and one recovers the flat direction associated with the dilaton.

We have followed a very crude approach and there are, of course, many possible refinements: one may include supergravity corrections, the effect of other scalar fields such as moduli, as well as corrections which may be needed to stabilize the potential for small values of s (that is, in the regime of strongly coupled string [8]). For example, in a given model [9], the potential reads, in terms of the field l which precisely describes the string gauge coupling,

$$V(l)=\frac{M_P^4}{16e^2 l}\left\{\left(1+f-l\frac{df}{dl}\right)\left(1+\frac{2}{3}b_0 l\right)^2 - \frac{4}{3}b_0^2 l^2\right\}e^{g-3(f+1)/2b_0 l}, \tag{4}$$

where $f(l)$ and $g(l)$ appear as nonperturbative contributions to the Kähler potential. The dilaton s is related to the field l as $s=(1+f)/2l$. One recovers, in the limit of large s (small string coupling l), a leading behavior in e^{-3s/b_0}.

Since there are obvious power law corrections to this behavior, we will consider a toy model of a dilaton field s with a Lagrangian

$$\mathcal{L}=-\frac{1}{4s^2}\partial^\mu s\partial_\mu s-V(s), \tag{5}$$

where

$$V(s)=V_0(s)e^{-3s/b_0}. \tag{6}$$

The noncanonical kinetic term for s is characteristic of the string dilaton and accounts for the nonflat Kähler metric.

The cosmological evolution of the s field is described by the following set of equations ($\kappa=1$):

$$\frac{\ddot{s}}{2s^2}-\frac{\dot{s}^2}{2s^3}+3H\frac{\dot{s}}{2s^2}+\frac{dV}{ds}=0,$$

$$H^2=\frac{1}{3}(\rho_B+\rho_s), \tag{7}$$

where ρ_B is the background energy density associated with matter ($w_B=0$) or radiation ($w_B=1/3$) and $\rho_s=\dot{s}^2/(4s^2)+V(s)$.

If we first consider that $V_0(s)$ is a constant and solve these equations assuming that ρ_B dominates for some time, there exists a scaling solution with the following behavior:[2] the field s evolves down the exponentially decreasing potential as $(t/t_1)^{(1-w_B)/(1+w_B)}$ as long as s remains smaller than $s_1\equiv(2b_0/3)(1+w_B)/(1-w_B)$, reached at $t=t_1$; for larger values, there exists a scaling solution [11–13] where the field evolves logarithmically as $s=s_1+(2b_0/3)\ln(t/t_1)$. The ratio ρ_s/ρ_{tot} starts at $3(1-w_B)^2/16$ for $t\leqslant t_1$ and from then on slopes down to zero as $(b_0^2/6s^2)(1+w_B)$ for large values of s. Finally, $w_s=p_s/\rho_s$ starts at a value of 1 and decreases monotonically towards w_B as s increases. There is therefore no hope of using the dilaton for the dynamical component of quintessence since w_s never reaches a negative value. Power law corrections $[V_0(s)\propto s^\alpha]$ do not change this conclusion.

This might be in some sense a welcome conclusion since the vacuum expectation value $\langle s\rangle$ provides, after renormalization down to low energy, the fine structure constant $1/\alpha$. A sliding dilaton would make the fine structure constant vary with time at an unacceptable rate [14].

Similar conclusions can be reached with other types of weakly coupled scalar particles, such as the moduli of string theories. For example, in a model with several gaugino condensates and a modulus field t describing the radius of the six-dimensional compact manifold, the scalar potential scales for large values of t as [9]

$$V=\sum_a t^{(b+b_a)/b_a}e^{-\pi[(b-b_a)/(3b_a)]t}e^{-2\langle s\rangle/b_a}, \tag{8}$$

where the sum runs over the different condensates (one for each group G_a, with corresponding beta function coefficient b_a). We have fixed the dilaton field s at its ground state value. Let us note that, although the modulus t definitely cannot be used for quintessence (since, as above, the corresponding w_t reaches asymptotically w_B), a large value of $\langle s\rangle$ may contribute to giving a small contribution from t to the vacuum energy.

MODEL OF FERMION CONDENSATES

We now turn to a model which yields inverse powers of fields in the potential, a welcome situation for quintessence models [6]. It is based on the gauge group SU(N_c) and has $N_f\leqslant N_c$ flavors: quarks $Q^i, i=1,\cdots,N_f$, in fundamentals of SU(N_c) and antiquarks $\tilde{Q}_i, i=1,\cdots,N_f$ in antifundamentals of SU(N_c).

Below the scale of dynamical breaking of the gauge symmetry Λ, the effective degrees of freedom are the fermion condensate ("pion") fields $\Pi^i_j\equiv Q^i\tilde{Q}_j$. The dynamically generated superpotential reads [15]

$$W=(N_c-N_f)\frac{\Lambda^{(3N_c-N_f)/(N_c-N_f)}}{(\det\Pi)^{1/(N_c-N_f)}}. \tag{9}$$

[2]For a similar analysis, although in a different context, see Ref. [10].

Usually, one allows a term linear in Π in the superpotential in order to stabilize this field. We will instead assume here that a discrete symmetry ensures that no linear term is allowed by the Abelian symmetry. Let us note that this symmetry cannot be a continuous gauge symmetry since this would yield in the scalar potential D terms with positive powers of Π which would stabilize the field.

The effective Lagrangian reads

$$\mathcal{L}=-\frac{1}{2}\mathrm{Tr}[(\Pi^\dagger\Pi)^{-1/2}\partial_\mu\Pi\partial^\mu\Pi^\dagger] + 2\,\mathrm{Tr}\left[(\Pi^\dagger\Pi)^{-1/2}\frac{(\Lambda\Lambda^\dagger)^{(3N_c-N_f)/(N_c-N_f)}}{(\mathrm{Det}\,\Pi^\dagger\Pi)^{1/(N_c-N_f)}}\right], \quad (10)$$

where the potential originates from the F term for the field Π. For simplicity, we will take Π^i_j to be diagonal and write $\Pi^i_j \equiv \Phi^2\delta^i_j$ with Φ real. One obtains

$$\frac{1}{4N_f}\mathcal{L}=-\frac{1}{2}\partial^\mu\Phi\partial_\mu\Phi+V(\Phi), \quad (11)$$

where

$$V(\Phi)=\lambda\frac{\mu^{4+\alpha}}{\Phi^\alpha}, \quad (12)$$

with $\mu=(\Lambda\Lambda^\dagger)^{1/2}$ and

$$\alpha=2\frac{N_c+N_f}{N_c-N_f}. \quad (13)$$

The corresponding potential has been studied in Ref. [11] in the case where ρ_B dominates over the energy density ρ_Φ of the Φ field. One obtains

$$\frac{\rho_\Phi}{\rho_B}=\left(\frac{a}{a_Q}\right)^{6(1+w_B)/(2+\alpha)}. \quad (14)$$

Hence ρ_Φ decreases less rapidly than ρ_B until it dominates it for values of the cosmic scale factor larger than a_Q. Throughout this period (which must obviously include nucleosynthesis), one has

$$\rho_\Phi=\frac{2(2+\alpha)}{4+\alpha(1-w_B)}\left(\frac{3(1+w_B)}{\alpha(2+\alpha)}\right)^{\alpha/2}\times\lambda\frac{\mu^{4+\alpha}}{M_P^\alpha}\left(\frac{a}{a_Q}\right)^{6(1+w_B)/(2+\alpha)}$$

$$\Phi=M_P\sqrt{\frac{\alpha(2+\alpha)}{3(1+w_B)}}\left(\frac{a}{a_Q}\right)^{3(1+w_B)/(2+\alpha)}. \quad (15)$$

The equation of state for the Φ field has [6]

$$w_\Phi=-1+\frac{\alpha(1+w_B)}{2+\alpha}. \quad (16)$$

Thus, in a matter-dominated universe $(w_B=0)$, $w_\Phi=-1/2+2N_f/N_c$ which is between $-1/2$ and 0 for $N_f\leqslant N_c$. This provides a candidate for the dynamics of quintessence.

Once Φ/M_P has reached the value $\sqrt{\alpha(2+\alpha)/3(1+w_B)}$, we enter a different regime where ρ_Φ dominates the energy density. The field Φ slows down and one may solve for it neglecting the terms $\ddot{\Phi}$ in its equation of motion and $\dot{\Phi}^2/2$ in ρ_Φ. One obtains

$$\Phi=\Phi_0\left[1+\frac{1}{2\sqrt{3}}\alpha(4+\alpha)V(\Phi_0)(t-t_0)\right]^{2/(4+\alpha)}, \quad (17)$$

where Φ_0 is the present value for Φ, and one obtains

$$w_\Phi\sim-1+\frac{\alpha^2}{3\Phi^2}. \quad (18)$$

If ρ_Φ at a_Q is already close to the present value (this occurs typically for $\mu\sim10^{-12+30\alpha/(4+\alpha)}$ GeV), this second period is short $(a_Q\sim a_0)$ and w_ϕ will be given approximately by Eq. (16). For simplicity, we will suppose from now on that this is so. In this case, the value of w_Φ might prove to be too small to account for the data [16].

However, larger values for w_Φ may be obtained by complicating slightly the model and introducing other fields. As an example, we will assume the presence of a dilaton field, much in the spirit of the models of the previous section (although the dilaton is this time not sliding but fixed at its ground state value). The dynamical scale Λ is expressed in terms of the dilaton through Eq. (2) with $b_0=(3N_c-N_f)/(16\pi^2)$. This induces a new term in the scalar potential:

$$\delta V=4s^2|F_s|^2, \quad (19)$$

with

$$F_s=\frac{dW}{ds}=-8\pi^2\frac{\Lambda^{(3N_c-N_f)/(N_c-N_f)}}{(\mathrm{Det}\,\Pi)^{1/(N_c-N_f)}}. \quad (20)$$

that is, an extra term of the form $\mu^{4+\beta}/\Phi^\beta$ with

$$\beta=\frac{4N_f}{N_c-N_f}. \quad (21)$$

Since $\beta<\alpha$, this term dominates for large values of the condensate Φ and, for $w_B=0$,

$$w_\Phi=-1+\frac{2N_f}{N_c+N_f}, \quad (22)$$

which precisely lies between -1 and 0: taking, for example, $N_c=5$ and $N_f=1$ yields $w_\Phi=-2/3$.

There could be other contributions to the F-term auxiliary field for S, say, F_0 (which will contribute to supersymmetry breaking). If so, the leading term in δV for large Φ is $F_s^\dagger F_0+F_sF_0^\dagger$ and $\beta=2N_c/(N_c-N_f)$, in which case $w_\Phi=-1+N_f/N_c$. This time, one may even obtain $w_\Phi=-2/3$ with $N_c=3$ $(N_f=1)$.

Strictly speaking, the leading term is $|F_0|^2$ and thus of the cosmological constant type. But this is an artifact of global supersymmetry and it is well known that, by going to supergravity, we may cancel this cosmological constant term, while keeping a nonvanishing contribution F_0 to the F term of the S field. Such a study goes beyond the framework of this paper. This stresses, however, an important fact: even if we deal here with a dynamical component (Φ) which may account for a cosmological-constant-type behavior of the cosmological parameters, it is important that the Φ energy density eventually dominate over all other forms and thus that these other components do not produce a significant cosmological constant of their own. Thus, the cosmological constant remains a problem for all other components.

Likewise, the amount of supersymmetry breaking due to the fact that Φ has not reached an infinite value (and thus its F term is not vanishing) is not sufficient to account for the amount of supersymmetry breaking observed in nature. There must be other sources (e.g., F_0 in our example) which may produce unwanted amounts of cosmological constant if care is not taken.

In other words, there is still a ''cosmological constant problem'' in the models studied here (that is to say, from the point of view of the quantum theory) but the interest of such models lies in the fact that they can successfully account for the recent cosmological data on supernovas of type Ia, if confirmed.

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