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Light refraction: Right or left turn of rays as polarization effect

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Abstract

Media with negative dispersion are represented as a set of specialized scatterers that will change, at waves' absorption and their reemission, the sign of magnetic strength parallel to surface. In metamaterials such scatterers (turn elements) are represented by the open oscillatory contours. In natural substance (metals) their role can execute magnetic dipoles and electrical quadropoles induced by polarization. Their converting features become decisive in sufficiently thin films at suppressing dipole polarization, i.e., such phenomena are shown the dimensional (size) effects of media polarization. This approach leads to estimations of optical thickness of medium, in which a turn of refracted beam is observable.
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1. Introduction

In recent years a big attention of researchers is attracted to problems of existence and peculiarities of so-called left-hand materials, LHM, with a negative index of refraction (such abbreviations as NIMs, negative index metamaterials, or MMs, metamaterials, are also used). The existence of substances with negative refraction or waves with negative direction of the group speed is analyzed and reviewed in general form in [1]. The opportunity of their usage was considered by Veselago for media with negative dielectric and magnetic susceptibilities in some frequencies region [2]. These ideas had been used for proposal of "perfect" lens by Pendry [3]. Such artificial systems have been created as metamaterials [4], and on their border, really, the right triad of vectors (**E**, **H**, **k**) had been transformed into the left triad [5] (e.g., the general reviews [6,7]).

The treating of these phenomena is usually carried out within the frame of theory of surface plasmons. However, we shall consider these phenomena via description of elementary processes of wave and photon interactions with definite elements of media. Both approaches have the advantages and can supplement each other.

The general macroscopic description of considered effects goes on such a way. Materials with structural units much smaller than the wavelength are characterized by a dielectric permittivity $\varepsilon(\omega) = \varepsilon' + i\varepsilon'' \equiv |\varepsilon(\omega)| \exp(i\phi_\varepsilon)$ and a magnetic permeability $\mu(\omega) = \mu' + i\mu'' \equiv |\mu(\omega)| \exp(i\phi_\mu)$. If the periodicity of phases are mutually coordinated and the real parts of material functions are negative, these phases are represented as $\phi_{\varepsilon,\mu} = \pi + \Delta\phi_{\varepsilon,\mu}$ and therefore

$$n(\omega) = \sqrt{\varepsilon(\omega)\mu(\omega)} \equiv \sqrt{|\varepsilon||\mu|} \exp\bigl(i\pi + i\Delta(\phi_\varepsilon + \phi_\mu)/2\bigr), \quad (1)$$

i.e., the real part of refraction index can be negative at definite values of these phases (the case of $\Delta(\phi_\varepsilon + \phi_\mu) \sim 0$ is the most illustrative, but not an exclusive one). In accordance with the Descartes–Snell law it means the "left" turn of light rays, but does not explain physical mechanisms of this phenomenon. Therefore the elementary acts of turn of waves (photons) and their features must be analyzed. In the simplest case of artificial scatterers (aerials) this effect is described in Section 2.

However, creation of metamaterials with artificial electric and magnetic oscillators in substance is not the only possible for research and usage of this effect. The similar result can

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be achieved if light is passing through a layer of substance, in which the current loops of corresponding frequency, equivalent to magnetic elements, can be induced. Discussion of this opportunity is the basic purpose of the Letter.

The essential role thus will be caused by compatibility of the electric and magnetic moments induced into medium. For their definition and specification some peculiarities of the general theory of optical dispersion may be reexamined.

According to the strict classical theory of dispersion, the Ewald–Oseen theory (e.g., [8]), the phenomena of light propagation in media, refraction and reflection are determined by the properties of elementary scatterers of substance and the interference of arising elementary waves at each stage. In the classical theory all scatterers participate in these processes, but in the scope of this theory the substance depth, within which refracted waves are formed, remains undefinable.

At the approach based on the quantum theory of scattering, photons must be absorbed and reemitted, with some temporary delay, by scatterers that are on distances of photons free paths ([9], some applications in [10], more complete consideration in [11]). Therefore the depth of single reflection and/or refraction layer must be defined via the photons free path in considered medium.

It means that such approach implies an existence of intermediate optical near-surface layer where the wave would be transformed. Note, that the intermediate optical near-surface layer as the certain stationary formation had been introduced by Drude [12] and then was considered by Born [13], see also [14]. But this old problem had not sufficient examination and at times is only marked in the literature, e.g., [15].

In Section 3 we describe this layer as an effective formation induced by singularities of reflected and refracted waves via the generalization of the known method of fields' discontinuities consideration [8, Appendix 6]. The type of polarization in this layer or in the equivalent thin film/rod can be changeable, in dependence of its sizes, with manifestation of higher moments if possibilities of electrical dipoles inducing will be suppressed. Thus the dimensional (or size) effect of media polarization appears and just it can be responsible for rays turn within thin films.

In Section 4 are considered, for partial substantiation of used mathematical method, the optical property of films as spatially restrained objects. It is shown that the formation of refracted waves is not strictly local and therefore electric and magnetic absorber–emitter can be spatially separated (note that there is a conjecture that LHM can be formed by separated negative ε and μ layers [16]).

The results of examinations and some perspectives are summed in the conclusions.

2. Elementary scatterers of metamaterials

Artificial metamaterials are constructed as systems of open oscillatory contours which can be excited by magnetic field of an incident wave, and/or by its electric field.

Let us consider the classical Hertzian vibrator, the LC-oscillator. It represents an elementary scatterer composed from magnetic (more correctly, diamagnetic) and electric dipoles with the common resonance frequency $\omega_0 = 1/\sqrt{LC}$. If the inductance is excited by a variable magnetic field (by its part parallel to the axis of contour), the induction current of oppositely direction (the Lenz's rule) charges the condenser. Therefore this element will radiate an electromagnetic wave with the magnetic strength (its definite component) inversed relative to incident wave.

Such selective refraction of field strength component changes the sign of corresponding component of the Poynting's vector, i.e., changes the wave direction. Thus, the oscillatory contour can be considered as the element turning a resonant wave or its harmonics onto certain angle.

The transfer of excitation from one part of this hybrid system to another is executing in a near field and, probably, instantaneously [10,17], i.e., this system can be considered as an elementary hybrid scatterer. The near field propagator describing this transfer is proportional to ω^{-2} [18] and corresponding probability is therefore proportional to $(LC)^2$.

If the metamaterials will compose a bulk medium, then at the distance, in mean, of two free path ℓ this wave will be scattered and transformed in the second time, i.e., will be reverted into the initial direction. Therefore in a sufficiently bulk medium refracted rays will propagate, in mean, along the normal to refracted surface. It leads to peculiarities of practical usage of thick media with $n < 0$, or, by the slightly another words, the variability of their optical features with thickness (see as examples the direction of rays after two interactions in [19]).

Therefore the thickness of metamaterials d used for turn formation must be limited as

$$\ell \leqslant d < 2\ell, \tag{2}$$

i.e., the Veselago lenses must be strictly restrained over their thickness. But the estimation of free path length for such artificial elements requires their concrete consideration. This problem represents easier and more practical for natural media.

3. Fields discontinuity

If the flat surface $z = 0$ separates substances with different physical parameters, this mathematical 2D plane must be replaced by an effective intermediate layer, in which parameters of fields are sufficiently smoothly altering. Such approach requires the splitting of all space onto three parts along z-axis, describable by the partition of unity onto three parts smoothly transforming an each into another instead of rigidly limited two parts [17]:

$$\theta(z|a) + \theta(-z|b) + \vartheta(z|a,b) = 1. \tag{3}$$

Elements of (3) can be considered as "smoothed projectors" that at $a, b \to 0$ transfer into the Heaviside units or zero correspondingly. These quasi-projectors can be constructed as limits of integrals over δ-sequences, conforming to the condition (3) at each step:

$$\{\theta(z|a); \theta(-z|b); \vartheta(z|a,b)\} = \left\{ \int_a^\infty ; \int_{-\infty}^{-b} ; \int_{-b}^{a} d\xi\, \delta(x-\xi,\eta) \right\}. \tag{4}$$

The most useful Cauchy–Lorentz δ-sequence $\delta(z,\eta) = \eta/\pi(z^2+\eta^2)$ with $\eta \to 0+$ leads to such quasi-projectors:

$$\theta(z|a) = 1/2 + (1/\pi)\tan^{-1}(z/a),$$
$$\vartheta(z|a,b) = (1/\pi)\{\tan^{-1}(-z/a) + \tan^{-1}(z/b)\}. \tag{5}$$

Let us consider an electromagnetic wave that falls from free space ($z > 0$) onto medium ($z < 0$). For any component of **E**, **B**, **D**, **H**, **j** or of **E**, **B**, **D**, **j** at another description of fields in substance [20] such its representation can be written:

$$V(t,\vec{r}) = (V_I + V_R)\theta(z|a) + V_T\theta(-z|b) + V_0\vartheta(z|a,b), \tag{6}$$

where the subscripts $I, R, T, 0$ are referring, correspondingly, to the initial, reflected, transmitted and intermediate fields and currents, a and b are depths of an intermediate layer (its spreading) in both half spaces. At $a = b = 0$ this decomposition transfers into the usual boundary conditions. (Note that at the asymmetric approach to Maxwell equations in substance the opportunity of occurrence of singular currents along an intersurface is considered for the problems of light reflection without obvious introduction of an intermediate layer. The used boundary conditions are similar to the limiting form of (6) [21].)

Let us describe the TE wave in the (x,z) plane that is entering into electrically neutral and optically passive substance. Since all fields, except $\vec{V}_0$, satisfy the equations $\nabla\cdot\vec{D}_i = 4\pi\rho_i$ ($\rho_i \to 0$ for metals), the operations of divergence of (6) with $V \to \mathbf{D}$ leads to the relation:

$$\nabla\cdot\vec{D}(z) = D_{0z}\big(\delta(z,a) - \delta(z,-b)\big) + 4\pi\rho_0^{(e)}(z)\vartheta(z|a,b) = 0. \tag{7}$$

Hence the intermediate layer represents an oscillating double electric layer of strength D_{0z} with induced charges of density $\rho_0^{(e)}$.

For the case of TM wave in the incident plane, we apply (6) to the magnetic field, $V \to \mathbf{B}$, with taking into accounts that $\nabla\cdot\vec{B}_i = 0$ for all fields except $\mathbf{V}_0$:

$$\nabla\cdot\vec{B}(z) = B_{0z}\big(\delta(z,a') - \delta(z,-b')\big) + 4\pi\rho_0^{(m)}(z)\vartheta(z|a',b') = 0, \tag{8}$$

where $\rho_0^{(m)}$ is the density of "magnetic charges" inducing near surface by transition effects. In diamagnetics, e.g., in Ag, this field increases the value of existing $\mu < 0$.

Similar substitutions in the other Maxwell equations describe the evanescent, oppositely directed currents in this layer. Thus, oscillating dipoles and currents on frequencies of incident fields absorb falling radiation and generate the reflected and refracted waves.

Therefore the light flux of arbitrary polarization induces in the refracting near-surface stratum the double electric and magnetic layers of dipoles or higher moments oscillating with external frequency. In this layer an incident flux must be absorbed and the reflected and refracted fluxes will be generating. In bulk media the photons' absorptions–reemissions are the most probable by the electric dipole transitions, but in a case of very thin films/rods situation can be more complex.

In films of small thickness, significantly less 100 nm, the mobility of electrons across a film decreases onto some orders of values (e.g., [22]; recent consideration with taking into account percolation phenomena [23], see also, e.g., [24]). But it in turn means complication of electric dipoles inducing and should lead to growth of significance or even to dominance of magnetic dipoles and electric quadropoles in optical processes for evanescent formation of which electrons can be excited along the z-plane. As they oscillate with equal frequencies, the transferring of excitations between them is possible and can be instantaneous [10]. The probability of such hybrid processes can be appreciated by comparison of probabilities of separate processes.

By virtue of the parity conservation such transitions are possible between magnetic dipole and electrical quadrupole or vice verse. Probabilities of their emissions (and, correspondingly, of absorption) are related as $w_{M1}/w_{E2} \sim (\alpha/kd)^2$, where $\alpha = e^2/\hbar c$, d is the size of emitter. Thus their equivalence is possible at

$$d \sim \lambda/20. \tag{9}$$

This condition determines possible thickness of films or diameter of rods, in which the considered effect can be observed. On the other hand, its thickness must be bigger the photon free path ℓ, but lesser it is twice value for preventing subsequent turning.

Free path is determined as $\ell = 1/N\sigma_{\text{tot}}$ via the density of scatterers and the total cross-section of scattering. It can be assumed, that this process takes place via scattering on the magnetic moment caused by a mean electron excitation in atom $|\vec{\mu}| \sim ea_B/2$, where a_B is the Bohr radius of atom and $1/2$ is taken by analogies with determination of the Bohr magneton. With the amplitude of photon scattering on magnetic moment $A = 4|\vec{\mu}|^2\omega/\hbar c^2$ and by the optical theorem of quantum electrodynamics it leads to

$$\sigma_{\text{tot}} = (4\pi c/\omega)\operatorname{Im} A(0) = 4\pi\alpha\cdot a_B^2 = 2.5\times10^{-16}\ \text{cm}^2. \tag{10}$$

As for Ag films density of scatterers $N \sim 6\times10^{22}$, the free length path of photons must be of order

$$\ell \sim 68\ \text{nm}. \tag{11}$$

This numerical value may be examined over the order of magnitude only. But the set of conditions (2), (9) and (11) allow a certain estimation of the admissible thickness of silver films working as "perfect lens" in the sense of Pendry.

4. Certain properties of thin optical films

Thin films have specific optical properties some of which must be considered here.

Let us begin with consideration of an indefinite flat and uniform substance layer of thickness $2a$ located at $-a < z < a$ with substances of indices of refraction n_1 and n_2 above and below this layer. Such property can be expressed via the equation of restriction [10,17]:

$$n(\omega, z) = n(\omega, z)\theta\left(a^2 - z^2\right) + n_1 \cdot \theta(z - a) + n_2 \cdot \theta(-z + a) \tag{12}$$

and analogical for other material functions (rods and wires instead of films can be also considered). The Fourier transformation of (12) reveals its features conditioned by the film thickness and features of surrounding substances:

$$n(\omega, q) = \frac{1}{\pi} \int_{-\infty}^{\infty} dk \frac{\sin ak}{a} n(\omega, k - q) + n_+ \cdot \left(\delta(q) - \frac{\sin qa}{\pi q}\right) + n_- \cdot \frac{\cos qa - 1}{\pi i q}, \tag{13}$$

where $n_\pm = (n_1 \pm n_2)/2$. It means that regardless of the proposed strict borders of different substances their optical properties depend on properties of surrounding media: note that the additional terms give contributions in both real and imaginary parts of n. This fact can be considered as a certain explanation of the procedure used in the preceding section, the replacement of unit projectors in (12) on quasi-projectors of (4)-types does not seriously change results.

Let us examine more complicated properties. The definition $n^2(\omega, \vec{k}) = \varepsilon(\omega, \vec{k})\mu(\omega, \vec{k})$ corresponds to such partial Fourier convolution:

$$\int d\vec{r}_1\, n(\omega, \vec{r}_1) n(\omega, \vec{r} - \vec{r}_1) = \int d\vec{r}_1\, \varepsilon(\omega, \vec{r}_1)\mu(\omega, \vec{r} - \vec{r}_1). \tag{14}$$

The substitution of decompositions (12) into (14) leads to the representation ($n_1 = n_2 = 1$ and so on, sign of ω and insignificant terms are omitted):

$$\int_{m_-}^{M_+} d\xi \left\{n(\xi)n(z - \xi) - \varepsilon(\xi)\mu(z - \xi)\right\} = \int_{m_+}^{M_-} d\xi \left\{\varepsilon(\xi) + \mu(\xi) - 2n(\xi)\right\}, \tag{15}$$

where $M_\pm = \min(a, z \pm a)$, $m_\pm = \max(-a, z \pm a)$. At $z = 0$ the right side equals zero, this expression characterizes possible spatial disjointing of electric and magnetic components of hybrid scatterer. It evidently shows, in particular, that if both ε and μ negative, n also will be negative. The form of the left part is close to some microscopic models of type $2n = \varepsilon + \mu$ (compare [25]).

These representations show possibility of formation of the integrated optical centers with spatially partitioned electric and magnetic points (dipoles, etc.). But the space extent between these partial centers, if the transferring of excitation between them must be executed in the near field, cannot be bigger half wavelength.

5. Conclusions

Let us enumerate the results of the consideration.

1. Turn of light rays direction is executable by hybrid scatterers with the integrated together electric and magnetic constituents. The turn of rays can be observed in media with odd number of scattering processes on such scatterers. The even number of these processes must lead to rays propagation along normal to refracting surface.

2. The constituents of hybrid scatterers can be as artificial (macroscopic) elements, so the virtual polarizing formations induced by the incident light wave.

3. Natural hybrid scatterers can execute the basic role, only if inducing of electrical dipoles, which usually dominate in polarization effects, is suppressed by abrupt decreasing of electron mobility in definite direction of media. Thus this phenomenon represents the dimensional or size effect of medium polarization. In other words along with usual polarization (by induced electric dipoles) may exist polarizations of higher orders by excitation of the electric and magnetic moments of higher orders that in usual conditions are weaker on many orders. Within the framework of quantum electrodynamics they would be described by analogs of the Kramers–Heisenberg formulae with corresponding replacement of moments.

4. In metal films of sufficient thickness, more 100 nm, the moments of higher orders are also induced, but are not appreciable on a background of usual polarization. However, with reduction of thickness their role grows and consequently both mechanisms of refraction can simultaneously be revealed. Hence, there should be films' thicknesses at which the refracted beam would be forked on two, right and left, with gradual growth of intensity of the left beam at reduction of the film thickness and/or at decreasing of falling angle. Such effects, apparently, are not still observed.

5. The executed consideration confirms the advantages of scattering theory methods in the theory of dispersion [9], i.e., the description of light propagation as saltatory process with taking into account features of elementary acts of scattering. The offered method of determination of intermediate surface optical layers can be useful in description of certain other optical phenomena also.

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