Effect of an electric field on the Bohm-Aharonov oscillations in the electronic spectrum of a quantum ring

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(Received 18 January 2002; revised manuscript received 6 March 2002; published 6 May 2002)

We have studied the effects of an external electric field on the electronic and optical spectra of a semiconductor quantum ring threaded by a magnetic flux. The calculations were performed within a single-particle formalism using the effective-mass approximation. An electric field applied in the ring plane destroys the rotational invariance, mixing the states with different angular momentum and suppressing the oscillations of the ground-state energy as a function of the magnetic field. However, for excited states whose electronic wave function are allowed to spread out along the ring, periodic Bohm-Aharonov-type oscillations are found. The range of energies for which these oscillations occur can be controlled by changing the electric-field strength. To explore the possibility that this effect can be observed in future optical measurements we have calculated the absorption coefficient for different configurations of magnetic and electric fields. Our results show very rich spectra for quantum rings in the presence of electric fields, which effectively displays the signature of the Bohm-Aharonov effect.

DOI: 10.1103/PhysRevB.65.193307 PACS number(s): 78.66.−w

Systems with ring-confining geometries have been the object of considerable attention during the last few years due to the possibility of experimental observation of the Aharonov-Bohm[1] effect in these structures. In mesoscopic rings threaded by a magnetic field,[2] a so-called "persistent current" has been measured, which is a periodic current in the magnetic flux with a period given by the elemental quantum flux. Recent advances in the fabrication of semiconductor nanostructures have made possible the realization of nanoscopic quantum rings in which the electronic states are in the true quantum-confinement limit.[3] By employing spectroscopy techniques Lorke *et al.* have investigated the ground-state and the excitations in self-assembled InGaAs quantum rings subjected to magnetic fields oriented perpendicular to the plane of the rings. They found that, when approximately one flux quantum threads the ring, a ground state transition from angular momentum $l=0$ to $l=-1$ take place.

The effects of the electron correlations and the presence of impurities in quantum rings subject to a perpendicular magnetic fields have been investigated.[7,5,6,4,8] Also theoretical studies on the optical properties, such as the influence of the different geometric-confinement parameters on the absorption spectrum[9] have been reported.

The purpose of this paper is to report the effects of an external in-plane electric field on the electronic and optical spectra of a semiconductor quantum ring threaded by a magnetic flux. We perform our calculations within a single-particle formalism using the effective-mass approximation. It is found that the electric field breaks the rotational symmetry by creating a pocketlike potential at one side of the ring where the electronic wave function becomes localized. The periodic Bohm-Aharonov-type oscillations are suppressed for the low-lying energy states but surprisingly they are exhibited for those higher excited states that have electronic wave functions spread out along the ring. The range of energies for which these oscillations occur can be controlled by changing the electric-field strength. To explore the possibility that this effect can be observed in future optical measurements we have calculated the absorption coefficient for different configurations of magnetic and electric fields. Our results show very rich spectra for quantum rings in the presence of electric fields, which effectively displays the signature of the Bohm-Aharonov effect.

The effective-mass Hamiltonian for an electron in a quantum-ring structure subjected to external magnetic field perpendicular to the ring axis and an electric field applied along one direction in the ring plane can be written as

$$H=\frac{p_z^2}{2m_z^*}+\frac{1}{2m^*}\left(\vec{p}_\rho+\frac{e}{c}\vec{A}_\rho\right)^2+V_{conf}(\vec{\rho},z)+e\vec{F}\cdot\vec{\rho}. \quad (1)$$

Here m^* and m_z^* represent the in-plane and perpendicular effective masses respectively, and $V_{conf}(\vec{\rho},z)$ is a confinement potential modeling the quantum ring. The potential vector has been taken in the symmetric gauge, then $\vec{A}_\rho=(B/2)(-y,x)$. We assume that the dependence in z of the eigenfunctions of the Hamiltonian (1) is described by the solutions of a quantum-well potential in that direction. In this way, the z motion can be integrated out by obtaining an effective ring-confinement potential depending only on the in-plane coordinates. For modeling the in-plane electronic ring confinement we use a potential generated by a rotation, around the ring axis, of a one-dimensional parabolic potential centered at a distance $\rho=\rho_0$ of the ring center, $\frac{1}{2}m^*\omega_g^2(\rho-\rho_0)^2$. We will refer to ρ_0 as the radius of the ring. ω_g is a characteristic frequency of the lateral geometric confinement, in terms of which we define the ring width as $R_W=\sqrt{\hbar/m^*\omega_g}$.

The electric field breaks the azimuthal symmetry and mixes the eigenfunctions with different angular momentum. In order to calculate the single-electron energy levels and wave functions in a quantum ring in the presence of both

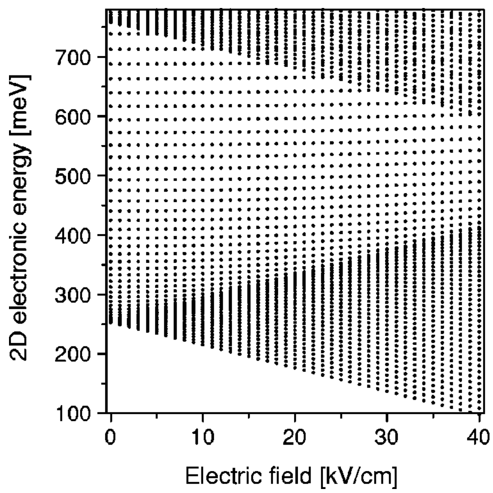

FIG. 1. In-plane energy spectrum as a function of the electric-field strength for a $GaAs/Al_{0.3}Ga_{0.7}As$ quantum ring of radius $\rho_0 = 400$ Å and width $R_W = 20$ Å. The magnetic field is zero.

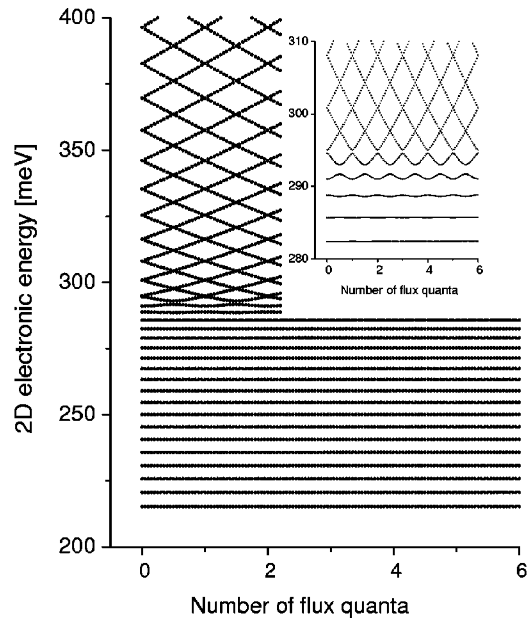

FIG. 2. The magnetic-field dependence of the electronic energy spectrum of a quantum ring for $F = 10$ kV/cm. The 2D electronic energies are plotted versus the number of flux quanta. The inset shows an expanded view on a shorter energy scale, in which the Aharonov-Bohm-type oscillations are clearly seen.

external fields we write the two-dimensional solutions as a linear combination of the eigenfunctions of the orbital angular-momentum operator, $\psi(\rho,\phi) = \Sigma_m C_m(\rho) e^{im\phi}$. The coefficients of this expansion are solutions of the coupled set of differential equations:

$$-\frac{\hbar^2}{2m^*}\left[\frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho} - \frac{m^2}{\rho^2}\right] C_m(\rho) + \left[\frac{1}{2}\hbar\omega_c m - E + \frac{1}{2}m^*\omega^{*2}(\rho-\rho_0^*)^2 + \frac{1}{2}m^*\omega^{*2}\left(\frac{\omega_c}{2\omega_g}\right)^2 \rho_0^{*2}\right] C_m(\rho) + \frac{eF\rho}{2}[C_{m+1}(\rho) + C_{m-1}(\rho)] = 0. \tag{2}$$

Here ω_c is the cyclotron frequency, $\omega^* = \sqrt{(\omega_c^2/4) + \omega_g^2}$ is an effective frequency, and $\rho_0^* = \rho_0(\omega_g/\omega^*)^2$ is an effective radius describing the lateral confinement potential.

To solve Eq. (2) we expand the coefficients $C_m(\rho)$ in terms of a basis of harmonic oscillator solutions:

$$C_m(\rho) = \sum_j N_{jm}\rho e^{-b^2(\rho-\rho_0^*)^2/2} H_j[b(\rho-\rho_0^*)]. \tag{3}$$

Here, $H_j(x)$ are the Hermite polynomials and $b = \sqrt{m^*\omega^*/\hbar}$ is the inverse of an effective ring width. The linear factor ρ has been included to assure a regular behavior of the solution at $\rho = 0$. The selection of this expansion was based in the case of large and narrow rings, $\rho_0 \gg R_W$, in the absence of any external field. In this limit, the bidimensional problem for the ring can be thought as a superposition of a one-dimensional harmonic potential and an infinite quantum-well potential with an effective width of $\mathcal{L} = 2\pi\rho_0$.

By replacing the expansion for $C_m(\rho)$ in Eq. (2), we derive an infinite coupled set of homogeneous equations for the coefficients N_{jm},

$$\sum_j N_{j,m}\left\{-6j\mathcal{S}(2,j',j-1) + (m^2-1)\mathcal{S}(1,j',j) - 3b\rho_0^*\mathcal{S}(2,j',j) + \left[4 + 2j - \left(\frac{2E}{\hbar\omega^*}\right) + \left(\frac{m\omega_c}{\omega^*}\right) + \left(\frac{\omega_c}{2\omega_g}\right)^2 (b\rho_0)^2\right]\mathcal{S}(3,j',j)\right\} + \frac{eF}{b\hbar\omega^*}\sum_j [N_{j,m-1} + N_{j,m+1}]\mathcal{S}(4,j',j) = 0, \tag{4}$$

where the integral $\mathcal{S}$ is defined as

$$\mathcal{S}(i,j',j) = \int_0^\infty \exp\{-[b(\rho-\rho_0)]^2\}\left(\frac{\rho}{\rho_0}\right)^i \times H_{j'}[b(\rho-\rho_0)] H_j[b(\rho-\rho_0)] d\rho.$$

To obtain accurate energies and wave functions from Eq. (4) we truncate the basis by choosing an adequate set of Hermite polynomials and eigenfunctions of the orbital angular-momentum operator. The set of angular-momentum quantum numbers m, is chosen starting from a particular value of m of the ground-state energy in the absence of an electric field. For the cases studied very good accuracies were reached with a set up to $j = 6$ and up to $m = 101$.

To show the main features of the effects of the external fields on the electronic and optical spectra of a quantum ring we have chosen a representative $GaAs/Al_{0.3}Ga_{0.7}As$ quantum ring of radius $\rho_0 = 400$ Å and width $R_W = 20$ Å corresponding to a radial confinement energy of $E_o = \hbar\omega_g/2 \approx 250$ meV.

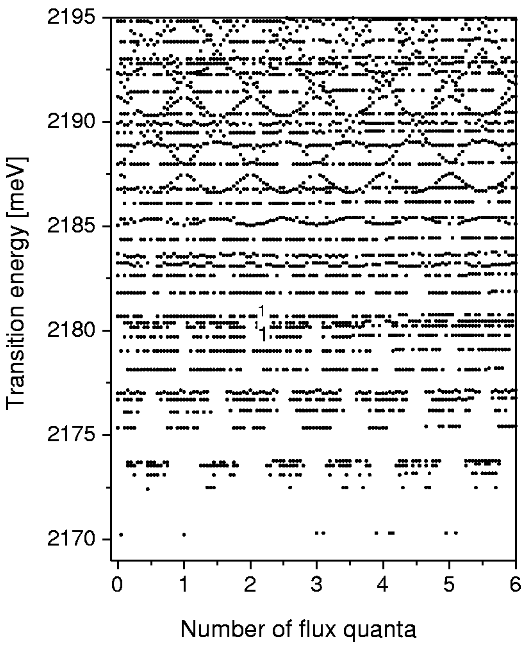

FIG. 3. Energies corresponding to the position of each resonance maximum in the absorption spectrum, as a function of the number of flux quanta.

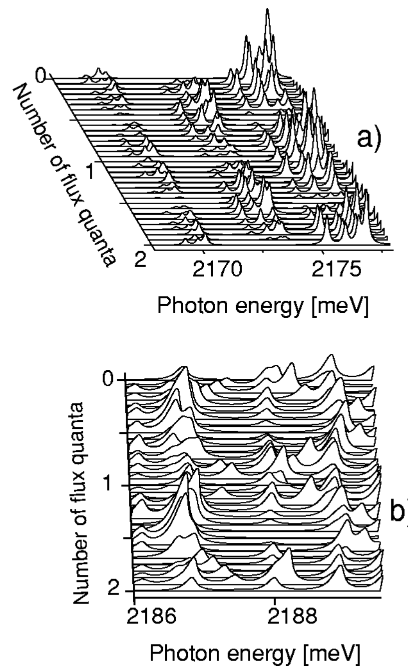

FIG. 4. Absorption coefficient as a function of the photon energy and as a function of the number of flux quanta. The figures show the region for photon energies (a) below and (b) above the optical gap in absence of the electric field. The amplitudes in (a) are amplified by a factor of 5.

Figure 1 shows the in-plane energy spectrum for a quantum ring, for zero magnetic field, as a function of the electric-field strength. When the electric field is zero one can observed the expected behavior for the energies of large and narrow rings. The ground state in this case corresponds to an angular momentum $m=0$, and its energy is roughly the characteristic energy of the radial confinement E_o. The low-lying excited states correspond to different values of the angular momentum with energies doubly degenerated in $\pm m$. A change in the spectrum is apparent when the second level associated with the radial confinement arises, and the levels become superposed.

As the electric field increases, the ground-state energy decreases with a slope determined by the ring radius. The same behavior is observed for that set of energies, starting roughly at $3E_o$, which is related to the first excited state of the radial confinement. We can also observe that for increasing electric fields the density of states decreases for energies nearby the ground state, while it strongly increases, for energies nearby $\Delta E=2eF\rho_0$ over the ground-state value, displaying a fringe with a positive slope. We think that this effect is a consequence of the bias induced by the electric field on the ring potential, the difference between the minimums of the potential along the field direction being approximately ΔE. Thus, the electron can be trapped in a pocketlike potential formed in the region of lowest energies. The density of states increases for higher energies where the pocket has larger angular extension because the electronic confinement is lower. Over the fringe, the density of states becomes nearly independent of the particular value of the electric field until the next set of energies associated to the lateral confinement is reached. This can be associated to the spread of the electronic wave function along the ring. The fundamental state of the electron is confined in the potential pocket, while for the excited state over the fringe region, the probability density is significant all along the ring.

The magnetic-field dependence of the electronic energy spectrum of the quantum ring in the presence of an electric field of $F=10$ kV/cm is displayed in Fig. 2. The two-dimensional (2D) electronic energies are plotted versus the number of flux quanta. This is defined as the ratio of the magnetic flux ϕ through a one-dimensional ring of radius ρ_0, and the fundamental flux quantum $\phi_o=hc/e$. One can see that the electric field destroys the well-known oscillations of the ground-state energy as a function of the magnetic field. The low-lying energy levels are almost independent of the magnetic field, up to an energy region where new periodic Aharonov-Bohm-type oscillations appear. These oscillations are clearly developed for that energy at which the electronic wave function is completely expanded along the ring. The inset shows in detail the region of energies where the oscillations arise.

To explore the possibility that these Aharonov-Bohm-type oscillations can be seen in optical measurements, we have calculated the absorption coefficient for dipole-allowed interband transitions as a function of the photon energy and for an electric field of $F=5$ kV/cm.

The electron-hole transitions are calculated by considering a confinement energy in the z direction of $E_z=121$ meV for electrons and $E_z=38$ meV for holes. These correspond to the ground-state energy of a 35 Å width quantum well, with barriers of 300 meV and 165 meV, respectively. For simplicity we have considered equal in-plane effective masses, $m^*=0.067m_o$, for electrons and holes. Figure 3 displays the photon energies corresponding to the position of the maximum of each resonance in the absorption spectrum, as a function of ϕ/ϕ_o. Let us note that the optical-absorption edge, which corresponds to an energy of $E_g^*=2185$ meV for zero electric field, is effectively shifted to lower energies in the presence of the field. However, the first significant resonances appear only 15 meV under E_g^*. This happens because in the ground state the electron and the hole are localized in opposite regions of the ring, along the electric-field direction. In fact, the oscillator strength corresponding to the lowest-energy transitions vanishes. The zero-field optical-absorption edge separates two different regimes related with the behavior of the energy transitions as a function of the magnetic field. It is readily seen that for energies lower than E_g^* the energy position of all resonances is almost independent of the magnetic field. For higher energies, however, transitions displaying the Aharonov-Bohm period are clearly visible in the plot.

To display the features of the amplitude of the resonances we show in Figs. 4(a) and 4(b) the absorption coefficient as a function of the number of flux quanta and as a function of the photon energy. In both figures the magnetic field ranges from $B=0$ to $B=2.47$ T, equivalent to two flux quanta. We have used a broadening parameter $\Gamma=0.1$ meV and the intensities in Fig. 4(a) have been amplified by a factor of 5 relative to those in Fig. 4(b). In Fig. 4(a), we show spectra for photon energies below E_g^*. We see there that the intensities of the resonances with positions independent of the magnetic field, show a peculiar dependence on the magnetic field, vanishing with the Aharonov-Bohm period. In Fig. 4(b) are shown transitions corresponding to energies above the optical gap E_g^*. There, stronger absorption peaks with positions oscillating with the magnetic field, correspond to transitions between the fundamental localized state and one of the states extended along the ring. Resonances of lower intensities but which depend on the magnetic field, are also seen in that photon energy region.

In summary, we have studied the effects of an external electric field on the Bohm-Aharonov oscillations in quantum rings. We have found that, in the absence of any impurity and neglecting excitonic effects the periodic Bohm-Aharonov-type oscillations are suppressed for the low-lying energy states but they are exhibited for higher excited states whose electronic wave functions spread out along the ring. The range of energies for which these oscillations occur can be controlled by changing the electric-field strength. To explore the possibility that this effect can be observed in future optical measurements we have calculated the absorption coefficient for different configurations of magnetic and electric fields. Our results show very rich spectra for quantum rings in the presence of electric fields, which effectively displays the signature of the Bohm-Aharonov effect.

This work was partially supported by Millenium Cientific Nucleus ICM P99-135-F, Catedra Presidencial F. Claro 1998, and by Fondecyt, Grant No. 1990271 and Grant No. 1010429.

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