Forward scattering probe of edge-state coupling in the quantum Hall regime

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(Received 26 March 2001; published 18 September 2001)

We report on magnetoconductance measurements of a submicrometer-wide constriction in a two-dimensional electron gas with a tunable central gate. For negative central gate biases, an edge channel loop forms around the central-gate potential hump and couples to reflected edge channels, opening a forward scattering pathway. From the conductance contribution of this pathway we deduce that the transmission probability for tunneling between adjacent edge channels of different Landau levels is 0.5, corresponding to the strong-coupling limit, for a coupling region of only a few hundred nanometers in length.

DOI: 10.1103/PhysRevB.64.153303 PACS number(s): 73.23.−b, 73.40.−c, 73.43.−f

Transport through edge states, which form at high magnetic fields in two-dimensional electron gas (2DEG) devices, is responsible for the quantum Hall effect,[1,2] as well as the complete suppression of Aharonov-Bohm oscillations in annular 2DEG structures with a magnetic flux through the ring.[3] This can be explained by the fact that the transmitted edge states are confined to a single edge of the Aharonov-Bohm ring and therefore there is no electron path which encloses the magnetic flux to cause Aharonov-Bohm oscillations. In small structures where the edge channels are brought into close proximity, resonant tunneling (''coupling'') between oppositely propagating edge channels can give rise to backscattering of the current-carrying electrons, resulting in the breakdown of the two-terminal quantized conductance, as well as emergence of Aharonov-Bohm oscillations in the quantum Hall regime.[4–6] Resonant tunneling between edge channels arises when edge-state wave functions at the Fermi level have nonvanishing overlap, which is aided by inhomogeneities of the electrostatic potential, such as those associated with accidental impurities, gate electrodes, or nanostructured device geometries.[7–9] A powerful method to study edge-state coupling effects involves devices with a tunable ''artificial impurity'' to control the scattering potential,[10–16] as well as devices in which transport proceeds through a tunable quantum dot.[14,17–19] In such devices, edge states typically form mesoscopic loops (magnetically bound states) around a potential modulation within a constriction in the 2DEG. In the presence of edge-state coupling, transport through the loop states may either lead to scattering from one edge to the opposite (and oppositely propagating) edge, which amounts to backscattering, or to source-to-drain scattering, i.e., forward scattering. Because the loop channels enclose magnetic flux, Aharonov-Bohm oscillations can result as the magnetic field is varied.

Whether edge-state coupling leads primarily to backscattering or forward scattering depends in large part on the sign of the potential perturbation. In the case of a potential depression (''quantum dot''), the loop channel has the same handedness as the edge channels along the outer sample edge, and near the sample edges necessarily has regions of close proximity with any transmitted edge channels, where inter-Landau level scattering may take place without requiring much momentum transfer. The net result is coupling between opposite edges via the loop, leading to backscattering.[18,19] Reflected edge channels, on the other hand, are separated from the loop channel around the depression by a region of flat potential, i.e., these channels are on opposite slopes of a potential barrier. This much greater distance (in combination with the fact that large momentum transfer is required as they propagate in opposite directions[20]) greatly suppresses tunneling from reflected edge states to the loop, making forward scattering insignificant. In the case of a potential hump (''quantum antidot'') the argument is reversed, i.e., close coupling of any reflected edge channels to the loop states (propagating in the same direction) gives rise to forward scattering,[10,12,13] while the transmitted edge states are separated from the loop states (propagating in the opposite direction) by a potential valley, suppressing backscattering.

The observation of Aharonov-Bohm oscillations in such ''loop-state'' devices implies some tunneling between adjacent edge state channels. However, the strength of this coupling has been subject to debate.[10–25] Here we report clear evidence from a forward-scattering experiment that in our device the tunneling probability between adjacent edge channels of different Landau levels is very large, such that electrons passing through a submicrometer-long coupling region emerge from this region with essentially equal probability in either channel.

Our experiment was performed on a device consisting of a tunable electrostatic potential scatterer inside a variable-width split-gate constriction in a 2DEG.[26] The 2DEG was confined to a GaAs/$Al_{0.3}Ga_{0.7}$As interface in a GaAs/AlGaAs heterostructure. The heterostructure consisted of a GaAs substrate with buffer layers, followed by a 1 μm thick GaAs layer, a 20 nm thick $Al_{0.3}Ga_{0.7}$As layer, an 8×10^{12} cm^{-2} Si δ-doped plane, a 13 nm thick $Al_{0.3}Ga_{0.7}$As layer, and a 7 nm thick GaAs cap layer. The 2DEG had a density of $n=4.0\times10^{11}$ cm^{-2} and a mobility of $\mu_e=3.5\times10^5$ cm^2/Vs at 4 K. The split gates were 400 nm apart and 600 nm long, and the tunable central gate was 130 nm in diameter. The split gates were biased negatively to create a constriction and to modulate the density of the 2DEG in the constriction. The central gate was also biased negatively, independently of the

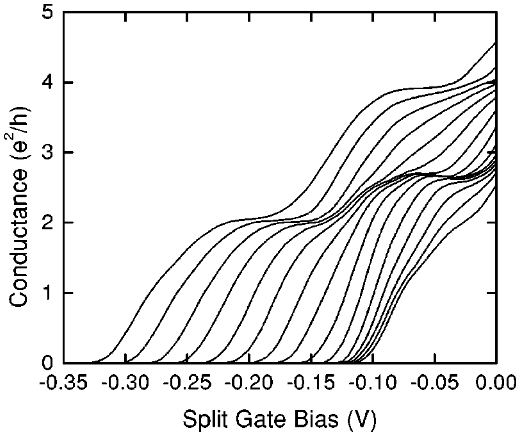

FIG. 1. Conductance vs split gate voltage at various central gate biases ranging from 0 V (leftmost curve) to -0.7 V (rightmost curve). The magnetic field was 3 T. A contact resistance of 500Ω has been subtracted. With the central gate voltage held at zero, two plateaus in the conductance were observed, near $2e^2/h$ and $4e^2/h$. As the bias of central gate was increased the behavior changed drastically. The two plateaus disappeared and a new plateau formed at $G\approx 2.7e^2/h$.

split gates, to create a tunable potential hump in the constriction. Two-terminal conductance measurements through the channel were performed at a temperature of 0.3 K with a magnetic field of 3 T perpendicular to the two-dimensional electron gas.

The conductance was measured as a function of split gate voltage, with various central gate bias voltages. By sweeping the split gate voltage, the Landau levels were depleted one by one. With the central gate voltage held at zero, two plateaus in the conductance were observed, near the quantized values of $2e^2/h$ and $4e^2/h$ (Fig. 1).[27] This behavior changed drastically as the negative bias of the central gate was increased. The two plateaus disappeared and a new plateau formed at $G\approx 2.7e^2/h$ (Fig. 1), starting at a central gate bias of -0.2 V.

This remarkable result is explained as follows. As the central gate is biased negatively, a potential maximum forms in the middle of the constriction.[28] The new step at $G \approx 2.7e^2/h$ is attributed to the formation of an edge channel of the first Landau level which loops around the central gate potential peak (Fig. 2). As explained in the introduction, this geometry promotes coupling between the loop state channel and the reflected edge state channels of the second Landau level, due to their proximity, and since the propagation direction is the same (Figs. 2 and 3). In this way, the second Landau level can contribute to the conductance by forward scattering through the loop channel.[12,13] Since both conduction pathways (via transmitted edge states, and forward scattering through the loop) involve conduction through states of the first Landau level, the conductance goes to zero in a single step as the first Landau level is pinched off with increasing split gate bias (Fig. 3). This explains the absence of a plateau at $2e^2/h$ at large central gate biases.

Assuming no loss of phase coherence, the total transmission probability T for an electron in the second Landau level on one side of the potential barrier to scatter into the corresponding state on the other side of the barrier is given by $T=|t_{\mathrm{tot}}|^2$, where t_{tot} is the total transmission amplitude[5]

$$t_{\mathrm{tot}}=te^{i\phi/2}t(1+e^{i\phi}r^2+e^{i2\phi}r^4+\cdots). \qquad (1)$$

Here, t is the hopping amplitude for an electron in the second Landau level to scatter to the loop state of the first Landau level and vice versa, r is the transmission amplitude of an electron in the loop state to remain in the loop state when emerging from the coupling region (with $|t|^2+|r|^2=1$), and $e^{i\phi}$ is the phase winding factor of the electron corresponding

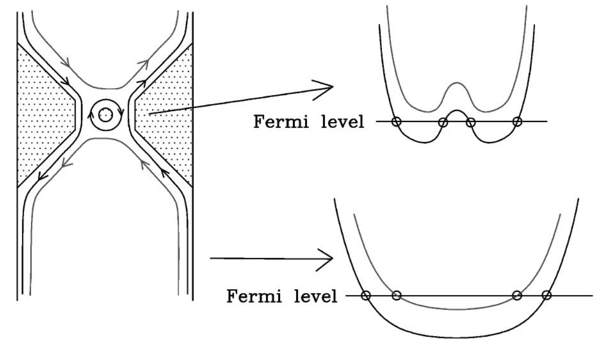

FIG. 2. Schematic diagram of the loop state around the potential hump. The dark and shaded lines represent the edge states of the first and second Landau levels, respectively.

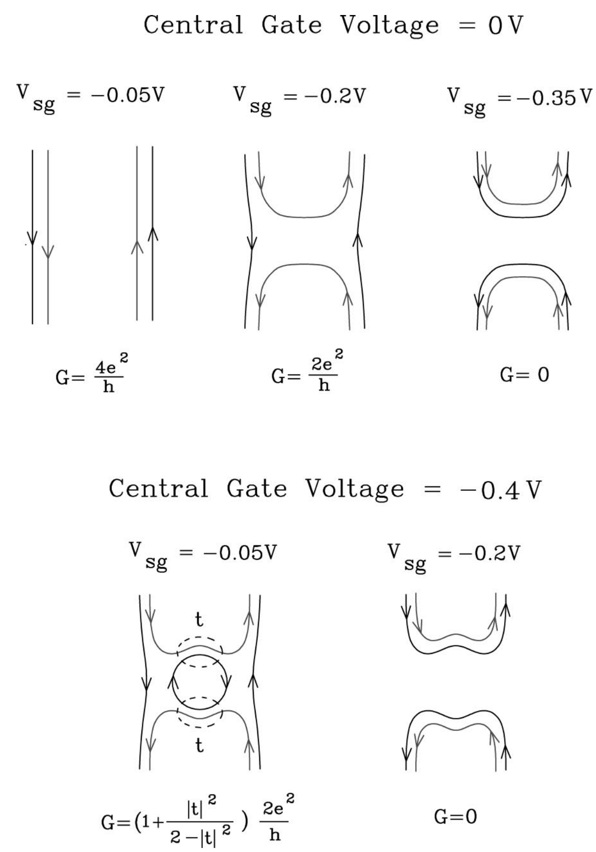

FIG. 3. Schematic diagram of the conducting edge-state channels at the indicated split gate (V_{sg}) and central gate biases. The dark and shaded lines represent the edge states of the first and second Landau levels, respectively.

to one cycle around the loop. The different terms in the series in Eq. (1) are the contributions to the forward scattering amplitude corresponding to paths involving 0.5, 1.5, 2.5, or more trips around the loop. For each round trip the amplitude picks up an Aharonov-Bohm phase factor of $e^{i\phi}$. The value of the round-trip phase ϕ depends sensitively on the energy of the incident electron, as well as on the magnetic field. Whenever the incident electron energy is degenerate with one of the single-particle loop states in the first Landau level, ϕ becomes equal to an integer multiple of 2π, resulting in a transmission resonance ($|t_{\mathrm{tot}}|^2=1$).[29] As the flux through the loop is varied, Eq. (1) predicts Aharonov-Bohm oscillations from interference between paths involving different numbers of trips around the loop, in perfect analogy with the transmission of a resonant-tunneling diode or a Fabry-Perot interferometer.[30]

In order to average out these Aharonov-Bohm oscillations, which were not the subject of the present study, we chose the temperature (0.3 K) and the excitation voltage (0.1 mV) large enough to suppress the coherence length to a value shorter than the loop circumference. (It was noted in Ref. 17 that Aharonov-Bohm oscillations disappeared when the temperature was raised to 0.2 K or the excitation voltage exceeded 40 μV.) Lacking phase coherence, the total transmission probability T is thus calculated by averaging the absolute square of Eq. (1) over the phase ϕ,

$$T=|t|^4(1+|r|^4+|r|^8+\cdots)=\frac{|t|^2}{2-|t|^2}. \tag{2}$$

Including the quantized conductance of the transmitted channel of the first Landau level, the above model predicts a conductance of

$$G=\left(1+\frac{|t|^2}{2-|t|^2}\right)\frac{2e^2}{h}, \tag{3}$$

where $|t|^2$ is the probability of an electron tunneling from the second Landau level edge channel into the first Landau level loop channel and vice versa. As the negative split-gate bias is increased, the reflected edge states of the second Landau level move away from the central gate region. However, since the radius of the loop channel grows by the same amount, $|t|^2$ remains approximately constant, resulting in an apparent conductance plateau.

From Eq. (3) and the measured conductance at the plateau of $G\approx 2.7e^2/h$, we conclude that $|t|^2\approx 0.5$. This is interpreted as a situation in which the electron makes many transitions back and forth between the two coupled channels, and, having lost its "memory" of which channel it originated from, emerges from the coupling region with equal probability in either channel. The value $|t|^2=0.5$ thus corresponds to the strong-coupling limit, in which increasing the coupling will not increase the transmission probability any further. In other words, the current-carrying edge states equilibrate remarkably efficiently in a very short coupling region ($\sim$ a few hundred nanometers in length).

To understand this result, we consider the expected resonant tunneling rate between edge states. For rapidly decaying wave functions separated by a distance Δy, the tunneling matrix element is expected to decay with distance at approximately the same rate as the wave function overlap, which for the harmonic-oscillator wave functions of the edge states is dominated by the exponential factor

$$\exp\left[-\left(\frac{\Delta y}{2l_{\mathrm{B}}}\right)^2\right], \tag{4}$$

where $l_{\mathrm{B}}=\sqrt{\hbar/eB}$ is the magnetic length, defining the length scale of the width of the edge-state wave functions. Because of this dependence on distance, it has been suggested that the tunneling is "exponentially" small in the absence of potential irregularities that increase the overlap.[31] However, this assumes Δy to be much larger than l_{B}, which is not the case in our experiment. For an electrostatic potential gradient $\nabla\phi$ the spatial separation between the edge states is given by $\Delta y=\hbar\omega_{\mathrm{c}}/e\nabla\phi$, where ω_{c} is the cyclotron frequency. At the experimental magnetic field of 3 T, the magnetic length is $l_{\mathrm{B}}\approx 15$ nm, and the Landau level splitting is $\hbar\omega_{\mathrm{c}}\approx 5.2$ meV (using an effective electron mass of $0.067m_{\mathrm{e}}$). From numerical simulations of the multilevel structure, we estimate the potential gradient of the slopes of the central-gate potential hump to be of the order of 10^{-1} mV/nm in the plane of the 2DEG,[26,32] implying an edge-state separation of the order of 50 nm. This is comparable to the quantity $2l_{\mathrm{B}}\approx 30$ nm appearing in Eq. (4), i.e., the overlap and tunneling matrix elements are not expected to be drastically suppressed by the decay of the wave functions. Efficient edge-state coupling in our experiment can thus be explained without invoking potential irregularities from defects and impurities.

In conclusion, strong coupling between Landau levels of different indices has been observed in a forward scattering mechanism through a loop state channel around a central gate. The coupling strength between edge states of adjacent Landau levels could be determined, showing that complete inter-Landau-level mixing occurs within a length scale of a few hundred nanometers. The efficiency of coupling can be understood in the framework of resonant tunneling due to wave function overlap.

This work was supported by the National Science Foundation through the Materials Science Center under Grant No. DMR- 9121654 and the Cornell Nanofabrication Facility.

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[27] A contact resistance of 500Ω has been subtracted from the data.

[28] According to numerical simulations on the multilevel structure, a central gate bias of -0.2 V corresponds to a potential hump of $\approx 7-10$ meV in the two-dimensional electron gas (Ref. 26), which is on the order of the energy spacing between Landau levels at 3 T.

[29] If the scattering electron is degenerate with one of the loop states, the phase ϕ in Eq. (1) is 2π times the number of magnetic flux quanta passing through the state into which the loop state would adiabatically evolve if the electrostatic potential was slowly turned off.

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