

## Dephasing Times in a Nondegenerate Two-Dimensional Electron Gas

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Studies of weak localization by scattering from vapor atoms for electrons on a liquid helium surface are reported. There are three contributions to the dephasing time. Dephasing by the motion of vapor atoms perpendicular to the surface is studied by varying the holding field to change the characteristic width of the electron layer at the surface. A change in vapor density alters the quasielastic scattering length and the contribution to dephasing due to the motion of atoms both perpendicular and parallel to the surface. Dephasing due to the electron-electron interaction is dependent on the electron density.

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Weak localization of degenerate electrons by elastic scattering from static impurities has been a topic of serious study for the last two decades [1]. Recently there has been a revival of interest in the damping of weak localization in these systems by the electron-electron interaction [2–4]. In comparison there have been few studies of weak localization in nondegenerate systems in which localization is due to quasielastic scattering from slowly moving impurities [5–7]. Adams and Paalanen explored both weak and strong localization of electrons on a solid hydrogen surface [6–8]. Localization occurred as a result of scattering from surface imperfections and from helium atoms that were introduced above the surface. In our system electrons are confined to two dimensions above a liquid helium surface. Weak localization results from quasielastic scattering from slowly moving helium vapor atoms. This system is particularly interesting because it possesses an unusual mechanism for damping quantum effects, namely, the motion of the vapor atoms. At the same time there are a number of experimentally tunable parameters (the electron density, the holding field that helps confine the electrons to the helium surface, and the vapor density) that can be varied to separate various damping mechanisms. In this Letter we report a systematic investigation of the dephasing times in this nondegenerate two-dimensional electron gas.

Weak localization is a quantum effect that results from constructive interference between closed electron paths and their time-reversed counterparts. This constructive interference increases the probability of backscattering and results in an increase in resistivity over the classical Drude value. In addition to the electron-electron interaction, in our system weak localization is damped by the slow motion of the helium vapor atoms. The velocity of thermal helium atoms is 1% of the electron velocity and the fractional change in electron energy in a collision is  $\approx 10^{-2}$ . There is an important distinction in the way in which weak localization is suppressed by the vertical and horizontal motions of the helium vapor atoms. Horizontal motion changes the lengths of the paths introducing a random relative phase between a path and its approximate time-reversed counterpart, thereby washing out their interference [9,10]. In con-

trast vertical motion suppresses weak localization because roughly speaking the scattering atom may not be present for both the forward and return paths, thereby reducing the weight of the interference contribution. Below we estimate the dephasing rate due to vertical motion of the vapor atoms; more details will be given elsewhere [11]. Similar ideas have been expressed in Ref. [12], but the precise formula we obtain is different. The corresponding discussion of horizontal motion is given in Refs. [9,10].

We use a Corbino geometry consisting of four electrodes located beneath the helium liquid. The resistivity is measured by capacitively coupling a low frequency ac current through the electron layer [13]. A normal field is applied to the inner three electrodes which are used for the resistivity measurement. When the holding field  $E_{\perp}$  is greater than the saturated field  $E_s = ne/2\epsilon_0$ , a positive voltage  $V_0$  is applied to the outermost guard electrode to compensate for fringing fields. We adjust the guard voltage to maximize the signal. The signal amplitude decreases if either the area of the third electrode covered with electrons is reduced (which reduces the capacitance between the electrons and the electrode) or if the diameter of the electron pool becomes sufficiently large that it capacitively couples to the guard ring. Numerical calculation with this optimum value of  $V_0$  indicates a nearly uniform electron density above the three inner electrodes.

For a nondegenerate, two-dimensional electron gas, the longitudinal conductivity in a magnetic field  $B$  perpendicular to the plane of electrons is given by [7,14]

$$\sigma_{xx} = \frac{-n_0 e^2}{m(k_B T)^2} \int_{E_c}^{\infty} dE \frac{e^{-E/k_B T}}{1 + (\mu B)^2} \times \left\{ E\tau_0 - \frac{\hbar}{2\pi} \left[ \Psi\left(\frac{\hbar m}{4eBE\tau_0^2} + \frac{1}{2}\right) - \Psi\left(\frac{\hbar m}{4eBE\tau_0\tau_{\phi}} + \frac{1}{2}\right) \right] \right\}. \quad (1)$$

Here  $n$  is the electron density,  $m$  is the electron mass,  $\mu$  is the mobility,  $\Psi$  is the digamma function,  $\tau_0$  is the quasielastic scattering time,  $\tau_{\phi}$  is the dephasing time,  $E$  is the energy, and  $E_c$  is the cutoff energy below which electrons are localized. The first term in the curly brackets

gives the Drude resistivity, and the second term gives the weak-localization correction. We assume that the total dephasing rate is given by

$$\tau_{\phi}^{-1} = \tau_{e-e}^{-1} + \tau_v^{-1} + \tau_h^{-1}. \quad (2)$$

Here  $\tau_{e-e}$ ,  $\tau_v$ , and  $\tau_h$  are the dephasing times due to the electron-electron interaction, vertical motion, and horizontal motion, respectively.

The dephasing time  $\tau_{\phi}$  is measured by fitting the longitudinal magnetoresistance. Figure 1 shows a graph of the longitudinal resistivity ( $1/\sigma_{xx}$ ) versus magnetic field. The solid line is given by Eq. (1) with the parameters  $\tau_0$  and  $\tau_{\phi}$  adjusted to give the best fit [15]. The dashed line is the Drude resistivity. In our analysis we calculate  $E_c$  self-consistently from the expression [6,10]

$$E_c = (\hbar/2\pi\tau_0) \ln(\tau_{\phi}/\tau_0). \quad (3)$$

The fits are relatively insensitive to the value of  $E_c$  since  $E_c \leq 250$  mK  $\ll T$ .

The dephasing due to the electron-electron interaction is caused by the fluctuations in the electric field due to other electrons. These fluctuations are controlled by thermally excited plasma oscillations. We therefore assume that the dephasing rate is inversely proportional to the characteristic plasma frequency

$$\omega_{cp} = \sqrt{n^{3/2}e^2/(2m\bar{\epsilon})}; \quad \bar{\epsilon} = (\epsilon + \epsilon_0)/2. \quad (4)$$

To verify this assumption and to separate the contribution of electron-electron interaction to  $\tau_{\phi}$  we measured the dephasing time as a function of electron density. These data are shown in Fig. 2 where  $\tau_{\phi}$  is plotted as a function of plasma frequency. The data are fit by

$$\tau_{\phi} = \tau_A/(1 + \alpha\omega_{cp}\tau_A). \quad (5)$$

This equation follows from Eq. (2) with  $\tau_A$  defined as  $\tau_A^{-1} \equiv \tau_v^{-1} + \tau_h^{-1}$  and  $\tau_{e-e} = 1/\alpha\omega_{cp}$ . The best fit for the parameter  $\alpha$  is  $1.1 \pm 0.1$ .

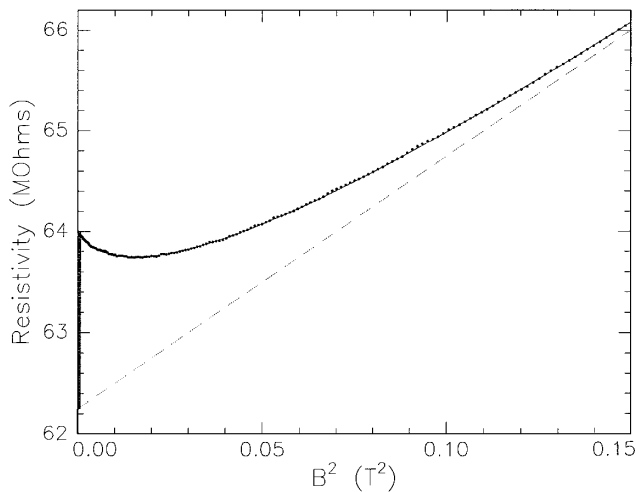


FIG. 1.  $\rho_{xx}$  versus  $B^2$ ;  $n = 1.7 \times 10^{11} \text{ m}^{-2}$ ,  $T = 2.15$  K,  $\mu = 0.7 \text{ m}^2/\text{V s}$ . The dashed line is the Drude theory.

Figure 2 shows that our data are consistent with the assumption that the time scale for dephasing via electron-electron interaction is set by the plasma frequency. Further support for this assumption comes from measurements of the electronic velocity autocorrelation time which is also found to be set by the inverse plasma frequency [17]. We hope this finding will stimulate the development of a theory of dephasing by electron-electron interaction that is applicable to a nondegenerate electron gas.

We turn now to the damping due to the motion of the vapor atoms. The theoretical expression for the dephasing time due to horizontal motion is [9,10]

$$\tau_h = (g\tau_0\tau_{\lambda}^2)^{1/3}; \quad \tau_{\lambda} = \lambda/\sqrt{2k_B T/M}. \quad (6)$$

Here  $\lambda$  is the de Broglie wavelength of the electron,  $M$  is the mass of a helium atom, and the theoretical value of  $g = 6$ . The analogous expression for vertical motion is

$$\tau_v = (f\tau_0\tau_z^2)^{1/3}; \quad \tau_z = b/\sqrt{k_B T/M}. \quad (7)$$

Here  $b$  is a measure of the width of the vertical subband wave function of the electrons and the theoretical value of  $f = 9/2$ . It is a known function of the holding electric field [see discussion following Eq. (9)]. Equation (7) has not appeared before in the literature; we sketch its derivation below. In both Eqs. (6) and (7) it is assumed that the motion of helium is ballistic on the dephasing time scale and that  $\tau_z, \tau_{\lambda} \gg \tau_0$ . Both conditions are met under experimental conditions.

To test the theoretical expressions, Eqs. (6) and (7), and to separate  $\tau_v$  and  $\tau_h$  we vary the characteristic width  $b$  of the electronic wave function by changing the holding field. The range of  $b$  was limited by microphonic induced instabilities of the charged surface at small  $n$  and large  $E_{\perp}$ . We calculate  $\tau_v$  from Eq. (2) using the empirical value of  $\tau_{e-e}$  and the theoretical value of  $\tau_h$  [Eq. (6)] but with  $g$  as an adjustable parameter. The calculated values of  $\tau_v$  are plotted as a function of  $(\tau_0\tau_z^2)^{1/3}$  (see Fig. 3) and the parameter  $g$

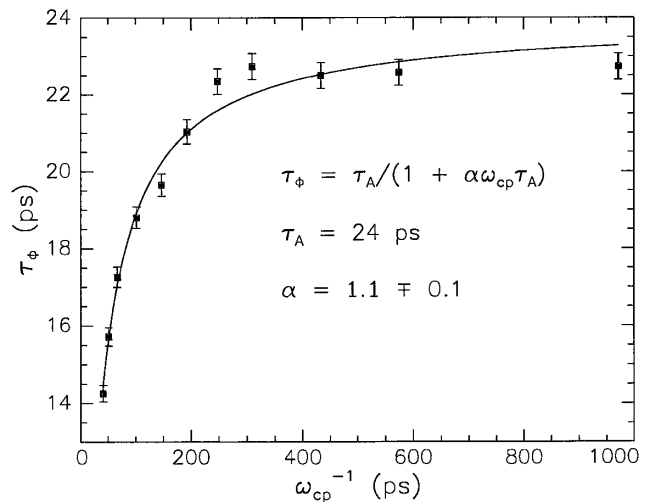


FIG. 2. Dephasing time versus inverse plasma frequency.  $T = 1.96$  K.

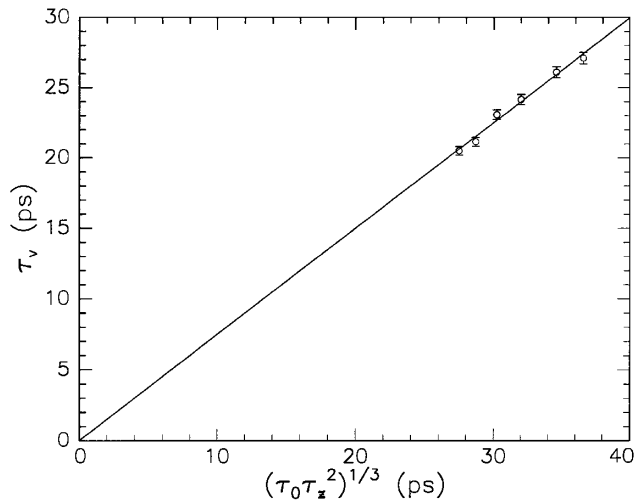


FIG. 3.  $\tau_v$  versus  $(\tau_0 \tau_z^2)^{1/3}$ .  $T = 2.165$  K. The solid line is a fit to theory with  $f = 0.4$  and  $g = 1.3$ .

is adjusted until the best linear fit through the data passes through the origin. This yields values of  $f = 0.4 \pm 0.1$  and  $g = 1.3 \pm 0.3$ . For the measurements shown in Fig. 3 the product  $k_T l$  was 1.6–2.4 with  $k_T = \sqrt{2mk_B T}/\hbar$ , and the product  $\omega_{cp} \tau_0$  was in the range 0.05–0.08. Thus, dephasing is dominated by dephasing due to the motion of helium atoms in these data. The measured dephasing times  $\tau_v$  and  $\tau_h$  are comparable because  $b$  and  $\lambda$  are comparable (at zero applied holding field  $b = 7.6$  nm while  $\lambda = 14$  nm at 2.1 K).

A second comparison to the theoretical expressions, Eqs. (6) and (7), comes from measuring the dependence of  $\tau_\phi$  on the electron-atom scattering time  $\tau_0$  studied by changing the vapor density which depends sensitively on temperature. Combining Eqs. (6) and (7) shows

$$\tau_A^{-1} = [(f\tau_z^2)^{-1/3} + (g\tau_\lambda^2)^{-1/3}]\tau_0^{-1/3}. \quad (8)$$

The coefficient of  $\tau_0^{-1/3}$  contains temperature dependent parameters  $\tau_z$  and  $\tau_\lambda$ . A graph of  $\tau_A^{-1}$  is shown in Fig. 4.

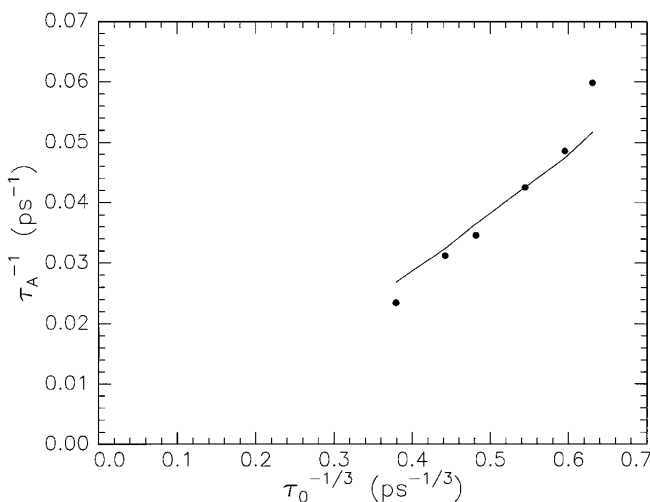


FIG. 4. The quantity  $\tau_A^{-1}$  versus  $\tau_0^{-1/3}$ .

The values of  $\tau_A^{-1}$  increase with increasing  $\tau_0^{-1/3}$ , but the fits to theory are poor. Figure 4 shows a fit with  $f = 0.8$  and  $g = 1.0$ . Similar fits can be obtained with  $f$  and  $g$  up to  $f = 0.4$  and  $g = 4$ , but the data cannot be fit to the values  $f = 0.4, g = 1.0$  obtained from the fit to Fig. 3. For these measurements the product  $k_T l$  was in the range 2.4–7.5.

We also studied the variation of the scattering time  $\tau_0 \propto \mu$  with the holding field  $E_\perp$ . Figure 5 shows a plot of the mobility as a function of the width  $b$  for electrons on both isotopes of helium. Curves represent scaled theoretical values. The data are inconsistent with theory, which predicts the mobility to be linear in  $b$  [18]. Values of  $k_T l$  were in the range 1.6–2.4 and 1.0–1.3 for  $^4\text{He}$  and  $^3\text{He}$ , respectively, and  $E_c$  was as large as 600 mK for  $^3\text{He}$ . The differences in behavior for the two isotopes may be related to the close approach to strong localization for  $^3\text{He}$ .

We turn now to the derivation of the formula for damping due to vertical motion of the vapor atoms [Eq. (7); see Ref. [11] for more details]. If we treat the helium vapor atoms as hard-core potentials, the contribution of a path to the return amplitude is a product of the amplitude to scatter off the first atom, multiplied by the amplitude to go to the second atom, multiplied by the amplitude to scatter off the second atom, and so on around the loop. Let  $A(z)$  be the amplitude to scatter from an atom at a height  $z$  above the liquid helium surface. We choose

$$A(z) = \frac{16\pi a \hbar^2}{mb^3} z^2 \exp\left(-\frac{2z}{b}\right) \quad \text{for } z > 0 \\ = 0 \quad \text{for } z < 0. \quad (9)$$

This is derived by taking the vertical subband wave function of the electrons to be of the variational form  $\psi(z) = 2b^{-3/2} z \exp(-z/b)$  [19]. The helium atoms are treated as hard-core potentials;  $a$  is the  $s$ -wave scattering length,

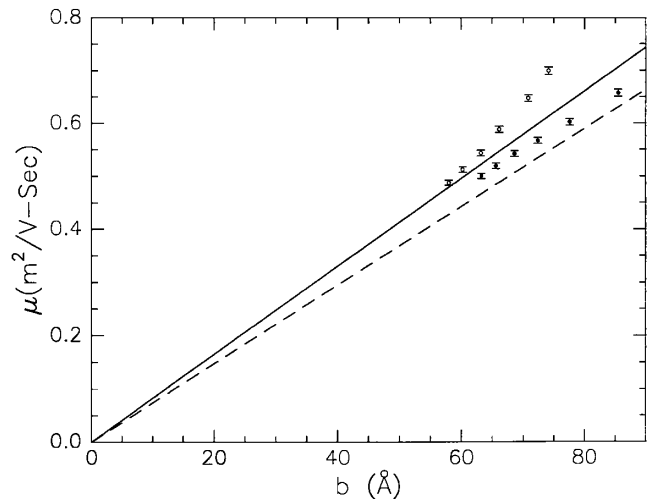


FIG. 5. Mobility versus the variational parameter  $b$ . Open symbols:  $^4\text{He}$  at  $T = 2.165$  K; closed symbols:  $^3\text{He}$  at  $T = 1.26$  K. Curves are theory/1.75: solid,  $^4\text{He}$ ; dashed,  $^3\text{He}$ .

and the variational parameter  $b$  is a function of the applied field.

We now assume the helium atoms are allowed to move vertically [20]. Since a given atom is encountered at different times on the forward and return paths we must consider

$$Q(t) \equiv \langle A(z)A(z + vt) \rangle. \quad (10)$$

Here  $t$  is the difference in the times at which the atom is encountered on the forward and return paths. The atom is assumed to move ballistically at vertical speed  $v$  for this time.  $\langle \dots \rangle$  denotes an average over all possible configurations of the helium atom (vertical position is assumed to be uniformly distributed and vertical speed is given by the Maxwell-Boltzmann formula). The interference between the forward and return paths is then reduced by the factor  $q(t) \equiv Q(t)/Q(0)$  due to the motion of this atom. A path of duration  $t$  encounters  $t/\tau_0$  atoms; hence its interference with its time-reversed partner is reduced by a factor of  $q(t)^{t/\tau_0}$  due to the vertical motion of all the atoms. This estimate is improved by noting that the difference in times at which an atom is encountered by the forward and return paths is not the same for all atoms: it varies from zero (for atoms in the middle of the path) to  $t$  (for atoms at the ends).

Using Eqs. (9) and (10) we find that the contribution of paths of duration  $t$  is reduced by  $\exp(-t^3/\tau_v^3)$  [21] and  $\tau_v^z$  is given by Eq. (7). In general, damping factors vary as  $\exp(-Ct^\gamma)$ . For electron-electron interactions,  $\gamma = 1$ ; for both horizontal and vertical motions of the helium atoms,  $\gamma = 3$ .

In conclusion, we have succeeded in separating the three contributions to the dephasing times. The times  $\tau_v$  and  $\tau_h$  are found to be consistent with the predicted functional forms. The experimentally determined values of the numerical coefficients are  $f = 0.4-0.8$  and  $g = 1.0-1.5$  (values based on the analysis of Fig. 3 which we believe provides a more reliable estimate than Fig. 4). These values are an order of magnitude smaller than the corresponding theoretical values. The reduction in  $\tau_v$  with an increase in holding field has a simple explanation. Increasing the field reduces the width of the volume occupied by electrons and, therefore, enhances the escape of atoms from this volume. Nothing was known regarding the dephasing time due to electric field fluctuations of other electrons. We found  $\tau_{e-e}$  empirically to be  $\approx \omega_{cp}^{-1}$ , the only obvious characteristic time associated with the electron gas.

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