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Superluminal propagation in resonant dissipative media

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ABSTRACT

We analyse the superluminal propagation of optical narrow-band pulses at resonances in dissipative media. We find that, for a broad class of optical systems holding this type of lossy faster-than-light transmission capability, the output waveform is an attenuated, time-advanced version of the input which can be interpreted as the result of the interference of two scaled replicas of the input having a positive relative delay. This analysis is shown to apply, among other scenarios, both in the propagation in a passive bulk medium at an electronic resonance and in a dielectric waveguide coupled to a lossy micro-ring resonator.

1. Introduction

Superluminal propagation is among the most striking phenomena associated to the propagation of electromagnetic waves. It has been known for long that it is possible to attain group velocities in excess of c when propagation takes place in anomalous dispersive media [1]. A large number of systems have been studied, both theoretically and experimentally, which hold group velocities larger than c. Garret and McCumber [2] considered a smooth pulse propagating through a dispersive absorbing medium and showed that the pulse peak moves at the group velocity classically defined, even if greater than c or negative. An experimental confirmation was given in [3]. Superluminal group velocity was studied in transparent media with inverted population by Chiao [4]. Steinberg et al. investigated photon tunnelling time [5] and superluminal propagation with negative group velocity in a medium with a gain doublet [6]. Bolda et al. considered propagation with negative group velocity due to a nearby gain line [7]. Later, Dogariu et al. demonstrated superluminal pulse propagation by transparent linear anomalous dispersion created through the use of two close Raman gain peaks [8], and Schweinsberg et al. observed superluminal (as well as slow) pulse propagation in an erbium-doped fibre (EDF) [9]. More recently, Jiang et al. have given an experimental confirmation that superluminality in an active Raman medium can be brought about by a single frequency pump field [10].

Although superluminal behaviour might seem unphysical, it is always found to be the result of some sort of artifact, so that the group velocity being greater than c or negative at certain frequencies does never imply an affront to relativity. In [11], a frequency-varying concept of group velocity is chosen that maintains the meaning of the function $d\omega/d[Re(k)]$ even for pulses that experience considerable distortion through propagation; an experimental demonstration using this description is presented in [12]. From a mathematical point of view, loss or gain resonances are responsible for the shaping of the linear dispersive properties of the medium which permit to produce spectral regions with associated superluminal or negative group delays in bulk dielectric media. On the other hand, superluminal propagation in metastructures such as those based on coupled micro-ring resonator (CMR) has also been predicted [13], so the question immediately arises of whether some connection exists, and to what extent, between both phenomena. As we shall presently see, the basic physics of the superluminal problem can be very insightfully addressed through the analysis of a simple CMR structure having a lossy ring waveguide. We will show that the model of interfering scaled self-replicas not only explains the superluminal operation of the CMR and similar structures, but can also be translated to the context of propagation through dielectric atomic (bulk) media near the absorption resonances. Indeed, superluminal linear propagation in such atomic media can be explained in terms of the interference

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with an echo of the input electromagnetic signal produced by the resonant coupling with the medium via the linear polarisation. We thus focus on the lossy CMR as a specially illustrative sample system.

2. Physical model for a class of superluminal lossy photonic systems

When only one mode of the electromagnetic (EM) field is involved, propagation problems can be addressed using a scalar approach. Two representative examples are the propagation of plane waves in homogeneous media (the vectorial formalism in [11], for example, is in practical terms restricted to this case) and that of fields in single-mode waveguiding structures. The pulse propagation problem with a plane wave is entirely equivalent to that of a waveguide mode, with the only proviso that the modal field profile across the waveguide is invariant for all frequencies in the pulse spectrum. This condition virtually applies in all cases of interest, even with ultrashort pulses containing very few cycles of the carrier. Therefore, in order to simplify the results, we use a general approach using abstract scalar signals and linear systems for the discussion of the propagation problems. This does not restrict the validity of the results and, further, shows that the same principles are applicable to any linear system.

We thus consider a generic optical superluminal device which consists of a section of a suitable photonic system. This could be, for example, an absorbing atomic medium of finite length or a waveguide mestastructure, but it does not need to be specified at this stage. We assume a time-localised propagating signal with a complex envelope (analytical signal) $x(t)$. We call $X(\Omega)$ the Fourier transform (FT) of $x(t)$, with $\Omega = \omega - \omega_0$ and ω_0 the carrier frequency. We will denote $y(t)$ the complex envelope of the output signal, $Y(\Omega)$ its FT, and $H(\Omega) = Y(\Omega)/X(\Omega)$ the system response. Our study deals with superluminal propagation in resonant structures, so we will assume that $H(\Omega)$ has a resonance at $\Omega = 0$. The "superluminal condition" is then defined as

$$\tau_g \equiv -\frac{d}{d\Omega}[\arg H(\Omega)]_{\Omega=0} < \tau_0, \tag{1}$$

including the case $\tau_g < 0$. For an atomic medium, τ_0 is the group delay experienced during the propagation in a medium of the same thickness and the same properties as the one considered, but with the resonance at ω_0 removed from its response. For a CMR such as that shown in Fig. 3, τ_0 would correspond to the straight waveguide alone, without the ring waveguide.

As seen below [c.f. expression (4)], the superluminal effect in this case arises from a distortion caused when the propagation in the medium essentially produces a first replica of the input pulse with a delay τ_0, and a second replica with a delay $\tau_0 + \Delta\tau$. It is necessary that the input pulsewidth be greater than $\Delta\tau$ so that the second replica can annihilate part of the first without significant distortion. As a result, the output pulse is apparently time-advanced with respect to the first replica. Calling $h(t) = \mathrm{FT}^{-1}[H(\Omega)]$, *i.e.* the system impulse response, this simply means that

$$h(t) = a\delta(t - \tau_0) - b\delta(t - \tau_0 - \Delta\tau), \tag{2}$$

where a and b are real positive constants. Taking the FT of Eq. (2) yields $H(\Omega) = a\exp(-i\Omega\tau_0) - b\exp(-i\Omega[\tau_0 - \Delta\tau])$. If the spectrum of $x(t)$ is much narrower than the resonance peak, the phase is given by

$$\arg H(\Omega) \simeq -\Omega\tau_0 + \arctan\frac{b\Omega\Delta\tau}{a - b}, \tag{3}$$

valid if $a \neq b$.

From expression (3), the relative group delay relative, defined as the delay increase with respect to that of the medium or metastructure without the resonance, is given by

$$\tau_g - \tau_0 = -\frac{b\Delta\tau}{a - b}. \tag{4}$$

For superluminal propagation, it is necessary that $b < a$. Eq. (4) also shows that, as $a \to b$, the magnitude of the delay τ_g increases and the amplitude, given by $|H(\Omega)| \simeq a - b$, decreases. Actually, it is trivial to see that the same superluminal situation can be accomplished, for example, in a Mach-Zender interferometer with a phase mismatch of π when the propagation delays (attenuation factors) of the two arms are τ_0 (a) and $\tau_0 + \Delta\tau(b)$, respectively. A detailed study of the distortion and pulse fractional advancement can be found in [14] for a generic linear system (not necessarily optical) described by Eq. (2) and several pulse shapes.

3. Retarded interference in a ring resonator structure

We now analyse a 1-ring "scissor" (side-coupled integrated spaced-sequence of resonators) as shown in Fig. 3. With an ideal lossless ring waveguide, it acts as an all-pass optical filter. However, the presence of propagation losses, for instance, due to the curvature, results in the appearance of a resonance in the transmission curve. Such losses play the same role as the absorption losses in an atomic medium, which, in the last analysis, are due to the electromagnetic energy being dissipated, e.g., radiated away by spontaneous emission. The response of the structure is given by [13]

$$H(\Omega) = \frac{\theta - \sigma e^{-ikl}}{1 - \sigma\theta e^{-ikl}}, \tag{5}$$

where $H(\Omega)$ relates the complex envelopes of the output and input modal electric fields, $0 < \theta < 1$ is the real transmission coefficient of the directional coupler (assumed lossless), l is the length of the ring waveguide, $0 < \sigma < 1$ is the attenuation factor due to the radiation losses in the curved sections, and $k = (\omega/c)\bar{n}$ is the real propagation constant of the waveguide, with $\bar{n}$ being the modal index and $\omega = \omega_0 + \Omega$. The carrier frequency is chosen at a resonance of the structure: $\omega_0 l = m2\pi$, with m an integer, thus $kl = m2\pi + \Omega\bar{n}l/c$. We then see that, if $\sigma \approx \theta << 1$ (but $\sigma < \theta$), expression (5) simplifies to yield, by inverse transformation, the result Eq. (2):

$$h(t) \simeq \theta\delta(t) - \sigma\delta(t - \Delta\tau), \tag{6}$$

with $\Delta\tau = l\bar{n}/c$ Thus, $\theta \equiv a$ and $\sigma \equiv b$. [Note that, in this case, $\tau_0 = 0$ as the straight, unloaded waveguide is immaterial in the model of Eq. (5).]

We study the propagation of optical Gaussian pulses using the Transmission-Line Model method [15] to solve the time-domain Maxwell equations in a two-dimensional geometry. We compare two structures: A waveguide section and the same waveguide loaded with a micro-ring resonator built with two straight segments of length 15.25 μm joining two semicircles of 18 μm radius. This design permits a better control of the coupling from the waveguide to the ring. For all the (straight and curved) waveguide sections, the core refractive index is $n_1 = 3.361$, the cladding refractive index is $n_2 = 3.168$ and the width of the core is 0.6 μm. The separation of the guides in the coupling region is 0.25 μm.

According to our model, the ideal conditions for the observation of superluminal propagation in the micro-ring resonator structure are defined by a large coupling to the ring so θ is small and, simultaneously, large enough losses so σ is comparable to (but smaller than) θ. We first inject a very short 5 fs pulse in order to compute the transfer function. We compare the signal at a given distance from the ring output with the reference signal at the same plane propagating in the unloaded waveguide; these are considered the output and input signals, respectively. Fig. 1 shows the amplitude response and the net group delay, computed from the phase response. The loss mechanism is supplied by the radiation in the curved sections which increases with frequency. The net group delay is negative, corresponding to superluminal propagation, up to a frequency limit when the loss increase sets $\theta > \sigma$. From that point onwards, subluminal propagation at the resonances is found. As we approach the critical frequency, the magnitude of the net group delay becomes larger, in agreement with Eq. 1, and the resonances become sharper.

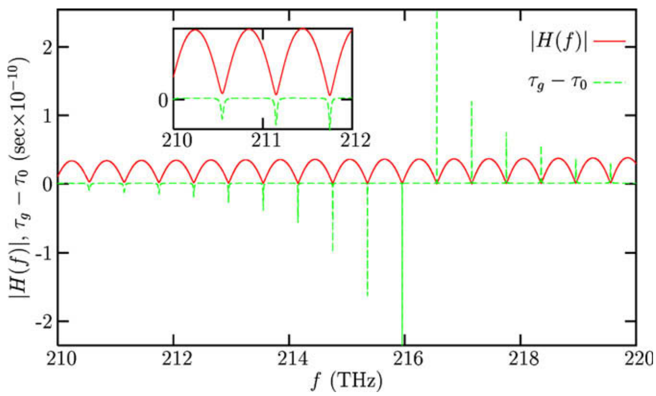

Fig. 1. Amplitude frequency response and relative group delay for the ring geometry described in the text.

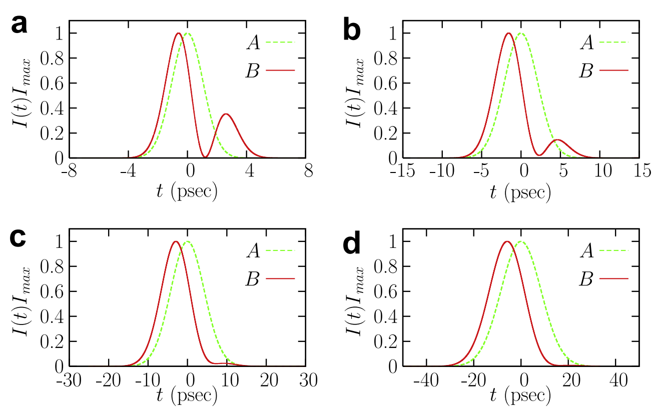

Fig. 2. Comparison of the normalised reference (A) and output (B) waveforms in the ring geometry for (a) 1.5, (b) 3, (c) 6 and (d) 12 ps $1/e$ input Gaussian pulses. $\Delta\tau \simeq 1.6$ ps.

Fig. 2 compares the normalised reference (A) and output (B) signal waveforms for four values of pulse duration: 1.5, 3, 6 and 12 ps. The carrier frequency of 210.55 THz is tuned close to the centre of the leftmost resonance in Fig. 1. The round-trip time in the ring is approximately 1.6 ps. We observe how, as the pulse spectrum narrows in relation with the resonance bandwidth, the propagation distortion decreases. The relation between the output/input peak signal levels ranges from −16 dB in case (a) to −27 dB in case (d).

Fig. 3 shows the field amplitude $|E(x, y, t = t_0)|$ distribution at time t_0 when the leading tail of the 6 ps pulse is entering the ring. For sufficiently long pulses, the shape of this distribution remains constant until the pulse finally leaves the system. This draws a quasi-stationary picture where, at each time instant, the whole structure is filled by a small fraction of the input pulse which can be regarded to hold a constant value. This whole picture evolves slowly in time as the input amplitude varies.

4. Retarded interference in an atomic medium

We now turn to the atomic media, where superluminality occurs for narrow-band pulses near a resonance. The dielectric susceptibility can be approximated by

$$\chi \simeq \chi_0 - \chi_r \frac{i + \Delta}{1 + \Delta^2}, \qquad (7)$$

where χ_0 (real and constant) is the background contribution to χ from all the resonances above the one considered, and $\Delta \equiv 2\Omega/\gamma$ is the detuning factor normalised to the resonance width $\gamma/2$. We will now see that the concept of retarded interference still applies. The key point is that the role of the ring waveguide in the CMR structure is played by the electric polarisation in the atomic medium. The atomic polarisability acts as an absorber and retarded reemitter of the propagating photons. Under certain conditions to be derived next, such function yields superluminal behaviour analogous to that of the electromagnetic field when fed back into the straight waveguide through the coupler in the CMR.

Starting from the wave equation for the electric field in the slab, $E(z, t)$, denoting $\tilde{E}(z, t)$ the complex field envelope, and using the slowly-varying envelope approximation (SVEA), one obtains

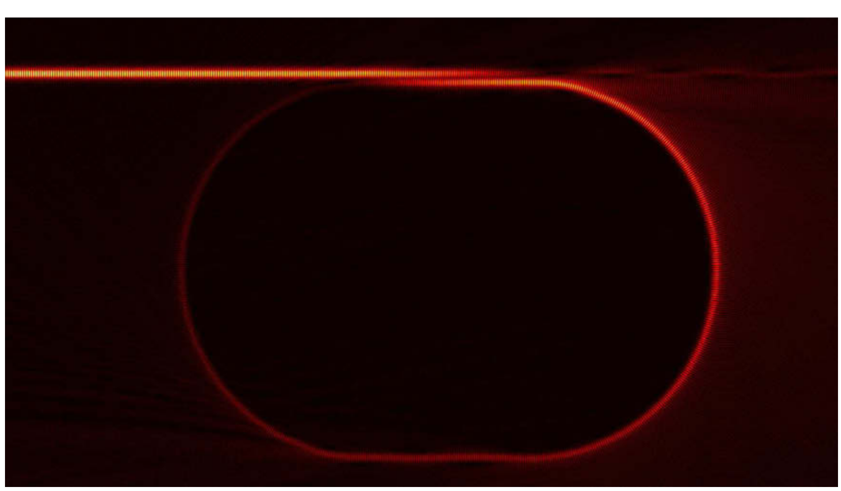

Fig. 3. Field amplitude distribution $|E(x, z, t = t_0)|$ at t_0 when the leading tail of a 6 ps Gaussian pulse is entering the ring.

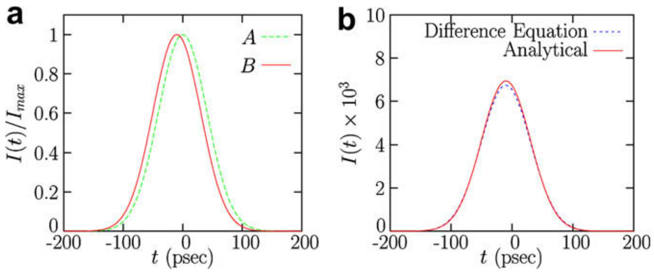

Fig. 4. (a) Normalised input and output waveforms for the superluminal propagation of a Gaussian input 58 ps pulse in a 9.5 μm thick medium with parameters obtained from Ref. [3]. (b) Output waveforms as obtained from the analytical solution and the difference Eq. 11.

$$\frac{\partial \tilde{E}(z,t)}{\partial z} + \left(\frac{n_0}{c} - \frac{2\alpha}{\gamma}\right)\frac{\partial \tilde{E}(z,t)}{\partial t} = -\alpha \tilde{E}(z,t), \tag{8}$$

where $n_0 = (1+\chi_0)^{1/2}$ is the background index of the material, $\alpha = \chi_r \bar{\omega}/(2n_0 c)$ is the (strong) amplitude attenuation factor, with $\bar{\omega}$ the carrier frequency, tuned to that of the atomic resonance. Eq. (8) is derived by considering the frequency dependence of the resonant polarisation up to first order in $\omega - \bar{\omega}$, which is valid for sufficiently slow (spectrally narrow) pulses. From Eq. (8) it follows that the resonant group velocity is given by

$$v_g^{-1} = \frac{n_0}{c} - \frac{2\alpha}{\gamma}, \tag{9}$$

where the negative term modifies the background, nondispersive value c/n_0, and may lead to negative group velocities.

Now, using the transformations $\tau = t - zn_0/c$ and $\zeta = z$, an equation is obtained for the amplitude $\tilde{E}'(\zeta,\tau) \equiv \tilde{E}(\zeta, \tau + \zeta n_0/c)$ in the frame moving at the background group velocity. Further, except for the exponential attenuation of the transmitted pulse, the spatial and temporal variations are very slow: The global picture of the evolution of the optical field in the medium is analogous to that given in Fig. 3 for the CMR case. This is similar to the quasi-stationary behaviour described in the lossless barrier tunnelling given by Winful [16], where the apparent superluminal tunnelling of narrow-band pulses through a barrier is shown to be caused by the modulation of a standing wave. So, by writing $\tilde{E}'(\zeta,\tau) = \exp(-\alpha\zeta) f(\zeta,\tau)$ the following equation is obtained for the slowly-varying amplitude $f(\zeta,\tau)$:

$$\frac{\partial f}{\partial \zeta} = \frac{2\alpha}{\gamma}\frac{\partial f}{\partial \tau}. \tag{10}$$

In order to bring the result calculations into the formalism of Eqs. (2)–(4), we approximate Eq. (10) by a difference equation, which can be done because $f(\zeta,\tau)$ is a slow function, with the proviso that the slab thickness L is sufficiently small. Writing $\partial f(\zeta,\tau)/\partial\zeta \simeq [f(L,\tau) - f(0,\tau)]/L$ and $\partial f(\zeta,\tau)/\partial\tau \simeq [f(0,\tau) - f(0,\tau-\Delta\tau)]/\Delta\tau$, where $\Delta\tau = Ln_0/c$, we finally come to the result

$$\tilde{E}(L,t) \simeq a\tilde{E}(0,t) - b\tilde{E}(0,t-\Delta\tau), \tag{11}$$

with

$$a = e^{-\alpha L}\left(1 + \frac{2\alpha L}{\gamma\Delta\tau}\right), \quad b = e^{-\alpha L}\frac{2\alpha L}{\gamma\Delta\tau}. \tag{12}$$

Expression (11) is entirely equivalent to expression (2), with $x(t) \equiv \tilde{E}(0,t)$ and $y(t) \equiv \tilde{E}(L,t)$. We see that $|b| < |a|$ and $|b| \to |a|$ as $(\gamma\Delta\tau)/(2\alpha L) \to 0$, which is the same regime as that considered for the CMR case. Fig. 4a shows the input and output normalised waveforms for a 58 ps $1/e$ width Gaussian pulse propagating in a 9.5 μm absorbing slab with the medium parameters obtained from [3]. The output waveforms obtained using the analytical expression $E(L,t) = E(0, t - L/v_g)\exp(-\alpha L)$ and the difference Eq. 11 are displayed in Fig. 4b illustrating excellent agreement.

5. Conclusions

We have analysed the superluminal behaviour of a lossy ring resonator structure by using a straightforward approach based on the generic concept of retarded interference in linear systems. We have established, in very simple terms, the conditions and parameter ranges necessary to obtain superluminality in such a waveguide resonator. Although we have focused on this particular device in this paper, the generic approach employed is useful for the study of other ring structures with different architectures, as well as other interference devices such as a Mach-Zender interferometer. We have shown that the formalism also explains, in a unified manner, the superluminal propagation in an thin dielectric slab, physically described by a linear wave equation near an atomic resonance. In this case, the retarded atomic polarisation is the agent that plays the role analogous to the ring waveguide, providing the suitable interference. The complex slow field envelope of the optical field thus evolves in a similar fashion as the field amplitude in the waveguide metastructure.

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