Quantum mechanical diffusion in complex surroundings

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The dynamics of a classical, heavy system coupled to a quantum mechanical light system is studied in a simple time-dependent random matrix model, where the degree of complexity can be changed by a ''chaoticity parameter.'' It is shown that the energy of the quantum mechanical system diffuses as it interacts with the classical system, providing it is chaotic, while a ballistic behavior appears in the ordered case. The energy diffusion of the chaotic system saturates, and it is shown that the saturation time is related to short-range correlations in the energy spectrum. In mixed systems the propagation of energy changes from a ballistic to diffusive behavior at a time related to long-range energy correlations. [S1063-651X(97)10809-1]

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I. INTRODUCTION

Many quantum mechanical systems can be separated into a few slowly varying and several fast degrees of freedom. To understand the influence on the heavy, slowly changing macroscopic system from the light quantum mechanical, fast system is a central issue in several branches of microscopic physics. In molecular physics the (slow) collective rotational and vibrational degrees of freedom are naturally separated from the (fast) motion of the electrons. In large-amplitude collective motion phenomena in nuclear physics, such as fission and fusion, the slow motion corresponds to the collective change of the shape of the nucleus, and the fast quantum mechanical motion to the motion of the individual protons and neutrons. Another example is the motion of electrons through a cavity, where the fast quantum mechanical system corresponds to electrons moving in a complex potential, and the slow motion corresponds to the imposed current.

The quantum mechanical system may give rise to energy diffusion and be the microscopic origin of dissipation and friction in the slow system. This question has been addressed by several authors; see, e.g., Ref. [1–5]. While previous studies assumed that the quantum mechanical system has chaotic properties we shall investigate how the degree of complexity of the quantum mechanical system influences the diffusion properties of the classical system. Our aim is thus to investigate consequences of chaotic and regular dynamics on energy diffusion in a rather general quantum mechanical system. Such a system may be simulated by a random matrix model where the Hamiltonians can describe a chaotic energy spectrum, a regular spectrum, or, interpolated in between.

In the present study we shall restrict the investigation to systems which have time-reversal symmetry, implying that the random matrices describing the chaotic limit are the Gaussian orthogonal ensemble (GOE); see, e.g., Ref. [6]. The classical, slowly varying system moves with a constant velocity, and has a linear time-dependent coupling to the quantum mechanical system. We thus study the (linear) response on the quantum mechanical system by numerically solving the time-dependent Schrödinger equation for the light system, and calculate how the energy diffuses, i.e., how the energy spreads out over instantaneous eigenstates of the time-dependent Hamiltonian. Similar studies have previously been performed by Wilkinson and co-workers [2,3], and by Bulgac, Do Dang, and Kusnetzov [4,5]. We shall study relatively small changes of the macroscopic variable, but which still is sufficiently large to cover different timescales.

One should also point out the relevance of the present type of investigations to understand and possibly justify the application of transport models to quantum mechanical systems. In such calculations, where, e.g., a Langevin type of force is added to the Boltzmann equation (and where all quantum mechanical phases are neglected), usually the explicit coupling to the quantum mechanical system is neglected, and the motion of the macroscopic system is simply treated by adding effective terms describing friction forces, stochastic forces, etc. A better understanding of the influence from the quantum mechanical system on the macroscopic variables in very simple models than the one here studied may ultimately lead to a better understanding also in more complex situations.

In Sec. II the time-dependent random matrix model is described and motivated. The results from the time-dependent evolution of the energy diffusion are shown in Sec. III particularly discussing the different time scales that are obtained from the calculation. Finally, in Sec. IV a short summary of the results is given.

II. MODEL

Our basic assumption is that the Hamiltonian describing the total system can be separated into one slow part and one fast part,

$$\mathbf{H}_{\text{total}}=\mathbf{H}_{\text{slow}}(q,\dot{q})+\mathbf{H}_{\text{fast}}(\dot{q};p_i), \tag{1}$$

where q is the (only) slow degree of freedom and p_i denotes all fast degrees of freedom. In our study we shall concentrate on the fast degrees of freedom, and assume that the coupling only occurs in terms of a constant velocity of the macroscopic variable. Will we thus assume a driven system and neglect the feed-back from the fast system on the slow system. Throughout the paper we shall write $\mathbf{H}\equiv\mathbf{H}_{\text{fast}}$.

We want to describe the intrinsic quantum mechanical system in a very simple model, containing only the very basic elements of physics. Our aim is not to consider a speci-

fied physical situation, but we generally want to study how the macroscopic system would respond to an underlying microscopic system of various dynamical nature. The microscopic system will be described by random matrix theory in a model that allows for a smooth parametric change from regular to chaotic dynamics. The time dependence is introduced by assuming a constant velocity of the macroscopic variable.

In Sec. II A the time-dependent random matrix model is described. The time evolution is obtained by numerically solving the time-dependent Schrödinger equation, and the energy diffusion is calculated as is shown in Sec. II B.

A. Simple time-dependent random matrix model

Assuming time-reversal symmetry, the fast quantum mechanical system may be described by the following time dependent random matrix model [7]:

$$\mathbf{H}(t)=\sum_{n=-N'}^{N'}(\varepsilon_n+\dot{q}tA_n)c_n^{\dagger}c_n+\sum_{n>k}V_{nk}(c_n^{\dagger}c_k+c_k^{\dagger}c_n), \tag{2}$$

where the creation and annihilation operators refer to the diabatic basis, $|n\rangle$. The basis states either may be thought of as describing single-particle states, or many-body configurations by Slater determinants. The time-dependent term $\dot{q}tA_n$ represents the coupling to the slow system. The interaction V, with matrix elements V_{nk}, describes a residual interaction that we shall control by a parameter, Δ. All matrix elements in Eq. (2) are defined as random numbers:

$$\varepsilon_n \in G\left[0,\left(\frac{2}{N}\right)^{1/2}\right], \tag{3}$$

$$A_n \in \sigma_A G\left[0,\left(\frac{1}{N}\right)^{1/2}\right], \tag{4}$$

$$V_{nk} \in \Delta G\left[0,\left(\frac{1}{N}\right)^{1/2}\right], \tag{5}$$

i.e., $\mathbf{H}(t)$ constitutes an ensemble of N-dimensional, time-dependent random matrices with Gaussian-distributed matrix elements. As a standard we shall consider 30 ensembles with dimensions $N=2N'+1=201$. The matrix elements ε_n, A_n, and V_{nk} are all *time independent*. Also, the velocity of the classical system, $\dot{q}$, is constant, i.e., the collective variable,

$$q=\dot{q}t. \tag{6}$$

The two parameters Δ and σ_A determine the properties of the system. By varying the strength of the residual interaction, ''the chaoticity parameter'' Δ between 0 and 1, the fluctuations of the energy spectrum as well as of the eigenfunctions smoothly change between Poisson (''regular'') to GOE (''chaos'') [7].

In the limit of no residual interaction ($\Delta=0$) each energy eigenvalue has a set of good quantum numbers, and the classical counterpart to the fast system is regular. The assumption in Eq. (2) is that the coupling to the classical system (represented by the external force $\dot{q}tA_n$) only has diagonal matrix elements, i.e. does not incorporate the breaking of the good quantum numbers. In many physical situations, e.g., nuclear fission, the degree of freedom expressed as the slow motion approximately has such a property. In the $\Delta=0$ limit the instantaneous energy eigenvalues of $\mathbf{H}(t)$ thus depend linearly on the collective variable q, i.e., (diabatic) straight lines with sharp crossings,

$$\widetilde{E}_n(q=\dot{q}t)=\varepsilon_n+qA_n. \tag{7}$$

The parameter σ_A determines the dispersion of the slopes, $\partial\widetilde{E}_n/\partial q$, and is a physically relevant parameter to be determined for each specific system. In the present model the properties are determined by the product, $\sigma_A\dot{q}$.

Our simplification to choose a linear dependence on the collective variable q is done to keep the number of parameters to a minimum. This can easily be changed, although we do not expect that our main results will be changed.

When $\Delta\neq 0$ the random residual interaction V is introduced, acting between all states. This interaction is assumed not to depend on q. This is a quite reasonable approximation, i.e., to assume the matrix elements of the residual interaction between unmixed (diabatic) eigenstates, $\langle n|V|k\rangle$, to be independent of q. In many realistic cases, as, e.g., for nuclear fission, the coupling matrix elements may, however, show a small but smooth variation with the collective variable (i.e., nuclear deformation).

After solving the time-independent Schrödinger equation,

$$\mathbf{H}|\mu(q)\rangle=E_{\mu}(q)|\mu(q)\rangle, \tag{8}$$

for different values of the collective variable a set of noncrossing energy levels $E_{\mu}(q)$ are obtained versus q. This implies that the good quantum numbers are not good any longer. In the classical counterpart this means that the corresponding constants of motion disappear, and chaos sets in smoothly according to the KAM theorem. Corresponding eigenstates $|\mu(q)\rangle$ (adiabatic basis states) have thus been given a q dependence that is fully controlled by the size of Δ (strength of residual interacion, ''chaoticity parameter''), as well as of the relative slopes of the unperturbed energy states (controlled by the parameter σ_A). The q dependence of the correlation function, $\langle\mu(q)|\nu(q')\rangle$, is consequently parametrized only through these two physically related parameters Δ and σ_A, while additional parameters were used to describe the correlation function in Refs. [4,5]. We shall generally choose $\sigma_A=0.1$ and $\dot{q}=0.001$, and study the system up to times $t=1000$, which means that few Landau-Zener crossings appear in the studied time interval.

B. Evolution with time

The time evolution of a prepared initial state is obtained by solving the time-dependent Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle=\mathbf{H}(t)|\Psi(t)\rangle. \tag{9}$$

Equation (9) is numerically solved with the initial condition that the middle eigenstate (state No. 0) of $\mathbf{H}(t=0)$ is occupied. This is done in the conventional way, i.e., by making the ansatz

$$|\Psi(t)\rangle = \sum_{\mu=-N'}^{N'} a_\mu(t) e^{-(i/\hbar)E_\mu t}|\mu\rangle, \tag{10}$$

which, inserted into Eq. (9), gives N coupled first-order differential equations,

$$\dot{a}_\mu(t) = \frac{i}{\hbar}\sum_\nu \langle\mu|\mathbf{H}_t|\nu\rangle e^{i\omega_{\mu\nu}t} a_\nu(t). \tag{11}$$

The basis states

$$|\mu\rangle \equiv |\mu(q=0)\rangle = \sum_n b_{\mu n}|n\rangle \tag{12}$$

and energies $E_\mu \equiv E_\mu(q=0)$ are eigenstates and eigenvalues of $\mathbf{H}$ at $q=0$, and the frequencies are defined as

$$\omega_{\mu\nu} = (E_\mu - E_\nu)/\hbar. \tag{13}$$

The matrix elements of the time-dependent part of the Hamiltonian $\mathbf{H}$,

$$\mathbf{H}_t = \sum_n \dot{q}tA_n c_n^\dagger c_n, \tag{14}$$

are transformed into the basis $\{|\mu\rangle\}$ giving

$$\langle\mu|\mathbf{H}_t|\nu\rangle = \dot{q}t\sum_n b_{n\mu}b_{\nu n}A_n \equiv \dot{q}tA'_{\mu\nu}. \tag{15}$$

The nondiagonal coupling of the time-dependent term thus solely comes from the complexity (mixing) of the adiabatic eigenstates, i.e., from the residual (time-independent) interaction. In order to compare the result for different Δ values, the energy levels E_μ are unfolded at $q=0$ so that the average level density is constant and equal to $\rho = 100$.

To study how the energy diffuses with time, we expand the calculated wave function in the basis states of instantaneous eigenvalues of $\mathbf{H}(q)$ [adiabatic basis; See Eq. (8)],

$$|\Psi(t)\rangle = \sum_\mu d_\mu(t)|\mu(q)\rangle, \tag{16}$$

and calculate the variance of the energy as

$$v_E(t) = \sum_{\mu=-N'}^{N'} |d_\mu(t)|^2[E_\mu(q)]^2 - \left(\sum_\mu |d_\mu(t)|^2 E_\mu(q)\right)^2. \tag{17}$$

III. RESULT

In Fig. 1 it is shown how the energy spreads out on the adiabatic eigenstates for four different values of the ''chaoticity parameter'' Δ. In general, it is seen that the chaotic system shows a diffusion linear in time [Fig. 1(a)], while quadratic diffusion (''ballistic behavior'') appears for the (almost) regular system [Fig. 1(d)].

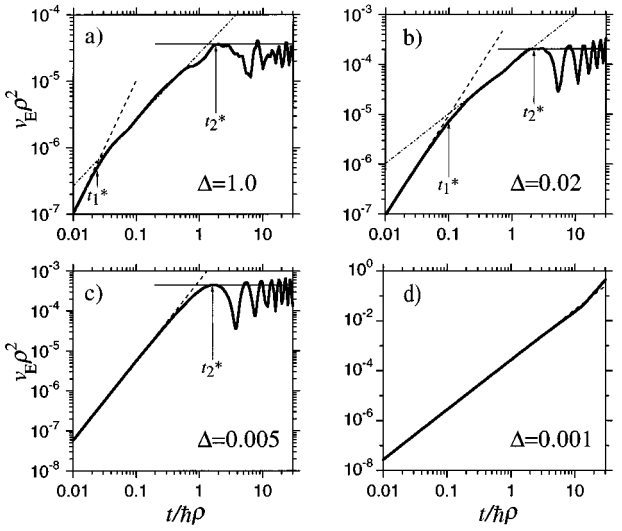

FIG. 1. The variance of energy v_E as a function of time (log-log scale) for (a) $\Delta=1$, (b) $\Delta=0.02$, (c) $\Delta=0.005$, and (d) $\Delta=0.001$, corresponding to chaotic, mixed, mixed, and regular energy spectra, respectively, are shown by thick lines. The four cases are selected from the ensembles. The thin lines show quadratic (dashed lines; ''ballistic behavior''), linear (dot-dashed) and constant (solid) diffusion, respectively. The time values where the diffusive behavior changes character are denoted as t_1^* and t_2^*. In (d) only quadratic diffusion appears. The change from quadratic to linear diffusion at $t_1^* \approx 2.5$ in (a) is an unphysical effect appearing due to the finite size of the system. In all cases $\sigma_A=0.1$ and $\dot{q}=0.001$. Dimensionless units are used for v_E as well as for t.

This behavior can be understood from the following simple picture. In the regular limit (small Δ) the interaction between energy levels occurs at rather sharp q values (time values), at so-called avoided crossings, while in the chaotic case (large Δ) the interaction between the different energy levels takes place over a very extended region in q (i.e., in time). In the regular case energy transport from one occupied level to other unoccupied levels can thus be considered to occur through a number of ''jumps'' in energy. If we denote the number of avoided crossings in the considered interval in q as n, and the spread in energy that occurs at each crossing as δE, we obtain a total spread of the energy, $\Delta E = n\,\delta E$. Since the number of avoided crossings is directly proportional to the size of the considered interval in q, i.e., the elapsed time, the total spread in energy is directly proportional to time, $\Delta E \propto t$, or the variance $v_E = \Delta E^2 \propto t^2$. This situation corresponds to a relatively small number of ''collisions,'' and may be termed ''ballistic.'' In the chaotic case, on the other hand, the energy spreads out to unoccupied levels in a continous way, resulting in a diffusive spread of the energy, $v_E = \Delta E^2 \propto t$.

If we write

$$v_E(t) = 2Dt^\alpha, \tag{18}$$

where D is a ''diffusion constant,'' the potency α is directly given by the slope (log-log scale). Three time scales can be identified from Fig. 1. For short time values $t < t_1^*$, the diffusion is seen to vary quadratically with time ($\alpha=2$); for

intermediate times $t_1^* < t < t_2^*$, it varies linearly with time ($\alpha = 1$); and after some larger time value $t > t_2^*$, the energy diffusion saturates ($\alpha = 0$). In Fig. 1 all three time scales only appear for the larger Δ values [Figs. 1(a) and 1(b)]. [Actually, the quadratic diffusion behavior seen for $t < 2.5$ in Fig. 1(a) corresponds to the finite size ($N = 201$) of the considered model, and is thus unphysical; see Sec. III A 2 below.]

Different time scales

The different types of diffusive behavior observed in the numerical calculations can be understood from the following approximations. The time-dependent problem of Eq. (9) with the initial condition $a_\mu(0) = \delta_{\mu,0}$ gives, in first-order perturbation, for $\mu \neq 0$,

$$a_\mu(t) = -\frac{i}{\hbar}\int_0^t e^{i\omega_{\mu 0}t'}\dot{q}t' A'_{\mu 0}dt' = -\frac{i}{\hbar}\dot{q}A'_{\mu 0}\left[\left(\frac{1}{\omega_{\mu 0}^2} - i\frac{t}{\omega_{\mu 0}}\right)e^{i\omega_{\mu 0}t} - \frac{1}{\omega_{\mu 0}^2}\right], \quad (19)$$

and, for $\mu = 0$,

$$a_0(t) = 1 - \frac{i}{2\hbar}\dot{q}A'_{00}t^2, \quad (20)$$

where $A'_{\mu\nu}$ is defined by Eq. (15). When $\dot{q}tA'_{\mu\nu}$ is small, the adiabatic wave functions $|\mu(q)\rangle$ can be expanded in $q = 0$ wave functions, $|\mu\rangle$, using second-order perturbation theory. This gives the amplitudes in the adiabatic basis [Eq. (16)],

$$|\langle\mu(q)|\Psi(t)\rangle|^2 = |d_\mu(t)|^2 = \left(\frac{\dot{q}}{\hbar}\right)^2 (A'_{\mu 0})^2 \frac{4}{\omega_{\mu 0}^4}\sin^2(\tfrac{1}{2}\omega_{\mu 0}t). \quad (21)$$

The spectrum is symmetric around $\mu = 0$ (at least for the ensemble average), implying that Eq. (21) inserted into Eq. (17) gives the variance in energy,

$$v_E(t) \approx 8\dot{q}^2 \sum_{\mu=1}^{N'} (A'_{\mu 0})^2 \frac{\sin^2(\frac{1}{2}\omega_{\mu 0}t)}{\omega_{\mu 0}^2}. \quad (22)$$

The distribution of the matrix elements, $A'_{\mu 0}$, thus determines the time dependence of the energy diffusion. Two limiting cases can be distinguished, corresponding to small and large values of Δ.

If Δ is small (regular spectrum) the transformation coefficients $A'_{\mu 0}$ can be calculated in second-order perturbation. This gives $\overline{A'^2_{\mu 0}} = \sigma_A^2\Delta^2/(N\hbar\omega_{\mu 0})^2$, and consequently

$$v_E(t) = 8\dot{q}^2\sigma_A^2\Delta^2 \frac{1}{N^2}\frac{1}{\hbar^2}\sum_{\mu=1}^{N'} \frac{\sin^2(\frac{1}{2}\omega_{\mu 0}t)}{\omega_{\mu 0}^4}. \quad (23)$$

To estimate $v_E(t)$, we assume a picket-fence spectrum, i.e.,

$$\omega_{\mu 0} = \mu\omega_0, \quad (24)$$

where μ is an integer. For large N' the summations can be carried out analytically [8], resulting in

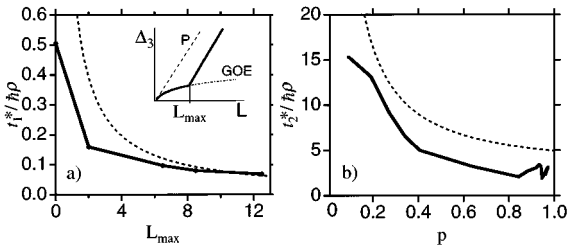

FIG. 2. Time scales t_1^* (a) and t_2^* (b), obtained from the simulations (ensemble averages; thick lines) vs the degree of complexity of the spectrum. In (a), t_1^* is shown vs $L_{\max}$, as defined from Δ_3 statistics by the schematic inset of the figure. Here P stands for Poisson, the ordered limit, and GOE is the chaotic limit. In the Δ region considered in (a), 0.01–0.05, $L_{\max}$ increases from 0 to 12.5. The dashed line shows the estimate $\pi\hbar\overline{\rho}/(4L_{\max})$ [Eq. (33)]. In (b), t_2^* is shown vs the Brody mixing parameter p ($p = 0$ corresponds to a regular spectrum and $p = 1$ to a chaotic spectrum), and the estimate from Eq. (31) is shown by the dashed line. $\Delta = 0.001$, 0.005 and 0.02 correspond to Brody mixing parameters $p = 0.1$, 0.4, and 0.85, respectively. The time scale is dimensionless, $t/\hbar\rho$.

$$v_E(t) = 2D_1 t^2\left[1 - \frac{\omega_0 t}{\pi} + \frac{3}{14}\left(\frac{\omega_0 t}{\pi}\right)^2\right], \quad (25)$$

where

$$D_1 = \frac{\pi^2}{6}(\sigma_A\dot{q})^2 \frac{1}{N^2}\left(\frac{\Delta}{\hbar\omega_0}\right)^2. \quad (26)$$

If Δ approaches 1 (chaotic spectrum) the coupling $A'^2_{\mu 0}$ is approximately equal ($=\sigma_A^2/N^2$) for all μ. In this case the energy denominator in Eq. (22) remains quadratic,

$$v_E(t) \approx 8\dot{q}^2\sigma_A^2 \frac{1}{N^2}\sum_{\mu=1}^{N'} \frac{\sin^2(\frac{1}{2}\omega_{\mu 0}t)}{\omega_{\mu 0}^2}, \quad (27)$$

and, after some algebra, using the same assumptions as above, we obtain the energy dispersion [8]

$$v_E(t) = 2D_2 t\left(1 - \frac{\omega_0 t}{2\pi}\right), \quad (28)$$

where the diffusion constant is

$$D_2 = \pi(\sigma_A\dot{q})^2 \frac{1}{N^2}\frac{1}{\omega_0}. \quad (29)$$

For finite N' a change from quadratic to linear time dependence of v_E in Eq. (27) appears at $t_1^* \approx \pi/(N'\omega_0)$. We have thus achieved an understanding of the diffusive behavior in the two limits, chaotic and regular energy spectrum, seen in Fig. 1, i.e., that $v_E \sim t$ in the chaotic case [Eq. (28)], and that $v_E \sim t^2$ in the regular case [Eq. (25)] [9]. We also see that in both cases the ''diffusion constant'' D_1, [Eq. (26)] as well as D_2 [Eq. (29)], is proportional to the square of the velocity, $\dot{q}^2$.

Ensemble averages of the two time values t_1^* and t_2^*, defined in Fig. 1, are shown in Fig. 2 for different degrees of

complexity of the system. We shall now relate the two transition times to the degree of complexity of the mixed system.

1. Relating t_2^* to the Brody mixing parameter

The time t_2^*, i.e., when the diffusion saturates, corresponds to relatively small energy distances in the spectrum. From Eq. (27) it is seen that up to some maximum time value each term in the sum is a monotonically increasing function. The term that reaches its maximum at the largest time value corresponds to the smallest energy excitation. Thus the diffusion increases with time, but saturates at latest at the time π/ω_{10}. The energy distance to the nearest-neighboring state $\hbar\omega_{10}$ thus mainly determines the time for saturation. For a mixed system several functions have been used to mimic the proper behavior of the nearest-neighbor energy distribution. Here we shall consider the Brody distribution [10], that gives a distribution of nearest-neighboring energies, s, at a mixing p between regularity and chaos,

$$P(s)=\gamma(p+1)s^p e^{-\gamma s^{p+1}}, \tag{30}$$

where γ is obtained from normalization, giving $\gamma=[\Gamma((p+2)/(p+1))]^{p+1}$. When $p=0$ $P(s)$ becomes exponential (i.e., a Poisson process), mimicking a regular energy spectrum, and a Wigner distribution that quite well approximates the chaotic energy spectrum (GOE) appears when $p=1$.

Utilizing Brody mixing, the expectation value of t_2^* in a mixed system may be estimated as

$$\begin{aligned}\langle t_2^*\rangle &= \left\langle \frac{\pi\hbar}{s}\right\rangle \overline{\rho}=\pi\hbar\overline{\rho}\int \frac{1}{s}P(s)ds \\ &= \pi\hbar\,\overline{\rho}\,\overline{\Gamma}\left(\frac{p+2}{p+1}\right)\Gamma\left(\frac{p}{p+1}\right),\end{aligned} \tag{31}$$

and is shown by a dashed line in Fig. 2(b). The function shows a strong increase for small p values, and saturates at $p=1$ to

$$\langle t_2^*\rangle=\pi^2\hbar\,\overline{\rho}/2. \tag{32}$$

This behavior is similar to what is extracted from the full calculations. For each value of the chaoticity parameter Δ, the Brody parameter p has been obtained from a fit to the nearest-neighbor energy distribution. Ensemble averages of t_2^* vs p are shown by a thick line in Fig. 2(b). The saturation time t_2^* is thus related to the complexity of the spectrum, as measured by the nearest-neighbor energy distribution, and is understood from Eq. (31). Also, the fact that no saturation is seen in Fig. 1(d) for the (more or less) regular spectrum (i.e., a small p value) is understood from Eq. (31): $\langle t_2^*\rangle$ goes to infinity as p goes to zero.

2. Relating t_1^* to Δ_3 statistics

In the perturbative regime, Eq. (23) was obtained, resulting in a quadratic time dependence of v_E, and for large couplings we obtained Eq. (27) and a linear time dependence. The time t_1^*, where the energy spread changes from ballistic (quadratic) behavior to diffusive (linear) behavior, is determined by the highest frequency (energy), giving a substantial contribution to the sum in Eq. (22), i.e., how far in energy the coupling matrix elements, $A'_{\mu 0}$, reach. For large values of Δ (chaotic spectrum) the width is so large that we can approximate the matrix element with a constant independence of energy, as was done above. In this case, $t_1^*\approx\pi/(N'\omega_0)$. This implies $t_1^*=0$ for a chaotic system with infinitely many levels. In real calculations finite-size effects always imply a small but finite t_1^*; cf. Fig. 1(a). For small values of Δ (near-regular spectrum) $A'_{\mu 0}$ is inversely proportional to the energy distance $\hbar\omega_{\mu,0}$, and $t_1^*\to\infty$, i.e., the energy transport follows a ballistic behavior for all times.

In the mixed spectrum the coupling matrix elements $A'_{\mu 0}$ are large in the energy interval Γ_A, giving $t_1^*\approx 2\pi\hbar/\Gamma_A$. Long-range correlations in the energy spectrum can be measured by Δ_3 statistics. In Ref. [7] it was shown that the mixed spectrum may show GOE properties restricted to an energy interval $L_{\max}/\overline{\rho}$, and nonchaotic correlations at larger energy distances; see the inset of Fig. 2(a). Furthermore, it was shown [7] that the energy region with GOE correlations is proportional to the spreading width of adiabatic wave functions on diabatic eigenstates, $\Gamma_\mu\approx 0.5\Gamma_A$, finally leading to the estimate

$$t_1^*\approx\frac{\pi\hbar\,\overline{\rho}}{4L_{\max}}. \tag{33}$$

In Fig. 2(a) this estimate is shown by a dashed line, while the thick line shows t_1^*, as obtained from the dynamical calculation for different Δ values. For each value of Δ the Δ_3 statistics was studied, and GOE properties were found in a certain energy range corresponding to $L<L_{\max}$. The ensemble averages of t_1^* and $L_{\max}$ are shown by the thick line in Fig. 2(a). The similarity between the two curves suggests that the time where the diffusion changes from a quadratic to a linear behavior is inversely proportional to $L_{\max}$. The estimate is not valid in the perturbative region (as discussed above), explaining the deviations at smaller $L_{\max}$ values.

IV. SUMMARY

In a simple model we have shown that a fast quantum mechanical system, driven with a constant velocity by a slow classical system, shows a diffusive behavior of the energy that strongly depends on the complexity of the quantum mechanical system. In a regular system energy is transported in a ballistic way (quadratic time dependence), while normal energy diffusion (linear time dependence) appears if the system is chaotic. In a system with mixed dynamics between regularity and chaos, a ballistic behavior is seen up to a time limit, t_1^*, determined by long-range energy correlations in the static spectrum $t_1^*\propto 1/L_{\max}$, where the Δ_3 statistics are of GOE type for $L<L_{\max}$. The energy then diffuses in a normal way over some time interval, and finally, saturates at a time value t_2^* determined by the nearest-neighbor energy statistics; see Eq. (31). The saturation situation corresponds to a localized wave function.

In addition, the energy diffusion of the completely chaotic system saturates. In the semiclassical limit the distance between energy levels go to zero, i.e., $\overline{\rho}\to\infty$, and conse-

quently, $t_2^* \to \infty$; see Eq. (32). The saturation is thus a pure quantum effect, and may be viewed as a kind of quantum mechanical suppression of classical chaos, similar to what has been found, e.g., in the quantum version of the kicked rotor [11].

We have also shown that the diffusion constant is proportional to the square of the velocity for all kinds of complexity of the system. All presented results are valid under the condition that the time-dependent perturbation is rather small.

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