



Electromagnetically induced absorption via spontaneously generated coherence of a Λ system

Cheng-pu Liu ^{a,*}, Shang-qing Gong ^a, Xi-jun Fan ^b, Zhi-zhan Xu ^a

^a Key Laboratory for High Intensity Optics, Shanghai Institute of Optics and Fine Mechanics, Shanghai 201800, China

^b Department of Physics, Shandong Normal University, Jinan 250014, China

Received 25 July 2003; received in revised form 5 December 2003; accepted 5 December 2003

Abstract

The effect of spontaneously generated coherence (SGC) on the pump–probe response of a nearly degenerate Λ system is investigated by taking into account the dephasing of the low-frequency coherence. It is found, in the case of small dephasing, that instead of electromagnetically induced transparency (EIT) at resonance, electromagnetically induced absorption (EIA) can occur due to the effect of SGC. We also study the effect of relative phase between the two applied fields and find that EIA and EIT can transform mutually by adjusting the relative phase.

© 2003 Elsevier B.V. All rights reserved.

PACS: 42.50.Gy; 42.50.Md; 42.50.Hz

1. Introduction

It is now well understood how the decay of a system with closely lying states induced by interaction with a common bath leads to one new type of coherence, generally called as spontaneously generated coherence (SGC). Recently, there is much interest in the study of this kind of coherence. It can be created by the interference of spontaneous emission of either a single excited level to two closely lying atomic levels (Λ -type atom) [1] or two closely lying atomic levels to a common atomic level (V-type atom) [2]. In a ladder-type system, it

can be also created in a nearly spaced atomic levels case [3,4]. The existence of such coherence effect depends on the nonorthogonality of the two dipoles transition matrix elements. Xia et al. [5] carried out the first experimental investigation of constructive and destructive interference effects in spontaneous emission. This experiment showed some interesting effects induced by SGC. The effects of SGC on absorption and dispersion, population inversion and lasing without inversion, or resonance fluorescent spectrum etc., have been extensively investigated recently [6–14].

Menon and Agarwal [6] investigated the effect of SGC on a Λ -type system with nearly degenerate lower levels and found that such coherence preserves both electromagnetically induced transparency (EIT) [15] and coherent population trapping (CPT)

* Corresponding author. Tel.: +86-02169918266; fax: +86-02169918800.

E-mail address: chpliu@yahoo.com.cn (C.-p. Liu).

[16] phenomena. But the system considered did not include the dephasing between the two ground states. In practice, the dephasing is present in realistic systems [17]. Tao et al. [18] found, due to the dephasing between the two ground states, the most effective enhancement of Kerr nonlinearity occurs. How does the dephasing alter the property of the medium? Some years ago, Friedmann et al. [19] investigated the properties of the gain, the refractive index and the noise for a degenerate Λ system interacting with a single pump and weak probe and found that they have the same linear dependence on the rate of the collisional relaxation rate between the two lower lying states due to the collisional transfer of population between the ground states leading to a deviation from exact CPT, so that there is a strong correlation between gain without inversion, enhanced refractive index and noise in these systems.

In this paper we investigate the effect of SGC on the pump–probe response of a nearly degenerate Λ system by taking into account the dephasing of the low-frequency coherence which is setting very small, and find that the presence of SGC will destroy the linear dependence of the absorption in this system on the collisional relaxation rate; moreover, under optimal SGC the absorption property of the medium can be significantly altered. Electromagnetically induced absorption (EIA) can occur instead of EIT at two photon resonance. The effect of relative phase between the two applied fields is also investigated and it is found that adjusting the relative phase can make EIT and EIA transform mutually.

The organization of this paper is as follows: In Section 2, we present the density-matrix equations for the atomic model. In Section 3, we show the effects of SGC on the absorption and dispersion line shapes. In Section 4, we discuss the effect of relative phase between the two applied fields. In Section 5, we develop an analysis which explains the numerical results of Sections 3 and 4. Finally, some conclusions are shown in Section 6.

2. Model and equations

Considering a closed Λ -type medium with excited state $|1\rangle$ and closely lying lower states, $|2\rangle$ and

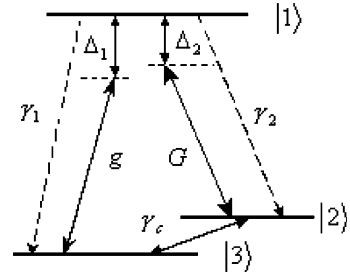


Fig. 1. Λ -type three-level atomic system. $2g(2G)$ denotes the Rabi frequency of the probe field (pump field); $2\gamma_1$ and $2\gamma_2$ are the decay rates, whereas collisional phase decay of the $|2\rangle \leftrightarrow |3\rangle$ polarization occurs at rate $2\gamma_c$; Δ_1, Δ_2 are the detunings of both fields.

$|3\rangle$, as illustrated in Fig. 1. Since the dipole moments are not orthogonal, we have to consider an arrangement where each field acts only on one transition. The excited state $|1\rangle$ decays to $|3\rangle$ and $|2\rangle$ with decay rates $2\gamma_1$ and $2\gamma_2$, respectively. A coherent probe field with Rabi frequency $2g$ drives the transition between states $|3\rangle$ and $|1\rangle$, and a pump field with Rabi frequency $2G$ is applied to the transition $|1\rangle$ and $|2\rangle$. Under the rotating wave approximation the density-matrix equation can be derived as,

$$d\rho_{11}/dt = -2(\gamma_1 + \gamma_2)\rho_{11} + ig\rho_{31} + iG\rho_{21} - iG^*\rho_{12} - ig^*\rho_{13}, \quad (1a)$$

$$d\rho_{22}/dt = 2\gamma_2\rho_{11} - iG\rho_{21} + iG^*\rho_{12}, \quad (1b)$$

$$d\rho_{33}/dt = 2\gamma_1\rho_{11} - ig\rho_{31} + ig^*\rho_{13}, \quad (1c)$$

$$d\rho_{12}/dt = -(\gamma_1 + \gamma_2 + \gamma_c + i\Delta_2)\rho_{12} + ig\rho_{32} - iG(\rho_{11} - \rho_{22}), \quad (1d)$$

$$d\rho_{13}/dt = -(\gamma_1 + \gamma_2 + i\Delta_1)\rho_{13} + iG\rho_{23} - ig(\rho_{11} - \rho_{33}), \quad (1e)$$

$$d\rho_{23}/dt = -[\gamma_c + i(\Delta_1 - \Delta_2)]\rho_{23} + 2p\sqrt{\gamma_1\gamma_2}\rho_{11} + iG^*\rho_{13} - ig\rho_{21}, \quad (1f)$$

with the closure relation $\rho_{11} + \rho_{22} + \rho_{33} = 1$. Here $\Delta_1 (= \omega_{13} - \omega_g)$ and $\Delta_2 (= \omega_{12} - \omega_G)$ are the frequency detunings of the two laser fields from their corresponding transitions. $\hbar\omega_{ij}(i, j = 1, 2, 3)$ is the energy separation between states $|i\rangle$ and $|j\rangle$, and ω_g (ω_G) is the frequency of the probe (pump) field. Note that we have included in Eq. (1) a collision-induced perturbation $2\gamma_c$ of the energy of level $|2\rangle$

leading to a dephasing of the polarizations of $|1\rangle \leftrightarrow |2\rangle$ and $|2\rangle \leftrightarrow |3\rangle$, but we have neglected the collisional dephasing of the $|1\rangle \leftrightarrow |3\rangle$ polarization. Fleischhauer and Scully [20] pointed out that this is a good approximation as long as under suitable conditions. The term $p\sqrt{\gamma_1\gamma_2}$ in Eq. (1f) represents the effect of SGC resulting from the cross-coupling between the transitions $|1\rangle \rightarrow |2\rangle$ and $|1\rangle \rightarrow |3\rangle$. It should be noted that only for nearly degenerated lower levels are the effects of generated coherence between $|2\rangle$ and $|3\rangle$ important, as for larger energy-levels separation the rapid oscillations in ρ_{23} will average out any such effects. The parameter p denotes the alignment of the two transition matrix elements determining the strength of the interference in spontaneous emission and is defined as $p = \vec{u}_{12} \cdot \vec{u}_{13} / |\vec{u}_{12} \cdot \vec{u}_{13}| = \cos \theta$, where θ is the angle between the two induced dipole moments \vec{u}_{12} and \vec{u}_{13} . The Rabi frequencies are connected to the p parameter by the relation $G = G_0\sqrt{1-p^2} = G_0 \sin \theta$ and $g = g_0\sqrt{1-p^2} = g_0 \sin \theta$, G_0 and g_0 are the Rabi frequencies corresponding to zero SGC. The parameter p plays an important role in the creation of coherence and has significant effect on the dynamics of systems, which we will show subsequently. The steady-state solutions can then be found by setting all the time derivatives in Eq. (1) to zero and reducing it to a set of coupled 9×9 algebraic equations after splitting into real and imaginary parts. In numerical calculations, we will use computation package Maple and choose the parameters to be dimensionless units by scaling with γ (and $\gamma = 1$).

3. Effect of spontaneously generated coherence on dynamics

It is well known that when the probe absorption spectrum is characterized by a deep dip at resonance, the phenomenon is called EIT. However, when the absorption spectrum is characterized by a sharp peak at resonance, the phenomenon is called EIA [21,22]. Generally, EIA cannot be generated in conventional three-level atomic system. While by taking into account the dephasing of the low-frequency coherence we will show EIA can occur in such a system under optimal SGC.

Firstly considering the ideal Λ system, that is, the collision-induced dephasing $\gamma_c = 0$. Setting parameters $G = g = 10 \sin \theta \gamma$, $\gamma_1 = \gamma_2 = \gamma$ and the pump field is in resonance $\Delta_2 = 0$, we give out the energy absorption from probe $\text{Im}\rho_{13}$ and dispersion plots for $\text{Re}\rho_{13}$ as function of probe detuning for different parameter p , shown in Fig. 2(a) and (b), respectively. The solid line, dashed line and dash dotted line correspond to $p = 0.99, 0.9, 0.0$, respectively. Fig. 2(a) shows that EIT is preserved regardless of the variance of p which is identical to that in [6]. Moreover the stronger the SGC, the narrower the transparency window is. When $p = 0.99$, the line-width is close to zero. Therefore

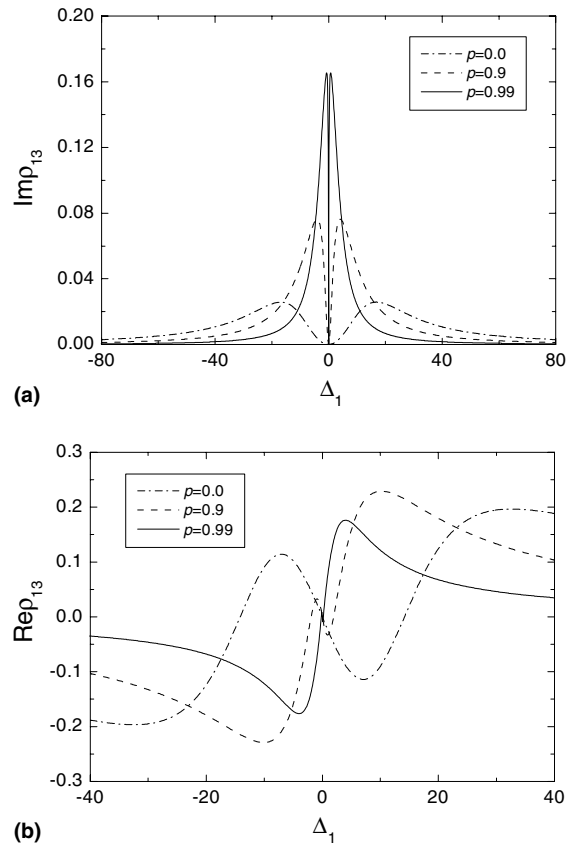


Fig. 2. Energy absorption from probe $\text{Im}\rho_{13}$ (a) and dispersion plots for $\text{Re}\rho_{13}$ (b) as function of probe detuning for different parameter p with $\gamma_c = 0$. The solid line, dashed line and dash dotted line correspond to $p = 0.99, 0.9, 0.0$, respectively. The other parameters are set as $G = g = 10 \sin \theta \gamma$, $\gamma_1 = \gamma_2 = \gamma$, $\Delta_2 = 0$.

it is very favorable for realization of efficient EIT-based nonlinear transformations and light storage. Fig. 2(b) shows that near the resonance domain of probe field, dispersion curve becomes much steeper as the parameter p increases. The combination of zero absorption and steep positive dispersion variance can lead to a dramatic slowing down of light and consequently large time delay [23].

In practice the dephasing of the low-frequency coherence exists in all realistic systems, though it is trivial ($\gamma_c \ll \gamma_1, \gamma_2$) [17]. Setting $\gamma_c = 0.05\gamma$ and the other parameters same as in Fig. 2, we get the energy absorption from probe $\text{Im}\rho_{13}$ for several different parameter p , shown in Fig. 3. Fig. 3 shows that, EIT can still occur in absence of the effect of SGC ($p = 0.0$). The behavior of the system for small values of p is similar to that for $p = 0.0$. However, in the case of large value p , such as $p = 0.9$, the inclusion of SGC obviously destroys EIT and obvious absorption at zero detuning appears. By increasing the p parameter to 0.99, the response of the system to the probe field can be completely altered, a sharp absorption peak centers at zero detuning (i.e. EIA) occurs. The authors of [21,22] proposed a simple theoretical model for EIA, which is in a four-state N -figuration system (Λ -type plus an auxiliary level transition driven by a control field) and found that the appearance of EIA is the consequence of coherence transfer from

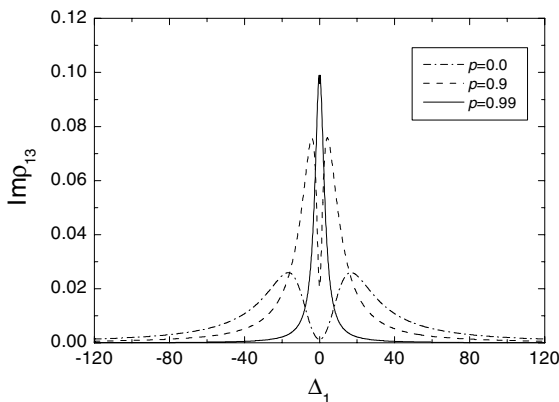


Fig. 3. Energy absorption from probe $\text{Im}\rho_{13}$ as function of probe detuning for different parameter p with $\gamma_c = 0.05\gamma$. The solid line, dashed line and dash dotted line correspond to $p = 0.99, 0.9, 0.0$, respectively. The other parameters are the same as in Fig. 2.

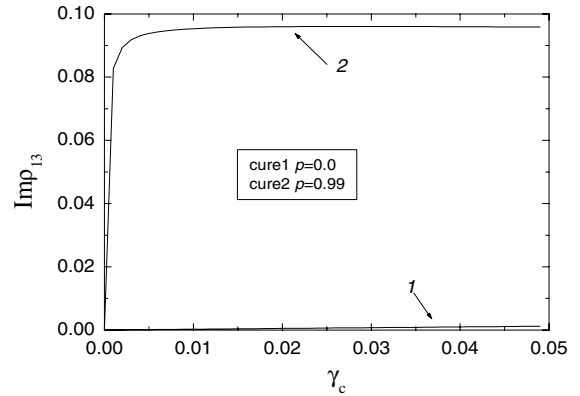


Fig. 4. Energy absorption from probe $\text{Im}\rho_{13}$ as function of the collisional relaxation rate γ_c and 2 correspond to $p = 0.0$ and $p = 0.99$, respectively. The other parameters are the same as in Fig. 2.

the excited levels to the lower ones via spontaneous emission, while here in the nearly degenerate Λ type system, the physical origin for generating EIA is obviously the existence of SGC.

Now we vary the collision-induced relaxation rate and investigate the changes in absorption (or gain) correspondingly. In Fig. 4, we plot the varying curves of absorption $\text{Im}\rho_{13}$ at $\Delta_1 = 0$ vs. the collisional relaxation rate γ_c . Firstly if the SGC is absent, from cure 1 we can see that the variance of $\text{Im}\rho_{13}$ vs. γ_c is linear which is similar to that in [19]. However if taking the SGC into account (taking $p = 0.99$ for example), the result is completely altered. Curve 2 shows that the variance trend of absorption vs. relaxation rate changes into nonlinear. Under condition that the collisional relaxation rate is zero, regardless of SGC, the absorption is always zero showing exact EIT preserved. With not too much relaxation rate, no SGC, the absorption is near to zero and the system shows good EIT; but when SGC is included and set optimal $p = 0.99$, a larger absorption occurs according to the same relaxation rate and EIT is destroyed significantly.

4. Effect of relative phase between the two coherent fields

The usual EIT experiments with well-separated ground levels in a Λ system do not depend on the

relative phase between the two applied fields. However, in the case of closely spaced levels, SGC makes the system quite sensitive to the relative phase between the two applied fields [6,7,24]. The p dependent terms are always accompanied by a phase dependent term $\exp(\pm i\Phi)$ where Φ denotes the relative phase between the two laser fields. Menon and Agarwal [6] pointed out that EIT is preserved regardless of what the relative phase is. But this is correct only in the limit when the relaxation of the low-frequency coherence vanishes. Adopting the same parameters as those in Fig. 3, we get two energy absorption curves from probe $\text{Im}\rho_{13}$ as function of probe detuning for two different relative phases Φ with $p = 0.99$, shown in Fig. 4. From this figure, we find that under optimal SGC, when Φ is π , the system takes on good EIT; while for Φ is zero or 2π , EIA appears. This acts like an optical switch. Tuning the relative phase between the two applied fields from π to zero, the medium becomes from EIT to EIA, vice versa.

5. Physical explanation of above numerical results

We now demonstrate how to understand the numerical results in above two sections by analyzing the original density matrix Eq. (1) in a field-dependent basis given by

$$|1\rangle, |+\rangle = \frac{G|2\rangle + g|3\rangle}{F}, \quad |-\rangle = \frac{g|2\rangle - G|3\rangle}{F}, \quad (2)$$

here $F = \sqrt{G^2 + g^2}$. We assume the CPT condition $\Delta_1 = \Delta_2$ throughout this section. Note that basis (2) is different from the dressed state basis which involving mixing of $|1\rangle$ and $|+\rangle$ states. For understanding the numerical results, basis (2) turns out to be useful. In this basis, we can easily get

$$\rho_{++} = (G^2\rho_{22} + g^2\rho_{33} + Gg\rho_{23} + Gg\rho_{32})/F^2, \quad (3a)$$

$$\rho_{--} = (g^2\rho_{22} + G^2\rho_{33} - Gg\rho_{23} - Gg\rho_{32})/F^2, \quad (3b)$$

with the relation $\rho_{11} + \rho_{++} + \rho_{--} = 1$. Here ρ_{++} (ρ_{--}) is the population of the bright (dark) state. It is well known that that all atoms populated in the dark state (that is $\rho_{--} = 1$, called as CPT)

corresponds to ideal EIT. While when atoms are partly populated in the dark state (that is $\rho_{--} < 1$) EIT is destroyed. So we can give out the reasonable explanation of phenomena by investigating the variance of population ρ_{--} . Adopting the parameters in Figs. 2, 3 and 5, respectively, we get the trapping population ρ_{--} and population ρ_{11} in the excited state as function of the probe detuning shown in Figs. 6–8 correspondingly. Fig. 6 shows

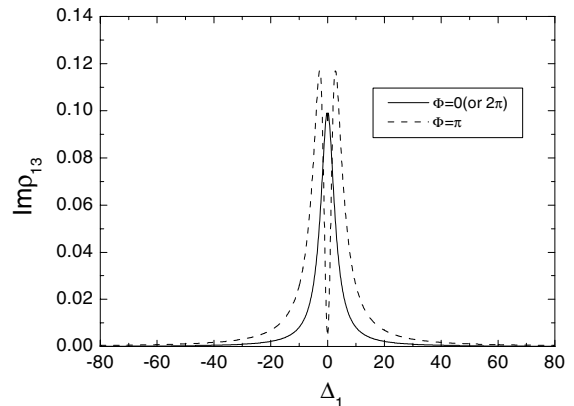


Fig. 5. Energy absorption from probe $\text{Im}\rho_{13}$ as function of probe detuning for different relative phase Φ with $p = 0.99$. The solid line and dashed line correspond to $\Phi = 0\pi$ (or 2π) and $\Phi = \pi$, respectively. The other parameters are the same as in Fig. 2.

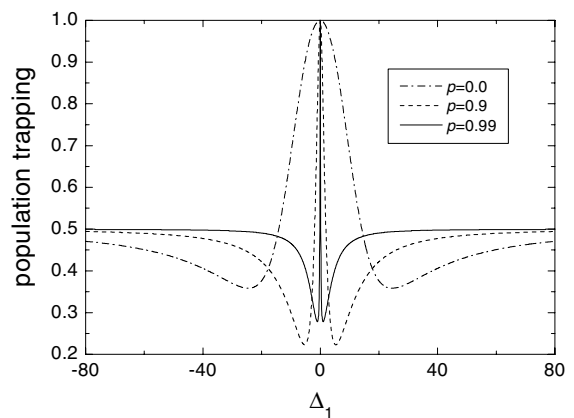


Fig. 6. Trapping population ρ_{--} as function of probe detuning for different parameter p with $\gamma_c = 0.0$. The solid line, dashed line and dash dotted line correspond to $p = 0.99, 0.9, 0.0$, respectively. The other parameters are the same as in Fig. 2.

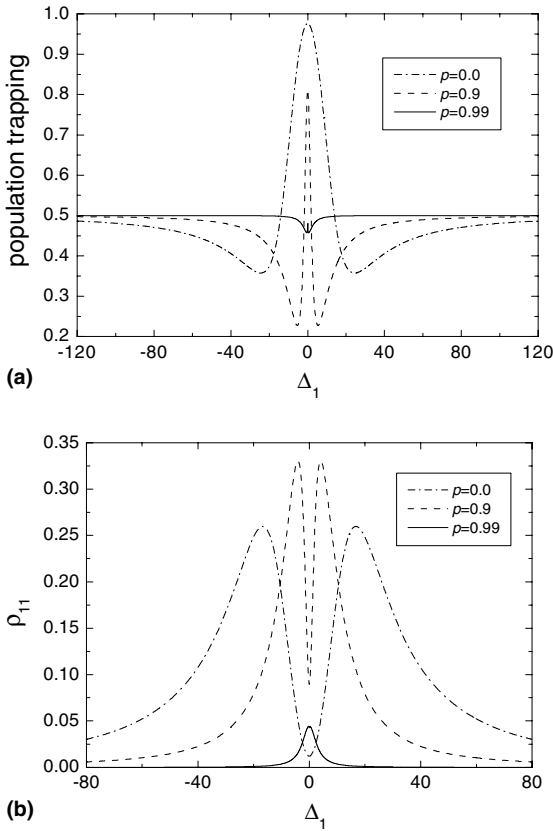


Fig. 7. Trapping population ρ_{--} (a) and population ρ_{11} (b) as function of probe detuning for different parameter p with $\gamma_c = 0.05\gamma$. The solid line, dashed line and dash dotted line correspond to $p = 0.99, 0.9, 0.0$, respectively. The other parameters are the same as in Fig. 2.

that CPT is preserved which is identical to that in [6], which leads to ideal EIT. From Fig. 7(a) we can see that ρ_{--} closes to 1 when $p = 0.0$ (in the absence of SGC), so the system can still take on good EIT. While p is large enough, the population in state ρ_{--} will obviously decrease. For the case $p = 0.99$, ρ_{--} closes to 0.5, that is, only one half of all atoms is trapped, and simultaneously ρ_{11} is smaller than 0.05 (shown in Fig. 7(b)). In this case, CPT is destroyed, so instead of EIT, EIA occurs. Considering the relative phase between the two fields, similar things appear. Tuning the relative phase from zero to π , ρ_{--} changes from approximate 0.5 to near 1, that is, EIA changes into EIT (see Fig. 8).

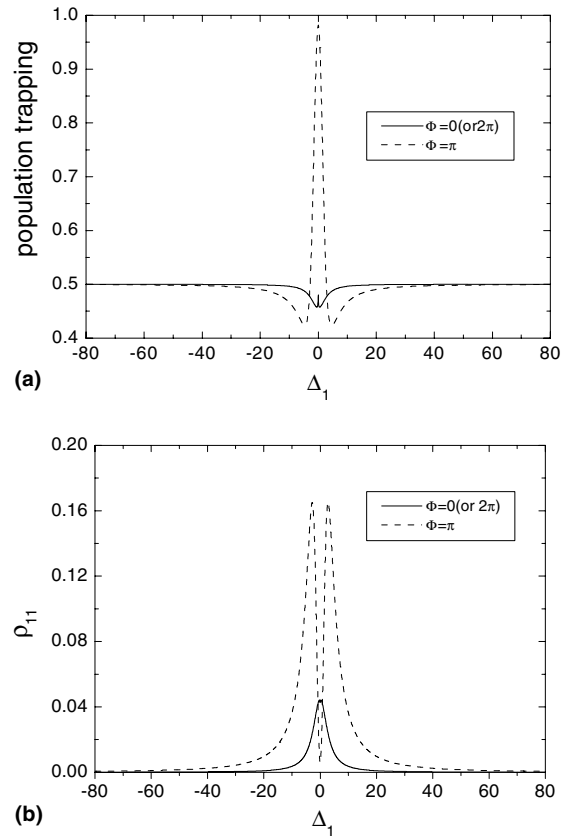


Fig. 8. Trapping population ρ_{--} (a) and population ρ_{11} (b) as function of probe detuning for different relative phase Φ with $p = 0.99$. The solid line and dashed line correspond to $\Phi = 0\pi$ (or 2π) and $\Phi = \pi$, respectively. The other parameters are the same as in Fig. 2.

6. Conclusions

In this paper, we investigated the effect of SGC on the pump–probe response of a nearly degenerate Λ system. Taking into account the dephasing of the low-frequency coherence, under optimal SGC, we found that EIA can occur instead of EIT at resonance. The effect of relative phase between the two applied fields was also given. Tuning the relative phase from π to zero, the medium becomes from EIT to EIA, vice versa. A simple physical explanation from a viewpoint of population trapping was given. These results could be experimentally observed provided that the dipole elements from the two optical transitions in the Λ

system are nonorthogonal. In fact such nonorthogonality has been obtained from the mixing of the levels arising from internal fields [5,25].

Acknowledgements

This work is supported by the National Natural Science Key Foundation of China (Grant No. 10234030).

References

- [1] J. Javanainen, *Europhys. Lett.* 17 (1992) 407.
- [2] P. Zhou, S. Swain, *Phys. Rev. Lett.* 77 (1996) 3995.
- [3] R.M. Whitley, C.R. Stroud Jr., *Phys. Rev. A* 14 (1976) 1498.
- [4] Z. Ficek, B.J. Dalton, P.L. Knight, *Phys. Rev. A* 51 (1995) 4062;
H.M. Ma, S.Q. Gong, C.P. Liu, Z.R. Sun, Z.Z. Xu, *Opt. Commun.* 223 (2003) 97.
- [5] H.R. Xia, C.Y. Ye, S.Y. Zhu, *Phys. Rev. Lett.* 77 (1996) 1032.
- [6] S. Menon, G.S. Agarwal, *Phys. Rev. A* 57 (1998) 4014.
- [7] W.H. Xu, J.H. Wu, J.Y. Gao, *Phys. Rev. A* 66 (2002) 063812;
Opt. Commun. 215 (2003) 345;
J.H. Wu, J.Y. Gao, *Phys. Rev. A* 65 (2002) 063807.
- [8] S.Q. Gong, Y. Li, S.D. Du, Z.Z. Xu, *Phys. Lett. A* 259 (1999) 43.
- [9] J. Evers, D. Bullock, C.H. Keitel, *Opt. Commun.* 209 (2002) 173.
- [10] E. Paspalakis, S.Q. Gong, P.L. Knight, *Opt. Commun.* 152 (1998) 293.
- [11] L. Li, X. Wang, J. Yang, G. Lazarov, J. Qi, A.M. Lyyra, *Phys. Rev. Lett.* 84 (2000) 4016.
- [12] J. Wang, H.M. Wiseman, Z. Ficek, *Phys. Rev. A* 62 (2000) 013818.
- [13] P. Zhou, S. Swain, *Phys. Rev. Lett.* 78 (1997) 832;
P. Dong, S.H. Tang, *Phys. Rev. A* 65 (2002) 033816.
- [14] S. Menon, G.S. Agarwal, *Phys. Rev. A* 61 (1999) 013807;
X.M. Hu, J.S. Peng, *Opt. Commun.* 170 (1999) 259.
- [15] S.E. Harris, *Phys. Today* 50 (1997) 36, and references therein;
S.E. Harris, *Phys. Rev. Lett.* 82 (1999) 4611;
S.Q. Gong, Z.Z. Xu, *Opt. Commun.* 110 (1995) 65;
A.G. Litvak, M.D. Tokman, *Phys. Rev. Lett.* 88 (2002) 095003.
- [16] G. Alzetta, A. Gozzini, L. Moi, G. Orriols, *Nuovo Cimento B* 36 (1976) 5;
E. Arimondo, in: E. Wolf (Ed.), *Progress in Optics*, vol. 35, Elsevier, Amsterdam, 1996.
- [17] M.M. Kash, V.A. Sautenkov, A.S. Zibrov, L. Hollberg, G.R. Welch, M.D. Lukin, Y. Rostovtsev, E.S. Fry, M.O. Scully, *Phys. Rev. Lett.* 82 (1999) 5229.
- [18] H. Tao, M.W. Jack, M. Yamashita, T. Mukai, *Opt. Commun.* 214 (2002) 371.
- [19] H. Friedmann, A.D. Wilson-Gordon, *Opt. Commun.* 98 (1993) 303;
A. Rosenhouse-Dantsker, A.D. Wilson-Gordon, H. Friedmann, *Opt. Commun.* 113 (1994) 226.
- [20] M. Fleischhauer, M.O. Scully, *Phys. Rev. A* 49 (1994) 1973.
- [21] A.M. Akulshin, S. Barreiro, A. Lezama, *Phys. Rev. A* 57 (1998) 2996;
A.V. Taichenachev, A.M. Tumaikin, V.I. Yudin, *Phys. Rev. A* 61 (1999) 011802;
C. Goren, A.D. Wilson-Gordon, M. Rosenbluh, H. Friedmann, *Phys. Rev. A* 67 (2003) 033807.
- [22] S.E. Harris, Y. Yamamoto, *Phys. Rev. Lett.* 81 (1998) 3611;
A. Lezama, S. Barreiro, A.M. Akulshin, *Phys. Rev. A* 59 (1999) 4732;
W.H. Xu, J.Y. Gao, *Phys. Rev. A* 67 (2003) 033816.
- [23] D. Bortman-Arbiv, A.D. Wilson-Gordon, H. Friedmann, *Phys. Rev. A* 63 (2001) 043818.
- [24] E. Paspalakis, P.L. Knight, *Phys. Rev. Lett.* 81 (1998) 293.
- [25] H. Schmidt, A. Imamoglu, *Opt. Commun.* 131 (1996) 333.