Two-dimensional electron transport with anisotropic scattering potentials

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Anisotropic scattering potential has been found at the heterointerface of a two-dimensional electron gas in low-temperature transport experiments. This scattering potential has various symmetries depending on the crystalline direction of the interface. The conductivity tensor of a two-dimensional electron gas is theoretically investigated with a Boltzmann equation. Several models of anisotropic elastic scattering potentials are examined. Explicit formulas of the conductivity tensor are given to the lowest order of the potential anisotropy. If isotropic scatterings and anisotropic scatterings coexist, Matthiessen's rule gives larger mobility than the exact value. The conductivity is isotropic if the number of the symmetry axes of the scattering probability is more than 2. [S0163-1829(98)04035-1]

I. INTRODUCTION

The possible origins of anisotropic conductivity are the anisotropic effective mass and the anisotropic scattering potential. Mass anisotropy is a dominant origin of the anisotropic conductivity, for example, in bulk Si or Ge (Ref. 1) or in Si metal-oxide-semiconductor structures at the (110) interface.[2,3] Recently, quite a large anisotropy has been found in the electron conductivity of an $Al_xGa_{1-x}As$/GaAs heterostructure on a (001) substrate[4–6] and also in that of a double heterostructure with a thin AlAs layer deposited in the well region.[7] More recently, intensive research has concentrated on the anisotropic electron transport on a (311)B GaAs substrate[8] and the anisotropic hole transport (with anisotropic hole mass) on a (311)A GaAs substrate.[9–12] For an electron on a (001) substrate, for example, the conductivity in the $[\bar{1}10]$ direction is higher than that in the $[110]$ direction. Since the low-energy effective mass of the conduction band is isotropic, the anisotropic transport originates from the anisotropy of the scattering potential. The effect of impurity scattering is assumed to be isotropic, and the phonon scattering is negligible in these experiments. Two candidates for the anisotropic potential are (1) interface roughness scattering from anisotropic islands and (2) chainlike scattering potential localized at the steps formed on a vicinal substrate. Both of these stem from the microscopic potential of the anisotropic structure in the two crystalline direction $[\bar{1}10]$ and $[110]$ of a (001) GaAs substrate. Assuming the anisotropic islands have some appropriate radial distribution and are located randomly at the interface, the mobility in the $[\bar{1}10]$ direction was shown to be larger.[4] In the nearly straight chainlike potential, the conductivity normal to the chain is smaller than that parallel to the chain.[5] An additional multiscattering effect has been discussed assuming that the spacing of these chains is almost periodic and shorter than the electron coherence length.[13]

Scanning tunneling microscopy (STM) or atomic-force microscopy have enabled us to inspect the surface structure in detail. Several experimental observations support the existence of an anisotropic (nearly elliptical) island that is longer in the $[\bar{1}10]$ direction. The surface diffusion constant of the adatoms is found to have an anisotropy in various directions, causing the shape of the island to be highly anisotropic. A stable two-dimensional island can have a variety of shapes depending on the direction of the crystal surface. An STM study of the GaAs (111)B surface has shown the existence of a triangular superstructure.[14] Both a tetrahedral structure[15] and a pyramid structure[16] have been realized on a (111)B or (001) GaAs surface. Recently, wire-shaped structure of Ge was realized on Si (113) surface.[17] It should be stressed that, although the microscopic shape and size of these scatterers may not be uniform, *the typical direction of these structures could be macroscopically homogeneous on the surface*, so that *the ensemble averaged scattering potential maintains the symmetry*. Therefore, the transport of a two-dimensional electron gas (2DEG) confined at the heterointerface will be anisotropic as a result of scattering by these interface structures. It has not yet been established, however, that the higher symmetric potential structures affect the electron transport confined in the heterostructure.

In this paper, we derive a general formula for the conductivity tensor of a 2DEG controlled by the anisotropic scattering potential under a zero magnetic field. The effect of weak magnetic field will be reported elsewhere. The calculation is restricted to the semiclassical transport regime. Quantum correction by a weak localization had been discussed in Refs. 18,19. This paper is organized as follows: In Sec. II a conductivity tensor is derived by using the Boltzmann equation. Section III shows the results for an anisotropic scattering probability with two symmetry axes. Conductivity tensors for the scattering probability with higher symmetry are derived in Sec. IV, and conclusion follows in Sec. V. The appendices provide detailed derivations of the formulas in the main text.

II. CONDUCTIVITY TENSOR

We start from a standard linearized Boltzmann equation for 2DEG under a uniform electric field $\vec{E}$,

$$-e\vec{E}\cdot\frac{\partial\varepsilon_k}{\hbar\partial\vec{k}}\frac{\partial f_0}{\partial\varepsilon_k}=\sum_{\vec{k}'}(g_{\vec{k}}-g_{\vec{k}'})\tilde{Q}(\vec{k},\vec{k}'),\qquad(2.1)$$

where $f_0 = [\exp(\beta(\varepsilon_k - \mu) + 1]^{-1}$ is the equilibrium distribution function at a temperature $1/(k_B \beta)$ and with the chemical potential μ, and $\varepsilon_k = \hbar^2 k^2/(2m^*)$ is the energy of an electron with a wave number $\vec{k}$. Here the effective mass m^* is assumed to be isotropic. The function $g_{\vec{k}} = f_{\vec{k}} - f_0$ is the change in the distribution function $f_{\vec{k}}$ by the electric field to a linear order, which is sustained by the elastic scattering with a probability $\tilde{Q}(\vec{k}, \vec{k}')$ from a state $\vec{k}$ to a state $\vec{k}'$. The scattering potential is assumed to have at least one symmetry axis and one of these axes is chosen for the x direction. Since we assume elastic scattering and take the change in the distribution function to the linear order of $\vec{E}$, the electron velocity $v_k = (1/\hbar)|\partial\varepsilon/\partial\vec{k}|$ is invariant and only its angle θ relative to the x axis is a variable. By defining

$$g_{\vec{k}} \equiv \left(-\frac{\partial f_0}{\partial\varepsilon} \right) e v_k \vec{\lambda}(\theta) \cdot \vec{E}, \qquad (2.2)$$

where $\vec{\lambda}(\theta)$ is the relaxation time vector to be solved, the Boltzmann equation [Eq. (2.1)] reduces to

$$\cos\theta = \int_0^{2\pi} d\phi [\lambda_x(\theta) - \lambda_x(\phi)] Q(\theta, \phi), \qquad (2.3)$$

$$\sin\theta = \int_0^{2\pi} d\phi [\lambda_y(\theta) - \lambda_y(\phi)] Q(\theta, \phi), \qquad (2.4)$$

where ϕ is an angle between the x axis and $\vec{k}'$. The scattering probability $Q(\theta, \phi)$, which is obtained by integrating $\tilde{Q}(\vec{k}, \vec{k}')$ by $|\vec{k}'|$, is defined from Fermi's golden rule, namely,

$$Q(\theta, \phi) \equiv \frac{L^2}{(2\pi)^2} \frac{2\pi m^*}{\hbar^3} |\langle\theta|U|\phi\rangle|^2, \qquad (2.5)$$

where U is the scattering potential and the bra $\langle\theta|$ denotes the plane wave $\Psi_\theta^\dagger(x,y) = e^{-i\vec{k}\cdot\vec{r}}/L = \exp[-i(xk\cos\theta + yk\sin\theta)]/L$ with the system size L^2. If it is necessary, the ensemble average of the potential distribution is used. Now, the problem is reduced to the integral equations for two independent functions λ_x and λ_y. Traditionally, the transport properties of the anisotropic scattering potential are investigated by using anisotropic scattering time $\tau(\theta)$ where the right hand side of Eq. (2.1) is replaced with $g_k/\tau(\theta)$ and then solving $\tau(\theta)$ by the iteration method or the variational method. This treatment is equivalent to ours by replacing $\lambda_x(\theta)$ with $\tau(\theta)\cos\theta$ and $\lambda_y(\theta)$ with $\tau(\theta)\sin\theta$. However, our method is advantageous in that the formulas are separated in the main two directions (x and y) and become simple as can be seen in the following. Our method is similar to that of Samoilovich and co-workers,[20,21] which expands the distribution function in spherical harmonics and applies it to the bulk transport with an anisotropic mass. There exists a rigourous method of dealing with the anisotropic scattering problem if the scattering probability is expressed by the sum of products of factors depending on $\vec{k}$ and of $\vec{k}'$, such as $\Sigma_i p_i(\vec{k}) q_i(\vec{k}')$.[22] However, such a method is only applicable in restricted cases.

The current density is given by

$$\vec{j} = \frac{2}{L^2} \sum_{\vec{k}} e\vec{v}_k g_{\vec{k}} \qquad (2.6)$$

$$= \frac{e^2}{2\pi^2} \int_0^\infty dk\,k \left(-\frac{\partial f_0}{\partial\varepsilon} \right) \int_0^{2\pi} d\theta v_k [\vec{v}_k \otimes \vec{\lambda}(\theta)] \vec{E}, \qquad (2.7)$$

where the factor 2 comes from the spin degeneracy. Then, the conductivity tensor is

$$\sigma = \frac{e^2 n_s}{m^*} \int d\varepsilon \frac{\varepsilon}{\varepsilon_F} \left(-\frac{\partial f_0}{\partial\varepsilon} \right)$$
$$\times \int_0^{2\pi} d\theta \begin{pmatrix} \lambda_x(\theta)\cos\theta & \lambda_y(\theta)\cos\theta \\ \lambda_x(\theta)\sin\theta & \lambda_y(\theta)\sin\theta \end{pmatrix}, \qquad (2.8)$$

where the electron density is $n_s = m^* \varepsilon_F/(\pi\hbar^2)$ with the Fermi energy $\varepsilon_F = \mu$ at zero temperature and the matrix in Eq. (2.8) is $(1/k)\vec{k} \otimes \vec{\lambda}(\theta)$. Henceforth, considerations are limited to zero temperature at which only the state $\vec{k}$ at the Fermi circle ($=k_F$) is relevant, i.e., $-\partial f_0/\partial\varepsilon = \delta(\varepsilon - \varepsilon_F)$. The extension to finite temperatures is straightforward since the effect comes only in a weighted integration by $[-(\partial f_0)/(\partial\varepsilon)]\varepsilon/\varepsilon_F$, if the change in the screening factor on the scattering potential is negligible.

Now the scattering probability $Q(\theta, \phi)$ is examined in detail. Since this term is a function of $\vec{k} - \vec{k}'$ in general, it is rewritten in terms of $q \equiv |\vec{k} - \vec{k}'| = 2k_F \sin(|\theta - \phi|/2)$ and the angle ψ between $\vec{k} - \vec{k}'$ and the x axis. The periodicity can be easily seen, $Q(q, \psi) = Q(q, \psi + 2\pi)$. Using microreversibility $Q(\theta, \phi) = Q(\phi, \theta)$, it is found that

$$Q(q, \psi) = Q(q, \psi - \pi) = Q(q, \psi + \pi). \qquad (2.9)$$

Since one symmetry axis in the x direction was chosen, the scattering probability between states $\vec{k}$ and $\vec{k}'$ is the same as the probability between states $\vec{\tilde{k}}$ and $\vec{\tilde{k}}'$ where $k_x = \tilde{k}_x$ and $k_y = -\tilde{k}_y$. Therefore,

$$Q(q, \psi) = Q(q, -\psi), \qquad (2.10)$$

and from Eqs. (2.9) and (2.10),

$$Q(q, \psi) = Q(q, \pi - \psi) = Q(q, -\pi - \psi). \qquad (2.11)$$

Hence, it is only necessary to know the function $Q(q, \psi)$ at $\psi \in [0, \pi/2]$ for a given q. Using these properties, by putting $(\theta, \phi) \rightarrow (-\theta, -\phi)$ in Eq. (2.3), it is found that $\lambda_x(-\theta) = \lambda_x(\theta)$, and in Eq. (2.4), $\lambda_y(-\theta) = -\lambda_y(\theta)$. Similarly, by setting $(\theta, \phi) \rightarrow (\pi - \theta, \pi - \phi)$, it is found that $\lambda_x(\pi - \theta) = -\lambda_x(\theta)$ and $\lambda_y(\pi - \theta) = \lambda_y(\theta)$.

We solve the integral equations [Eqs. (2.3) and (2.4)] by Fourier expansion. Using the above symmetries, λ_x and λ_y are expanded as follows:

$$\lambda_x(\theta) = \sum_{n=0}^\infty \lambda_x(2n+1)\cos[(2n+1)\theta], \qquad (2.12)$$

$$\lambda_y(\theta) = \sum_{n=0}^\infty \lambda_y(2n+1)\sin[(2n+1)\theta], \qquad (2.13)$$

with the inversion

$$\lambda_x(n) = \frac{1}{\pi} \int_0^{2\pi} d\theta \lambda_x(\theta) \cos(n\theta), \qquad (2.14)$$

$$\lambda_y(n) = \frac{1}{\pi} \int_0^{2\pi} d\theta \lambda_y(\theta) \sin(n\theta), \qquad (2.15)$$

for $n > 0$. Terms such as $\int d\theta \lambda_x(\theta) \sin(n\theta)$ or $\int d\theta \lambda_y(\theta) \cos(n\theta)$ are absent due to the symmetry. Hence, using Eq. (2.8), the conductivity tensor is diagonal, i.e.,

$$\sigma_{xx} = \frac{e^2 n_s}{m^*} \lambda_x(1), \qquad (2.16)$$

$$\sigma_{yy} = \frac{e^2 n_s}{m^*} \lambda_y(1), \qquad (2.17)$$

from which $\lambda_x(1)$ and $\lambda_y(1)$ can be interpreted to the relaxation times in the x and y directions, respectively.

The scattering probability is also expanded using the symmetry,

$$Q(q, \psi) = \sum_{m=0}^{\infty} Q_m(q) \cos(2m\psi), \qquad (2.18)$$

with the inversion

$$Q_m(q) = \frac{1}{(1 + \delta_{m,0})\pi} \int_0^{2\pi} d\psi Q(q, \psi) \cos(2m\psi), \qquad (2.19)$$

where $\delta_{m,n} = 0$ for $m \neq n$ and $\delta_{n,n} = 1$. If the scattering probability $Q(q, \psi)$ is periodic in ψ, the period should be π/s with an integer s because of the microreversibility [Eq. (2.9)] which requires the inversion symmetry through a point $q = 0$. Then, using Eq. (2.19), one can find that Q_m should be zero if $\cos(2m\pi/s) \neq 1$, or equivalently, if $m \neq ls$ with nonnegative integer l. In this case, one obtains that the number of symmetry axes N_Q of the scattering probability is $2s$ with considering an additional property, Eq. (2.10). The isotropic scattering probability is a special case when $s = \infty$ and all the terms Q_m with $m > 0$ vanish. The symmetry of the scattering *probability Q* should be clearly distinguished from the symmetry of the scattering *potential U*. If the potential has even number of symmetry axes N_U as found in a pyramid structure on a (001) surface ($N_U = 4$), the scattering probability has the same number of symmetry axes and only $Q_{lN_U/2}$ terms are nonzero. However, care is needed if the potential has odd number of symmetry axes, as found in a triangular structure on the (111)B surface ($N_U = 3$). Because of the microreversibility, the scattering probability has $N_Q = 2N_U$ symmetry axes and only Q_{lN_U} terms are nonzero.

By putting all the expansions into the integral equation [Eq. (2.3)], multiplying by $\cos(l\theta)$, and integrating over θ from 0 to $2\pi(l = 1, 2, \ldots)$, one obtains, after a straightforward calculation,

$$\delta_{l,1} = \sum_{n=1}^{\infty} \lambda_x(2n-1) K_{l,n}^x, \qquad (2.20)$$

where the symmetric matrix K^x is given by

$$K_{l,n}^x = \frac{(-1)^{l-n}}{2} [(1 + \delta_{l,n}) J_{|l-n|,n+l-1} - J_{n+l-1,|l-n|}], \qquad (2.21)$$

where

$$J_{n,m} = \int_0^{2\pi} d\zeta Q_n(q_\zeta) [\cos(n\zeta) - \cos(m\zeta)], \qquad (2.22)$$

and $q_\zeta = 2k_F \sin|\zeta/2|$. One finds a similar equation in the y direction,

$$\delta_{l,1} = \sum_{n=1}^{\infty} \lambda_y(2n-1) K_{l,n}^y, \qquad (2.23)$$

with

$$K_{l,n}^y = \frac{(-1)^{l-n}}{2} [(1 + \delta_{l,n}) J_{|l-n|,n+l-1} + J_{n+l-1,|l-n|}]. \qquad (2.24)$$

By defining the vector $\lambda_x = [\lambda_x(1), \lambda_x(3), \ldots]^T$ and $I = (1,0,0, \ldots)^T$ where T is the transpose, Eq. (2.20) is expressed by $I = K^x \lambda_x$. A similar equation is also obtained in the y direction. If the matrices K^x and K^y are not singular, the scattering times are obtained by the equations

$$\lambda_x(1) = (K^x)_{1,1}^{-1}, \qquad (2.25)$$

$$\lambda_y(1) = (K^y)_{1,1}^{-1}, \qquad (2.26)$$

where $M_{1,1}$ means the (1,1) element of a matrix M.

Here several comments are needed. Even if one of the matrices, for example, K^x, is singular, a physically sound relaxation time is obtained in the y directions, which will be demonstrated in Sec. III D. The order of matrices and vectors have not been mentioned. To undertake some numerical evaluation of this problem, we have to truncate the infinite expansion of Q to some order N_t. This approximation gives fairly good results if the expansion of Q converges absolutely, which is so in most cases except for that found in Sec. III D. If the scattering potential is isotropic, all Q_m values with $m > 0$ are zero, and, the matrices K^x and K^y are the same and are diagonal with the elements $K_{l,l} = J_{0,2l-1}$. The matrix inversion is trivial and the inverse of the scattering time, which is also isotropic, is given by

$$\lambda(1)^{-1} = J_{0,1} = \int_0^{2\pi} d\zeta Q_0(q_\zeta)(1 - \cos\zeta), \qquad (2.27)$$

which is a standard definition of the transport relaxation time in an isotropic system.

The scattering probabilities of the two main types of scattering at low temperatures are summarized here for later use. For screened Coulomb scattering, which is isotropic in general, the scattering probability is given by[23]

$$Q^{\text{Coulomb}}(q) = \frac{\hbar \pi n_I}{2m^*} \left(\frac{q_s}{q\kappa(q)} \right)^2 e^{-2qz_d}, \qquad (2.28)$$

where $q_s = m^* e^2 / 2\pi \hbar^2 \kappa^*$ is the screening constant with the effective dielectric constant κ^*, z_d is the distance of the

ionized impurities and the conducting layer, and n_I is the impurity areal density in the layer $z = z_d$. The polarization factor $\kappa(q)$ is assumed to be isotropic and is $1 + q_s/q$ by neglecting the form factor of the wave function.[23]

The scattering probability between the two-dimensional states $\vec{k}$ and $\vec{k}'$ due to a fluctuation in the interface height, $d(\vec{\rho})$ at a position $\rho = (x, y)$, is given by the first Born approximation,[23-25]

$$\tilde{Q}^{\mathrm{rough}}(\vec{k}, \vec{k}') = \frac{2\pi}{\hbar} \left\langle \left| \frac{\Upsilon \Delta_{k-k'}}{\kappa(|\vec{k} - \vec{k}'|)} \right|^2 \right\rangle \delta(\varepsilon_k - \varepsilon_{k'}), \tag{2.29}$$

where the local change in the potential caused by the shift of the interface is given by $\Upsilon \Delta(\vec{\rho})$ using a relative displacement $\Delta(\vec{\rho}) = d(\vec{\rho}) - \langle d(\vec{\rho}) \rangle$. $\Delta_{\vec{q}}$ is the Fourier transform of $\Delta(\vec{\rho})$, and the average $\langle \cdots \rangle$ is obtained for an appropriate roughness distribution. The interface correlation function G and its Fourier transform F (Ref. 26) are introduced by

$$G(\vec{a}) = \frac{1}{\Delta^2} \langle \Delta(\vec{\rho}) \Delta(\vec{\rho} + \vec{a}) \rangle, \tag{2.30}$$

$$F(\vec{q}) = \frac{1}{\Delta^2 \Lambda^2} \langle |\Delta_{\vec{q}}|^2 \rangle, \tag{2.31}$$

where Δ is the root mean square of $\Delta(\vec{\rho})$ and Λ is a scale characterizing the fluctuation in the lateral direction. The scattering probability is therefore

$$Q^{\mathrm{rough}}(\theta, \phi) = \frac{m^*}{2\pi\hbar^2} \left(\frac{\Delta \Lambda \Upsilon}{\kappa(q)} \right)^2 F(\vec{q}). \tag{2.32}$$

For a 2DEG confined to a single heterostructure, $\Upsilon = (e^2/\kappa^*)(\frac{1}{2}n_s + n_{\mathrm{depl}})$ where n_{depl} is the fixed-charge concentration in the depletion layer.[23]

III. TWO SYMMETRY AXES

In this section, we restrict the scattering probability to the least symmetry, i.e., the case of two symmetry axes ($N_Q = 2$ or $s = 1$).

A. Small anisotropy

For a small anisotropy, the expansion [Eq. (2.18)] of the scattering probability is truncated as $Q(q, \psi) = Q_0(q) + Q_1(q)\cos 2\psi$. The matrices $K^{x/y}$ are given by

$$K^{x/y} = \begin{pmatrix} J_{0,1} \mp \frac{1}{2}J_{1,0} & -\frac{1}{2}J_{1,2} & & & 0 \\ -\frac{1}{2}J_{1,2} & J_{0,3} & -\frac{1}{2}J_{1,4} & & \\ & -\frac{1}{2}J_{1,4} & J_{0,5} & -\frac{1}{2}J_{1,6} & \\ & & -\frac{1}{2}J_{1,6} & J_{0,7} & \ddots \\ 0 & & & \ddots & \ddots \end{pmatrix}, \tag{3.1}$$

where the upper and lower signs are for K^x and K^y, respectively. Evaluating the inverse of the matrix, it is found that

$$\frac{1}{\lambda_{x/y}(1)} = J_{0,1} \mp \frac{1}{2}J_{1,0} - \Gamma_3(1), \tag{3.2}$$

where the factor $\Gamma_3(1)$ is isotropic and is defined in a form of continued fraction as follows:

$$\Gamma_m(s) = \frac{\left(\frac{1}{2}J_{s,m-s} \right)^2}{J_{0,m} - \Gamma_{m+2s}(s)}. \tag{3.3}$$

Since $\Gamma_m(s)$ is a rapid decreasing function of m in most cases, we can have a reasonably approximated value of $\Gamma_3(1)$ by truncating the series at some point. If the isotropic term Q_0 is much larger than Q_1, $\Gamma_3(1)$ can be neglected in Eq. (3.2). In this approximation, Eq. (3.2) is in an explicit form,

$$\frac{1}{\lambda_{x/y}(1)} = \int_0^{2\pi} d\zeta Q_0(q_\zeta)(1 - \cos \zeta)$$

$$\pm \frac{1}{2} \int_0^{2\pi} d\zeta Q_1(q_\zeta)(1 - \cos \zeta). \tag{3.4}$$

If there are several scattering processes, Matthiessen's rule is often employed to evaluate the total relaxation time from the independently evaluated relaxation times. Takeda and Pearsall[27] and Saxena and Mudares[28] have shown that this is not correct when treating alloy disorder scattering if the temperature dependences of each relaxation time differ greatly. However, for low-temperature elastic scatterings, the relaxation time is almost temperature independent. Here, the validity of Matthiessen's rule is examined when isotropic and anisotropic scatterings coexist. The inverse of the total relaxation time is given by [cf. Eq. (2.27)],

$$\frac{1}{\lambda_{x/y}(1)} = J_{0,1}^I + J_{0,1}^A \mp \frac{1}{2}J_{1,0}^A - \frac{(\frac{1}{2}J_{1,2}^A)^2}{J_{0,3}^I + J_{0,3}^A - \Gamma_5^{I+A}(1)}, \tag{3.5}$$

where the suffixes I and A show the contribution from isotropic and anisotropic scattering, respectively. On the other hand, the relaxation times for the individual processes are

$$\frac{1}{\lambda_{x/y}^I(1)} = J_{0,1}^I, \tag{3.6}$$

$$\frac{1}{\lambda_{x/y}^{A}(1)} = J_{0,1}^{A} \mp \frac{1}{2} J_{1,0}^{A} - \frac{(\frac{1}{2} J_{1,2}^{A})^2}{J_{0,3}^{A} - \Gamma_5^{A}(1)}. \tag{3.7}$$

The failure of Matthiessen's rule is evaluated while neglecting the order of $\Gamma_5(1)$, namely,

$$\frac{1}{\lambda_{x/y}(1)} - \sum_i \frac{1}{\lambda_{x/y}^i(1)} = \left(\frac{1}{2} J_{1,2}^{A}\right)^2 \left(\frac{1}{J_{0,3}^{A}} - \frac{1}{J_{0,3}^{I} + J_{0,3}^{A}}\right). \tag{3.8}$$

Since $J_{0,3}$ is positive in most cases, the value of Eq. (3.8) is positive and the total relaxation time with Matthiessen's rule is overestimation. Therefore, care is needed to evaluate the anisotropic transport when other isotropic scattering is present and of the same magnitude. This error is demonstrated explicitly in Sec. III C.

B. Mass anisotropy

A system with a special type of anisotropic mass and with an isotropic scattering potential can be reduced to a system of an anisotropic potential with an isotropic mass.[1,20,21,29] We consider the system with ellipsoidal parabolic band structure as

$$\varepsilon_k = \frac{\hbar^2 k_x^2}{2m_x} + \frac{\hbar^2 k_y^2}{2m_y}, \tag{3.9}$$

and the scattering potential depends only on $q = |\vec{k} - \vec{k}'|$. Defining a new wave vector by scaling the k_y axis,

$$\vec{k}^* = \alpha \vec{k} \equiv \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{m_x/m_y} \end{pmatrix} \vec{k}, \tag{3.10}$$

an isotropic band with an isotropic mass m_x is obtained (Herring-Vogt transformation).[1] By noting

$$\frac{\partial \varepsilon}{\hbar \partial \vec{k}} = \alpha \frac{\partial \varepsilon}{\hbar \partial \vec{k}^*} = \alpha \vec{v}_k^*, \tag{3.11}$$

and $\vec{E}^* = \alpha \vec{E}$ and with the definition

$$g_{\vec{k}} = \left(-\frac{\partial f_0}{\partial \varepsilon}\right) e \vec{E}^* \cdot \vec{\lambda}(k^*) |v_k^*|, \tag{3.12}$$

the Boltzmann equation [Eq. (2.1)] reduces to

$$(\cos\theta, \sin\theta) = \sqrt{\frac{m_y}{m_x}} \frac{L^2}{(2\pi)^2} \int d^2 k'^* [\lambda(\vec{k}^*) - \lambda(\vec{k}'^*)]$$
$$\times \tilde{Q}(\alpha^{-1}\vec{k}^*, \alpha^{-1}\vec{k}'^*), \tag{3.13}$$

where the angle θ is determined by $\vec{k}^*$ and the x axis. By putting, for elastic scattering,

$$\tilde{Q}(\alpha^{-1}\vec{k}^*, \alpha^{-1}\vec{k}'^*) = \frac{\hbar^2 (2\pi)^2}{L^2 \sqrt{m_x m_y}} Q(\theta, \phi) \delta(\varepsilon_{k^*} - \varepsilon_{k'^*}), \tag{3.14}$$

the same equation is obtained for $\vec{\lambda}$ as Eqs. (2.3) and (2.4). The current density is similarly given as for an isotropic mass [cf. Eqs. (2.6), (2.7)],

$$\vec{j} = \frac{e^2}{2\pi^2} \sqrt{\frac{m_y}{m_x}} \int d^2 k^* \delta(\varepsilon_F - \varepsilon_{k^*}) |v_k^*| [\alpha \vec{v}_k^* \otimes \vec{\lambda}(k^*)] \vec{E}^*, \tag{3.15}$$

which reduces to the diagonal conductivities [cf. Eqs. (2.16), (2.17)]

$$\sigma_{xx} = \frac{e^2 n_s}{m_x} \lambda_x(1), \tag{3.16}$$

$$\sigma_{yy} = \frac{e^2 n_s}{m_y} \lambda_y(1), \tag{3.17}$$

using the electron density given by $n_s = \sqrt{(m_y/m_x)} k_F^{*2}/(2\pi)$ with $k_F^* = \sqrt{2m_x \varepsilon_F}/\hbar$. In a crude approximation, the conductivity tensor has been evaluated[30] using an isotropic relaxation time τ,

$$\sigma = \frac{2e^2 \tau}{(2\pi)^2} \int d^2 k f_0 \left(\frac{1}{m}\right), \tag{3.18}$$

where $(1/m)$ is the inverse mass tensor. This approximation corresponds to the conductivities with $\lambda_{x/y}(1)$ replaced with an isotropic relaxation time τ in Eqs. (3.16,3.17). In this scaled system,

$$q^2 = q^{*2} \left(\frac{m_x + m_y}{2m_x} + \frac{m_x - m_y}{2m_x} \cos 2\psi\right), \tag{3.19}$$

where ψ is the angle between $\vec{k}^* - \vec{k}'^*$ and the x axis and $q^* = 2k_F^* \sin|(\theta - \phi)/2|$. If the isotropic scattering probability $Q(q)$ does not depend on q, (short-range scattering), the scattering time $\lambda_{x/y}(1)$ is isotropic and, therefore, Eq. (3.18) is exact. However, if the q dependence of the matrix is not negligible, the reduced scattering probability $Q(\theta, \phi) = Q(q^*, \psi)$ is anisotropic in general. If the original scattering probability $Q(q)$ is a monotonically decreasing function of q, as with the ionized impurity scattering [see Eq. (2.28)], and in the case of $m_y > m_x$, the reduced probability $Q(q^*, \psi)$ is a monotonically decreasing function of ψ in the region $[0, \pi/2]$ for a fixed q^*. If one can approximate $Q(q^*, \psi)$ in the Fourier expansion up to the second term of Eq. (2.18), Q_1 (as well as Q_0) is positive. Using the result of Eq. (3.4), it is found that $\lambda_y(1) > \lambda_x(1)$. Similarly, if $m_y < m_x$, then $\lambda_y(1) < \lambda_x(1)$.[2,3,20] If $Q(q)$ is a monotonically increasing function of q, then the relations for $\lambda_{x/y}(1)$ are opposite.

C. Elliptic roughness potential

The anisotropic low-temperature Hall mobilities found in heterostructures with nominally no misorientation[4,7–12] are interpreted in terms of anisotropic interface roughness scattering. This anisotropy is examined by postulating a model for the epitaxially grown heterointerface, assuming the formation of anisotropic islands.

The standard approximation imposed on the interface correlation function [Eq. (2.30)] is Gaussian, $G(\vec{a}) = \exp[-(a/\Lambda)^2]$,[23–26,31–33] or exponential $G(\vec{a}) = \exp[-|a|/\Lambda]$,[26,33] in the isotropic system. Studies on the anisotropic roughness potential have used an anisotropic correlation length with a Gaussian form[7,12,8,13] such that
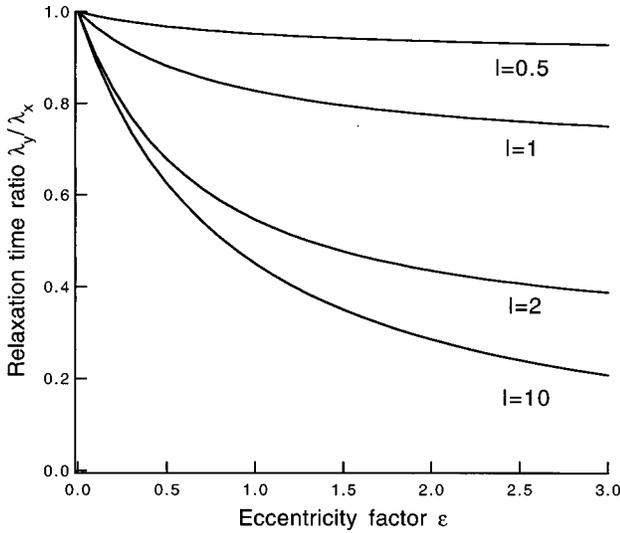
FIG. 1. Calculated ratio of relaxation times λ_y/λ_x as a function of eccentricity parameter ϵ. Several parameters $l=\Lambda k_F$ are chosen.

$$G(x,y)=\exp\left[-\left(\frac{x}{\Lambda_x}\right)^2-\left(\frac{y}{\Lambda_y}\right)^2\right]. \quad (3.20)$$

By setting, $\Lambda=\Lambda_x$ and $\Lambda_y/\Lambda_x=1/\sqrt{1+\epsilon}$, the scattering probability from Eq. (2.32) for this case is

$$Q(q,\psi)=P(q)\exp\left(-\frac{\epsilon}{8(1+\epsilon)}\Lambda^2q^2\cos 2\psi\right), \quad (3.21)$$

where

$$P(q)=\frac{P_0}{\kappa(q)^2}\exp\left(-\frac{2+\epsilon}{8(1+\epsilon)}\Lambda^2q^2\right),$$

with $P_0=m^*(\Delta\Lambda\Upsilon)^2/[2\hbar^2\sqrt{1+\epsilon}]$. The eccentricity factor ϵ shows the anisotropy rate, which is in the range of $-1<\epsilon<\infty$. The first two terms of the Fourier expansion of the scattering probability are given by

$$Q_0(q)=P(q)I_0\left(\frac{\epsilon}{8(1+\epsilon)}\Lambda^2q^2\right), \quad (3.22)$$

$$Q_1(q)=-2\,\text{sign}\,(\epsilon)P(q)I_1\left(\frac{|\epsilon|}{8(1+\epsilon)}\Lambda^2q^2\right), \quad (3.23)$$

where $\text{sign}(\epsilon)=\pm 1$ depending on the sign of ϵ and I_n is an nth order modified Bessel function of the first kind. Assuming $[|\epsilon|/(1+\epsilon)]\Lambda^2k_F^2\ll 1$ and for $q_s\ll k_F$ or $q_s\gg k_F$ where q_s is the screening constant, analytical expressions for the conductivity can be obtained, which are a sum of terms such as $I_n\{[2+\epsilon/4(1+\epsilon)]\Lambda^2k_F^2\}$. They are listed in Appendix A.

In Fig. 1, the ratio of relaxation times λ_y/λ_x is shown as a function of ϵ for several values of $l\equiv\Lambda k_F$ evaluated from Eq. (3.4) numerically. We used $n_s=4\times 10^{15}m^{-2}$, the effective mass 0.067 and dielectric constant 12.5 of GaAs. The reason for the smaller anisotropy for smaller l is quite simple, namely, that the islands are too small for the electrons to resolve their shape. For larger ϵ, the approximation of neglecting $\Gamma_3(1)$ and $Q_2,Q_3,\ldots$ may become questionable. We also show in Fig. 2 the relaxation time when the
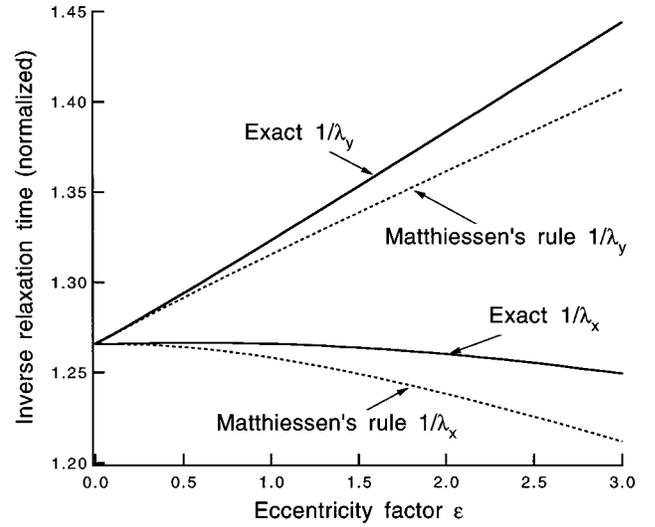
FIG. 2. Calculated anisotropic inverse relaxation times as a function of eccentricity parameter ϵ when the isotropic Coulomb scatterings and anisotropic elliptic roughness scattering are coexist. The solid lines are exact result and the dashed lines are evaluated with Matthiessen's rule.

isotropic Coulomb scattering and the anisotropic elliptic roughness scattering coexist. Using the scattering probability given in Eq. (2.28) with $z_d=0$, and the same parameters as used in Fig. 1, the inverse of exact total relaxation time evaluated from Eq. (3.5) and the one evaluated with the Matthiessen's rule using Eq. (3.7) are shown as a function of ϵ. The inverse relaxation times are normalized to $1/\tau_0\equiv\hbar\pi n_I/(2m^*)$ and we have used $P_0\sqrt{1+\epsilon}\tau_0=500$ and $l=10$. The error of the Matthiessen's rule increases with ϵ and the exact relaxation times are smaller than the approximated ones.

Although this parametrization $(\Delta,\Lambda_x,\Lambda_y)$ explains the experimental results well, the physical relation between these parameters and the microscopic structure is not clear. We have modeled in Ref. 4 the roughened heterointerface as randomly distributed elliptical two-dimensional islands with monolayer heights $[d_0=2.83$ Å for GaAs (001)].

$$d(\vec{\rho})=d_0\sum_i\eta_i\Theta[R_i^2-(x-x_i)^2-(1+\epsilon)(y-y_i)^2], \quad (3.24)$$

where ϵ is the eccentricity of the island, η is $+1$ (islands) or -1 (holes), and Θ is a step function. R_i and (x_i,y_i) are the radius and the center of the ith island. First the isotropic case, $\epsilon=0$, is considered. In this model, $\Delta^2G(a)=\langle\Delta(r)\Delta(r+a)\rangle=d_0^2n_N\langle I(a,R)\rangle_g$, where the overlapping function I of two circles with radius R with a separation a is given by

$$I(a,R)=\pi R^2D\left(\frac{a}{2R}\right), \quad (3.25)$$

$$D(x)=\frac{2}{\pi}(\cos^{-1}x-x\sqrt{1-x^2}), \quad (3.26)$$

with $D(x)=0$ for $1<x$ and n_N is the areal concentration of the islands. Here the correlation of the island centers (x_i,y_i)

is neglected. The amplitude of the roughness is $\Delta = d_0 \sqrt{n_N \pi \langle R^2 \rangle_g}$ and Δ^2/d_0^2 corresponds to the surface coverage, if there is no overlapping of islands. The average $\langle \cdots \rangle_g$ over the island radius R is undertaken with the radius distribution $g(R)$. Although the actual functional of $g(R)$ is not known, the following normalized functions are considered for a trial:

$$g_a(R) = \frac{2}{\sqrt{\pi} R_0} e^{-(R/R_0)^2}, \tag{3.27}$$

$$g_b(R) = \frac{2}{R_0^2} R e^{-(R/R_0)^2}, \tag{3.28}$$

with a characteristic length scale R_0. Roughly speaking, the function g_a corresponds to a nonequilibrium surface and g_b corresponds to an equilibrium surface. Approximating $D(x)$ as $1-x$ for $0<x<1$ and zero otherwise, the asymptotic form of $G(a)$ for the function g_a is found to be $\exp[-(a/2R_0)^2]/a$. Quite interestingly, a Gaussian form $\exp[-(a/2R_0)^2]$ is obtained not from g_a for the nonequilibrium surface but from the the function g_b for the equilibirum surface.

If a finite anisotropy is introduced, G and F also become anisotropic, where G is always larger and F always smaller in the direction parallel to the semimajor axis of the ellipse (the x direction for $\epsilon>0$). This anisotropy of G can be understood by noting that the overlap of two ellipses displaced in the x direction is larger than that in the y direction.[4] In general, the eccentricity is likely to be smaller for smaller islands because of the cost of surface energy for a fixed gain in the nucleus formation energy proportional to its area. However, here for simplicity the eccentricity of the islands ϵ is considered to be uniform, independent of the size. In this model, F is easily found by the scale transformation of F of the isotropic potential. Using the isotropic correlation function $G(a)$, one can define anisotropic function, $G^*(x,y)$ as $G(x,\sqrt{1+\epsilon}y)$. Therefore, after Fourier transformation, one has $F^*(q_x,q_y) = F(q_x,q_y/\sqrt{1+\epsilon})/\sqrt{1+\epsilon}$, where $F(\vec{q})$ is the Fourier transform of $G(\vec{a})$. The example of correlation functions $F^*(q_x,q_y)$ is shown in Fig. 2 in Ref. 4.

D. Chainlike potential

Recently, very active research has been conducted on the physics of surfaces vicinal from a low crystal index because their properties are substantially different from those of low index surfaces. Fractional layer superlattices[34,35] created using lateral growth, and the wirelike incorporation of Si dopant atoms[36] are examples.

We have found an anisotropy of mobilities (μ) in a (111)A vicinal GaAs (001) substrate at 1.5 K between two adjacent Hall bridges aligned in the $[\bar{1}10]$ and $[110]$ directions, i.e., $\mu_{[\bar{1}10]} > \mu_{[110]}$.[5] We could not find the anisotropy of μ in (111)B vicinal samples, the mobility values of which were between the two μ values of the (111)A vicinal sample. In the (111)B vicinal surface, no reflection high-energy electron diffraction oscillation originating from a regular terrace structure was found[37,38] but a fairly ragged step structure was found using STM.[39] On the other hand,

clear terrace structures and relatively straight steps were found in the (111)A vicinal surface. See Ref. 5 for details.

Since conductivity anisotropy is quite sensitive to step shapes, an effective scattering potential is proposed at the interface steps to account for the conductivity anisotropy. The scattering potential of one step, $U(\vec{r})$, is constructed by the superposition of potential V centered at the ith step point $\vec{r}_i = [x_i(u), y_i(u)]$ (u is a parameter), where the x axis is set in a tilting direction and y is the other axis at the interface. Therefore, each step extends roughly along the y direction. Assuming x_i is the single-valued function of y_i (neglecting step overhangs), the step position is redefined as $[f_i(y_i), y_i]$ using y_i as a parameter and the step front function f_i of the ith step. The chainlike potential is given by $U(\vec{r}) = \Sigma_i \int dy_i V(\vec{r} - \vec{r}_i)$.

There exists an approach to the array of chainlike potentials,[13] in which the interface correlation function G is described as the product of an anisotropic Gaussian function Eq. (3.20) and a periodic function with average step distance W.[40] However, in this work no correlation is assumed in the relative positions of the average step fronts and the effect of electron wave interference scattered by the adjacent steps is neglected. The average step position $\bar{x}_i = (1/L)\int dy_i f_i(y_i)$ distributes randomly with density $1/W$ where L is the size of the system. Here the statistical properties of step front function f_i are assumed to be independent of step suffix i. Therefore, the total scattering probability, which is the square of the matrix element of the total scattering potential, has no cross term contributed from different steps and is given by multiplying the number of steps L/W with the scattering probability of one step.

Here we postulate a simple model for V as $v\delta^{2D}(\vec{r})$, where δ^{2D} is a two-dimensional δ function. The origin of this potential v has been discussed in Ref. 5. By requiring the spatial average of scattering potential to be zero, the potential U is $v\delta^{1D}[x-f(y)]-(v/L)$. The absolute square of its Fourier transform is

$$|\langle \vec{k}|U|\vec{k}+\vec{q}\rangle|^2 = \frac{v^2}{L^4} \int dy\, dY \exp\{-iq_y y$$
$$-iq_x[f(y+Y)-f(y)]\}, \tag{3.29}$$

the value of which at $q=0$ is zero because of the term $-(v/L)$. In particular, when the step is straight, $f(y) = 0, \forall y$, one finds

$$Q(q,\psi) = \frac{\pi\Sigma}{8} \frac{k_F}{q} [\delta(\psi) + \delta(\psi - \pi)] \tag{3.30}$$

where $\Sigma = (8mv^2/\pi\hbar^3 k_F W)$. If the effect of screening is important, Q should be divided by $\kappa(q)^2$, however, in the following, this factor is omitted for simplicity. The relaxation time is easily evaluated as $\lambda_x(1) = (32/3\pi^2)/\Sigma$ by inserting Eq. (3.30) into Eq. (2.3) and integrating over θ. By physical inspection, $\lambda_y(1) = \infty$.

For finite step raggedness, the y integration in Eq. (3.29) is evaluated by expanding the exponential,

$$\frac{1}{L}\int dy\ e^{-iq_x(f(y+Y)-f(y))}\sim 1-\frac{q_x^2}{2!L}\int dy[f(y+Y)^2+f(y)^2$$

$$-2f(y+Y)f(y)]=1-q_x^2g^2$$

$$+q_x^2g^2C(Y), \qquad (3.31)$$

where $C(Y)=(1/g^2L)\int dy\ f(y+Y)f(y)$ is the step point autocorrelation function. The parameter g characterizes the amplitude of the step raggedness, and if $2k_Fg\ll1$, this expansion to the order of g^2 is good. The step point autocorrelation function $C(Y)$ satisfying $C(0)=1$ and $C(\pm\infty)=0$ is assumed to have a Gaussian form $\exp[-(Y/\Lambda)^2]$ using a single parameter Λ, which characterizes the step-position correlation in the y direction. Then the scattering probability is given by

$$Q(q,\psi)=\frac{\pi\Sigma}{8}\left[\frac{k_F}{q}(\delta(\psi)+\delta(\psi-\pi))(1-g^2q^2)\right.$$

$$\left.+\frac{\Lambda g^2}{2\sqrt{\pi}}k_Fq^2\cos^2\psi\exp\left(-\frac{1}{4}\Lambda^2q^2\sin^2\psi\right)\right].$$

$$(3.32)$$

The first term shows specular reflection at the steps and the second term corresponds to diffuse scattering at the steps because of the step undulations. Brataas and Bauer[41] considered a system with lines of impurities with short-range potential. Although the physics is closely related to the current problem, the main difference is that they assumed no correlation in the positions of the impurities ($k_F\Lambda\to0$). The Fourier expansion of Q to the first two terms is good since, even for $g=0$, $\lambda_x(1)=1/\Sigma$ to this order of approximation (cf. the exact value $(32/3\pi^2)/\Sigma$). The relaxation times in this approximation are given by

$$\frac{1}{\lambda_x(1)}=\Sigma(1-\alpha(l)\tilde{g}^2), \qquad (3.33)$$

$$\frac{1}{\lambda_y(1)}=\Sigma\beta(l)\tilde{g}^2, \qquad (3.34)$$

where dimensionless parameters $\tilde{g}=k_Fg$ and $l=k_F\Lambda$ were used. $\alpha(l)$ and $\beta(l)$ have analytical forms in two extreme cases: if $l\ll1$, $\alpha(l)=\frac{8}{3}-(8/\sqrt{\pi})(3\pi/16)^2l$, $\beta(l)=(8/3\sqrt{\pi})(3\pi/16)^2l$, and if $l\gg1$, $\alpha(l)=0$, $\beta(l)=2/l^2$. The definition of functions $\alpha(l)$ and $\beta(l)$, and numerical estimations of relaxation times as function of $\tilde{g}$ and l has been found in Ref. 5.

IV. HIGHER SYMMETRIES

In this section, the conductivity anisotropy is discussed for higher symmetric scattering probability ($N_Q>2$ or s >1).

First Q is expanded to the second order, Q_s, assuming that the anisotropy is small. A formula for $s=2$ is obtained as

$$[\lambda(1)^{x/y}]^{-1}=J_{0,1}-\Gamma_3(2)-\Gamma_5(2), \qquad (4.1)$$

and for $s=3$

$$[\lambda(1)^{x/y}]^{-1}=J_{0,1}-\Gamma_5(3)-\Gamma_7(3). \qquad (4.2)$$

The details of the derivation are given in Appendix B. Since $\Gamma_m(s)$ defined in Eq. (3.3) is isotropic, the conductivity is isotropic.

In general, we can prove that the conductivity is isotropic for a scattering probability with $s>1$. The details of the proof are given in Appendix C. Therefore, the conductivity of a 2DEG with a tetrahedral structure[15] or a pyramid structure[16] is isotropic.

V. CONCLUSION

A conductivity tensor was formulated for anisotropic elastic scattering potentials in the semiclassical transport regime using a Boltzmann equation. Explicit formulas of the conductivity tensor are given to the lowest order of the potential anisotropy. If isotropic and anisotropic scatterings coexist, Matthiessen's rule gives larger mobility than the exact value. The results are compared with those for an anisotropic mass. The models of anisotropic elastic scattering potentials, namely, elliptic roughness potential and chainlike potential, are examined using the formula. The conductivity is proved to be isotropic if the number of the symmetry axes of the scattering probability is more than 2.

APPENDIX A: ANALYTICAL EXPRESSION OF CONDUCTIVITY WITH ANISOTROPIC ROUGHNESS SCATTERING

Explicit analytical expressions for the conductivity dominated by anisotropic roughness scattering modeled in Eq. (3.20) are listed assuming $[|\epsilon|/(1+\epsilon)]\Lambda^2k_F^2\ll1$ for two limits of the screening constants q_s.

If $q_s\gg k_F$,

$$\lambda_{x/y}^{-1}(1)=\frac{\pi k_F^2}{q_s^2}e^{-b}P_0[(6I_0(b)-8I_1(b)+2I_2(b))$$

$$\mp(10I_0(b)-15I_1(b)+6I_2(b)-I_3(b))d]$$

$$\to\frac{6\pi k_F^2}{q_s^2}P_0[1-\tfrac{5}{3}b\mp\tfrac{5}{3}d] \quad \text{for} \quad b\ll1$$

$$\to3\sqrt{\frac{\pi}{2}}\frac{k_F^2}{q_s^2b^{5/2}}P_0\left(1+\frac{5}{8b}\mp\frac{5d}{3b}\right) \quad \text{for} \quad b\gg1,$$

$$(A1)$$

where $b=(k_F\Lambda)^2[2+\epsilon/4(1+\epsilon)]$, $d=(k_F\Lambda)^2[\epsilon/8(1+\epsilon)]$ and P_0 is given in the main text below Eq. (3.21).

Similarly, for $q_s\ll k_F$, we have

$$\lambda_{x/y}^{-1}(1)=e^{-b}P_0[(2I_0(b)-2I_1(b))\mp(3I_0(b)-4I_1(b)$$

$$+I_2(b))d]$$

$$\to 2\pi P_0[1-\tfrac{3}{2}b\mp\tfrac{3}{2}d]\quad\text{for}\quad b\ll 1$$

$$\to\sqrt{\frac{\pi}{2}}\frac{1}{b^{3/2}}P_0\left(1+\frac{3}{8b}\mp\frac{3d}{2b}\right)\quad\text{for}\quad b\gg 1.$$

$$(A2)$$

APPENDIX B: CONDUCTIVITY TENSOR FOR HIGHER SYMMETRIES

Here, an explicit formula of the conductivity tensor Eq. (4.1) for the lowest order of $s=2$ is derived. The extension of the method to Eq. (4.2) for the lowest order of $s=3$ or higher symmetries is straightforward.

First we notice the Woodbury formula,

$$(A+U\cdot V^T)^{-1}=A^{-1}-[A^{-1}U(E+V^TA^{-1}U)^{-1}V^TA^{-1}],$$
$$(B1)$$

where A is an $n\times n$ matrix and U,V are $n\times p$ matrices with integers $n>p$, and E is a $p\times p$ unit matrix. Now, $K^{x/y}$ with only nonzero Q_0 and Q_2 is the sum of A and $U\cdot V^T$ as follows:

$$A=\begin{pmatrix} J_{0,1} & 0 & \frac{1}{2}J_{2,3} & & 0 \\ 0 & J_{0,3} & 0 & \frac{1}{2}J_{2,5} & \\ \frac{1}{2}J_{2,3} & 0 & J_{0,5} & 0 & \ddots \\ & \frac{1}{2}J_{2,5} & 0 & J_{0,7} & \ddots \\ 0 & & \ddots & \ddots & \ddots \end{pmatrix},\quad(B2)$$

$$U^T=\begin{pmatrix} t & 0 & 0 & \cdots \\ 0 & t & 0 & \cdots \end{pmatrix},\quad(B3)$$

$$V^T=\begin{pmatrix} 0 & 1 & 0 & \cdots \\ 1 & 0 & 0 & \cdots \end{pmatrix},\quad(B4)$$

where $t=\pm\frac{1}{2}J_{2,1}$. The inverse of matrix A has nonzero elements $(A)_{i,j}^{-1}\equiv B_{i,j}$ with $i+j$ even. Directly applying the formula, we have

$$(E+V^TA^{-1}U)^{-1}=\frac{1}{\Xi}\begin{pmatrix} 1 & -tB_{2,2} \\ -tB_{1,1} & 1 \end{pmatrix},\quad(B5)$$

where $\Xi=1-t^2B_{1,1}B_{2,2}$. Therefore,

$$[\lambda^{x/y}(1)]_{1,1}^{-1}=B_{1,1}+\frac{t^2B_{1,1}^2B_{2,2}}{\Xi}\quad(B6)$$

$$=\frac{1}{B_{11}^{-1}-t^2B_{2,2}}.\quad(B7)$$

Since $B_{1,1}=1/[J_{0,1}-\Gamma_5(2)]$ and $B_{2,2}=1/[J_{0,3}-\Gamma_7(2)]$,

$$[\lambda^{x/y}(1)]_{1,1}^{-1}=J_{0,1}-\Gamma_5(2)-\frac{(\frac{1}{2}J_{2,1})^2}{J_{0,3}-\Gamma_7(2)}\quad(B8)$$

$$=J_{0,1}-\Gamma_3(2)-\Gamma_5(2),\quad(B9)$$

where the definition of $\Gamma_m(s)$ is used.

APPENDIX C: PROOF OF ISOTROPIC CONDUCTIVITY FOR A SCATTERING PROBABILITY WITH $S>1$

Here we prove that the conductivity is isotropic for a scattering probability whose Fourier terms Q_m are nonzero only for $m=ls$ where l is integer and $s>1$. We begin with a rather abstract algebra for a group G of certain kinds of matrices. Assume the members of G are not singular and divided into two types, say A and B, and define a function giving the type of a matrix such that if the matrix M is type A, $[M]=A$ and the matrix M is type B, $[M]=B$. The following properties are required. (1) For two matrices M_1 and M_2, the sum: $[M_1+M_2]=[M_1]+[M_2]$. If the matrix M is the sum of type A and type B matrices, $[M]=A+B$, however, this M is not a member of G. (2) For matrices with $[M_A]=A$ and $[M_B]=B$, the products have the following properties, $[M_AM_A]=A$, $[M_BM_B]=A$, and $[M_AM_B]=[M_BM_A]=B$. (3) The unit matrix E is of type A, hence, we have $[M_A^{-1}]=A$ and $[M_B^{-1}]=B$.

Now, the problem that we want to postulate involves the type of the inverse of the matrix M_A+M_B, if the inverse exists. We use the Woodbury formula, $(M_A+M_B)^{-1}=M_A^{-1}-M_A^{-1}(E+M_BM_A^{-1})^{-1}M_BM_A^{-1}$. By setting $C=M_BM_A^{-1}$, $[C]=B$ and $[(E+C)^{-1}]=[E-C+C^2-C^3+C^4-\cdots]=[E+C^2+C^4+\cdots]+[-C-C^3-\cdots]=A+B$. Finally, using $[M_A^{-1}]=A$ and $[M_A^{-1}(1+C)^{-1}C]=A+B$, we have $[(M_A+M_B)^{-1}]=A+B$. Therefore, the inverse of a matrix of type $A+B$ is of type $A+B$.

Let us apply the above formulas to a system with a scattering probability with four symmetry axes, where only terms such as $J_{n,m}$ with an even n exist. The general form of the matrix $K^{x/y}$ is

$$\begin{pmatrix} J_{0,1} & \pm\frac{1}{2}J_{2,1} & \frac{1}{2}J_{2,3} & \pm\frac{1}{2}J_{4,3} & \frac{1}{2}J_{4,5} & \cdots \\ \pm\frac{1}{2}J_{2,1} & J_{0,3} & \pm\frac{1}{2}J_{4,1} & \frac{1}{2}J_{2,5} & \pm\frac{1}{2}J_{6,3} & \cdots \\ \frac{1}{2}J_{2,3} & \pm\frac{1}{2}J_{4,1} & J_{0,5} & \pm\frac{1}{2}J_{6,1} & \frac{1}{2}J_{2,7} & \cdots \\ \pm\frac{1}{2}J_{4,3} & \frac{1}{2}J_{2,5} & \pm\frac{1}{2}J_{6,1} & J_{0,7} & \pm\frac{1}{2}J_{8,1} & \cdots \\ \frac{1}{2}J_{4,5} & \pm\frac{1}{2}J_{6,3} & \frac{1}{2}J_{2,7} & \pm\frac{1}{2}J_{8,1} & J_{0,9} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix},$$
$$(C1)$$

which can be expressed as the sum of M_A+M_B. Type A of matrix M_A is where M_A has a zero matrix element $M_{Ai,j}$ with $i+j$ odd. And type B of matrix M_B is where M_B has a

zero matrix element $M_{Bi,j}$ with $i+j$ even. The key point of this proof is that only the nonzero matrix elements of M_B change sign depending on the direction x or y. Since the products of these matrices have property 2 (above) and assuming the existence of the inverse, we have $[(K^{x/y})^{-1}]=A+B$. Therefore, the relaxation time given by $(K^{x/y})_{1,1}$ is

isotropic, since it is free from the element of matrix type B, whose sign depends on the directions x and y.

A similar argument can be applied to a system with a scattering probability with six symmetry axes, where only terms such as $J_{3n,m}$ with integer n exist. The general form of matrix $K^{x/y}$ is

$$
\begin{pmatrix}
J_{0,1} & 0 & \mp\frac{1}{2}J_{3,2} & -\frac{1}{2}J_{3,4} & 0 & \pm\frac{1}{2}J_{6,5} & \cdots \\
0 & J_{0,3}\mp\frac{1}{2}J_{3,0} & 0 & 0 & -\frac{1}{2}J_{3,6}\pm\frac{1}{2}J_{6,3} & 0 & \cdots \\
\mp\frac{1}{2}J_{3,2} & 0 & J_{0,5} & \pm\frac{1}{2}J_{6,1} & 0 & -\frac{1}{2}J_{3,8} & \cdots \\
-\frac{1}{2}J_{3,4} & 0 & \pm\frac{1}{2}J_{6,1} & J_{0,7} & 0 & \mp\frac{1}{2}J_{9,2} & \cdots \\
0 & -\frac{1}{2}J_{3,6}\pm\frac{1}{2}J_{6,3} & 0 & 0 & J_{0,9}\mp\frac{1}{2}J_{9,0} & 0 & \cdots \\
\pm\frac{1}{2}J_{6,5} & 0 & -\frac{1}{2}J_{3,8} & \mp\frac{1}{2}J_{9,2} & 0 & J_{0,11} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots &
\end{pmatrix},
\tag{C2}
$$

which can be expressed as the sum of M_A+M_B where M_A has a nonzero matrix element $M_{Ai,j}$ with $j=i+3n$ with an integer n, and M_B has a nonzero matrix element $M_{Bi,j}$ with $i+j=3n+1$ and with a sign that changes depending on the direction x or y. Since the products of these matrices have property 2 and assuming the existence of the inverse, $[(K^{x/y})^{-1}]=A+B$. Hence the relaxation time given by $(K^{x/y})_{1,1}$, is again isotropic.

Now, we prove that for a scattering probability with a nonzero $Q_{ls}(l=0,1,2,\ldots,$ and $s>1)$ the conductivity is isotropic. We define M_A and M_B by the first and second terms of Eqs. (2.21) and (2.24), respectively. Since the (i,j) element of the matrix product $M_A M_A'=\Sigma_k M_{Ai,k}M_{Ak,j}'$ is non-

zero only for $i-k=ls$ and $k-j=l's$ and hence $i-j=(l+l')s$, the product is type A. The (i,j) element of the matrix product $M_B M_B'=\Sigma_k M_{B(i,k)}M_{B(k,j)}'$ is nonzero only for $i+k-1=ls$ and $k+j-1=l's$. Therefore, the product is type A since $i-j=(l-l')s$. In the same way, the (i,j) element of the product $M_A M_B=\Sigma_k M_{Ai,k}M_{Bk,j}'$ is nonzero only for $i-k=ls$ and $k+j-1=l's$ and hence the product is type B since $i+j-1=(l+l')s$, which also holds for $M_B M_A$. Now, property 2 is fulfilled. Unit matrix E belongs to type A. Therefore, the matrix $(K^{x/y})^{-1}$ belongs to type $A+B$. Since the $(1,1)$ element of a type B matrix is zero for a scattering probability with a number of symmetry axes $2s$ larger than 2, the conductivity is isotropic.

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