Nonmagnetic Impurity Scattering in a $d_{x^2-y^2}$ Superconductor near a Van Hove Point: Zn versus Ni in the Cuprates

R. Fehrenbacher

Max-Planck-Institut für Festköperforschung, Heisenbergstrasse 1, D-70569 Stuttgart, Germany (Received 29 December 1995)

We consider the effect of nonmagnetic impurities in a $d_{x^2-y^2}$ superconductor with ε_F close to a Van Hove singularity. It is shown that the nontrivial density of states (DOS) allows for resonant scattering already at intermediate potential strengths $|u| \approx 1-2$ eV. The residual DOS at ε_F and the T_c suppression rate are found to depend strongly on the carrier concentration. Quantitative agreement with experiments on Zn and Ni doped cuprates is obtained by adjusting a single parameter, *u*. [S0031-9007(96)-00916-7]

PACS numbers: 74.62.Dh, 74.20.Mn, 74.72.-h

The effect of nonmagnetic impurities on the properties of a superconductor (SC) can provide useful information about the symmetry of its order parameter (OP). Anderson's theorem [1] states that nonmagnetic impurities affect neither the transition temperature T_c nor the density of states (DOS) $N(\omega)$ of a BCS SC with an *isotropic* OP. However, if the OP is anisotropic, possibly exhibiting nodes (such as, e.g., a $d_{x^2-y^2}$ OP), nonmagnetic impurities may lead to a large residual DOS $N_{\text{res}} \equiv N(\omega = 0)$ at the Fermi energy ε_F , and even to a complete suppression of T_c [2].

In the high- T_c copper oxides, Zn impurities are particularly harmful. For YBa₂(Cu_{1-x}Zn_x)₃O_{7-y}, for instance, where Zn substitutes primarily on *planar* Cu sites [3], a rapid suppression of T_c ($T_c = 0$ at $x \approx 0.08-0.1$), as well as a $N_{\rm res}$ growing as $x^{1/2}$ observed [4]. On the other hand, nominally magnetic Ni⁺² impurities (also believed to primarily substitute for planar Cu) have a much milder effect: While suppressing T_c at a rate 1/2-1/3 of Zn, they do not lead to a significant $N_{\rm res}$ up to $x \approx 0.05$ [4].

An appealing and simple interpretation of these results is possible in terms of a $d_{x^2-y^2}$ OP affected by nonmagnetic scattering, assuming that Zn acts as a strong, in fact, resonant scatterer [5]. However, one problem with this model comes from the fact that the growth in the normal state resistivity $d\rho_n/dx$ just above T_c is quite similar for Ni and Zn [6]. Given that, their different behavior in the SC state is puzzling, since, in standard models (for magnetic *and* nonmagnetic impurities) [7,8], the T_c suppression rate dT_c/dx and the residual resistivity are both determined by the same parameter, the normal state scattering rate. Also, it is not clear why Zn should scatter resonantly, since this is obtained only for scattering strength $|u| \rightarrow \infty$.

Reports about localized magnetic moments on Cu sites next to a Zn [9] further complicate the issue. The size of the moment depends on hole doping, and seems to be large in underdoped cuprates [$\approx (0.9-1.4)\mu_B$] [9,10], whereas in optimally doped YBa₂Cu₃O_{7- γ} (Y-O₇) it is small [9] or maybe absent [11]. Therefore, at least in Y-O₇, it seems justified to model Zn as a pure

potential scatterer. In underdoped samples, additional magnetic pair breaking might occur. Note, however, that even then it is not obvious that Abrikosov-Gorkov (AG) theory [7] applies due to (i) the existence of short-range antiferromagnetic (AF) correlations, and (ii) the fact that the "impurity" spins are probably not totally immobile, but part of the same SC spin fluid (Cu spin 1/2). For a strongly correlated system such as the t-J model, for instance, it was shown [12] that a localized spin 1/2impurity has only a small influence on magnetic and pairing properties when its coupling to the mobile spins J_i is the same or similar to the coupling J among the latter. This should be satisfied here. There is also the possibility that Zn might act on the pair potential itself as proposed recently [13] in the context of an AF spin fluctuation model. However, the presence of localized Cu moments nearest neighbor to Zn suggests that AF correlations are probably unaffected, or maybe even enhanced in the vicinity of the impurity site, since this region corresponds to a locally underdoped phase, i.e., is closer to AF.

As one possible route to resolve the discrepancies between theory and experiment described above, we explore the influence of a realistic quasiparticle dispersion on the effect of nonmagnetic impurity scattering in a model BCS superconductor with an assumed $d_{x^2-y^2}$ OP chosen as $\Delta_{\mathbf{k}} = \Delta \eta_{\mathbf{k}}$, with $\eta_{\mathbf{k}} = (\cos k_x - \cos k_y)/2$. Our main findings are the following: (i) The existence of a Van Hove singularity (VHS) just below ε_F [suggested by angular-resolved photoemission (ARPES) [14,15]] leads to a strong violation of particle-hole symmetry, and allows for the occurrence of resonant scattering, at realistic potential strengths $|u| \approx 1-2$ eV. (ii) The sign of u is crucial. For hole concentrations corresponding to Y-O₇, we find that resonant scattering occurs only for negative u (in electron notation). A large negative u is expected for Zn impurities, since an inert d shell strongly repels holes, i.e., attracts electrons. This is also consistent with the observed localized Cu moments. Ni, however, should create a weaker attractive potential for holes, i.e., positive u, since a Ni^{+III} oxidation state is formed more easily

than Cu^{+1II}. (iii) The VHS also results in a *strong frequency* dependence of the normal state quasiparticle lifetime. This might explain why the T_c suppression rate and increase of residual resistivity are not simply related, as observed for Zn and Ni. (iv) Both N_{res} , and dT_c/dx are strongly doping dependent, the latter becoming much larger in the underdoped regime away from the VHS. (v) Overall, good *quantitative* agreement with experiments is obtained with only one adjustable parameter, *u*.

We model the impurities by a short-range potential $V(\mathbf{r}) = u\delta(\mathbf{r} - \mathbf{r}_i)$, and apply the self-consistent *t*-matrix approximation [8,16]. This approach has recently been shown to yield accurate results in the dilute impurity limit [17]. Working in particle-hole space, the *t* matrix \hat{T} satisfies the following Lippmann-Schwinger equation (quantities with a hat represent matrices):

$$\hat{T}(\omega) = u \bigg[\hat{\sigma}_3 + \hat{\sigma}_3 \hat{T}(\omega) \frac{1}{\Lambda} \sum_{\mathbf{k}} \hat{g}(\mathbf{k}, \omega) \bigg].$$
(1)

Here we introduced the propagator

$$\hat{g}(\mathbf{k},\boldsymbol{\omega}) = \frac{\tilde{\omega}\hat{\sigma}_0 + \tilde{\Delta}_{\mathbf{k}}\hat{\sigma}_1 + \tilde{\xi}_{\mathbf{k}}\hat{\sigma}_3}{\tilde{\omega}^2 - \tilde{\Delta}_{\mathbf{k}}^2 - \tilde{\xi}_{\mathbf{k}}^2}, \qquad (2)$$

where $\tilde{\omega} = \omega - \Sigma_0$, $\tilde{\Delta}_{\mathbf{k}} = \Delta_{\mathbf{k}} + \Sigma_1$, $\tilde{\xi}_{\mathbf{k}} = \xi_{\mathbf{k}} + \Sigma_3$, $\xi_{\mathbf{k}}$ the quasiparticle energy. The self-energy (such as all other matrices) is expanded in terms of the identity and Pauli matrices $\hat{\sigma}_0, \dots, \hat{\sigma}_3$ as $\hat{\Sigma} = \Sigma_j \hat{\sigma}_j$, and is given by $\hat{\Sigma} = n_i \hat{T}, n_i$ the impurity concentration, and Λ the volume. For the $d_{\chi^2 - \gamma^2}$ OP, the off-diagonal self-energy vanishes by symmetry, and from (1), the nonzero components are

$$\Sigma_0 = \frac{n_i G_0}{(1/u - G_3)^2 - G_0^2}, \quad \Sigma_3 = \frac{n_i (1/u - G_3)}{(1/u - G_3)^2 - G_0^2}.$$
(3)

where $G_i(\omega) = \Lambda^{-1} \sum_{\mathbf{k}} g_i(\mathbf{k}, \omega)$. Previous approaches [5,8] assumed particle-hole symmetry in the normal DOS, which has the consequence that $\Sigma_3 = 0$ is a self-consistent solution of (3). Here, we relax this condition and take into account the full structure of the *t* matrix. Unfortunately, this also means that all \mathbf{k} sums have to be done numerically when iterating Eqs. (3) for a self-consistent solution. A $d \times d$ \mathbf{k} -space grid with d = 4000 for calculations of spectral quantities, and d = 200 to calculate T_c , proved to be sufficient to obtain good convergence.

For the quasiparticle dispersion ξ_k , we use a tight-binding fit to ARPES data on Bi₂Sr₂CaCu₂O₈ (Bi-2212) with real space hopping matrix elements $[t_0, \ldots, t_5] = [0.879, -1, 0.28, -0.087, 0.094, 0.087]$, $(t_0$ on-site, t_1 nearest neighbor, t_2 next nearest neighbor hopping etc.) [18,19]. All energies are measured in units of $|t_1| = 0.149$ eV. The value of t_0 corresponds to a hole doping of $\delta = 0.17$. In the calculations, we consider doping levels of $\delta = 0.25, 0.18, 0.13$ ($t_0 = 1.0, 0.9, 0.8$) which are representative of Y-O₇ ($\varepsilon_{\text{VHS}} \approx -16$ meV

[15]), Bi-2212 ($\varepsilon_{\rm VHS} \approx -31 \text{ meV}$ [14]), and Y-O_{6.6} (no precise value of $\varepsilon_{\rm VHS}$ is published to date), respectively, $\varepsilon_{\rm VHS}$ the position of the VHS with respect to ε_F . The experiments indicate that a rigid band picture is an acceptable approximation in this small doping interval. In the calculations of spectral quantities, we set $\Delta = 0.2$ (30 meV). The *only* free parameter in the theory is then the potential strength *u*, since the impurity concentration n_i is also fixed by experiment. We shall search for values of *u* to consistently explain the experiments for Zn and Ni.

The effect of the nontrivial DOS is rather dramatic, in particular, if a VHS is close to ε_F , as in the present case: (i) The s-wave scattering phase shift δ_0 acquires a strong frequency dependence already in the *normal* state, which also reflects itself in the scattering rate, hence, resistivity. Furthermore, the dependence on u is highly nontrivial. (ii) In the self-energy Eqs. (3), the cotangent $c = \cot \delta_0$ (being frequency independent in case of a constant normal DOS) that is usually used to parametrize the scattering strength is replaced by the (now *frequency*) dependent) quantity $\tilde{c}(\omega) \equiv -1/u + G_3(\omega)$. This leads to a strong sensitivity of the superconducting DOS and T_c on both u and the chemical potential. Resonant scattering is usually observed for $|c| \ll 1$ (corresponding to $|u| \approx \infty$ for constant DOS). In the present case, this translates to the condition $\tilde{c}(\omega = 0) \ll 1$, allowing for resonant scattering even if $|u| \ll \infty$. We shall illustrate these points below.

Figure 1(a) shows $N_{\text{res}} \equiv N(\omega = 0) = -\pi^{-1} \times$ $\operatorname{Im}\{G_0(\omega) + G_3(\omega)\}|_{\omega=0}$ as a function of the potential strength u for fixed $n_i = 0.002$. A broad peak is observed, the center of which shifts from negative $(1/u \approx -0.05 \text{ for } \delta = 0.25)$ to positive $(1/u \approx 0.1 \text{ for } \delta = 0.13)$ values as the hole doping is lowered. The broadness of the peaks shows that the signature of resonant scattering, i.e., a large $N_{\rm res}$ for very small n_i , occurs in a rather extended interval on the 1/uaxis. Using $\delta = 0.25$ to model Y-O₇, we find that a value as small as 1/u = -0.1, i.e., $|u| \approx 1.5 \text{ eV} \approx W$, the bandwidth, is sufficient to explain the values reported for Zn [4]. This assignment immediately leads to the prediction that the Zn induced $N_{\rm res}$ should decrease significantly for underdoped samples, e.g., for $\delta = 0.13, N_{\text{res}}(n_i = 0.002) \approx 0$. In Fig. 1(b), we plot the n_i dependence of $N_{\rm res}$ for selected values of 1/uand $\delta = 0.25$. The expected relation $N_{\rm res} \sim n_i^{1/2}$ holds for 1/u = -0.1, whereas for 1/u > 0.6, N_{res} is negligibly small up to concentrations of $n_i = 0.05$. The latter behavior was reported for Ni doped Y-O7, hence, we assign a value of $1/u \approx 0.7$ to this case. Note that, if substitution occurs primarily on the planar Cu site, $n_i \approx 1.5x$ in YBa₂(Cu_{1-x} T_x)₃O_{7-y} (T = Zn, Ni). However, at larger x, substitution is also expected on the Cu(1) chain sites. Figure 1(c) confirms the point made earlier, that the occurrence of resonant scattering is now linked to the condition $|\tilde{c}(\omega = 0)| \ll 1$. For



FIG. 1. (a) The residual DOS N_{res} as a function of 1/u and different doping levels. (b) N_{res} as a function of impurity concentration n_i for various values of 1/u. (c) The real and imaginary parts of $\tilde{c}(\omega)$ for values of 1/u near resonance.

 $\delta = 0.25$, $|\tilde{c}(\omega = 0)| = 0$, i.e., resonance, occurs for $1/u \approx -0.0425$.

Figures 2(a)-2(d) show the frequency dependence of the DOS near ε_F . For very small n_i [Figs. 2(a) and 2(b)], we obtain the characteristic hump at ε_F , provided the resonance condition $|\tilde{c}(\omega = 0)| \ll 1$ is satisfied. On the other hand, for weaker scattering 1/u = 0.7 [Figs. 2(c) and 2(d)], $N(\omega)$ vanishes linearly at ε_F for $n_i \leq 0.02$; and, for larger n_i , a sublinear dependence is found with a small N_{res} , which grows as δ is reduced. From this, we predict that a small N_{res} could appear in *underdoped* Ni substituted cuprates for $n_i < 0.05$.

To calculate T_c , we solve the linearized gap equation in the presence of impurities

$$1 = \frac{V}{4\Lambda\beta_c} \sum_{n,\mathbf{k}} \frac{(\cos k_x - \cos k_y)^2}{\tilde{\omega}_{nc}^2 + \tilde{\xi}_{\mathbf{k}}^2}.$$
 (4)

The pair potential is chosen as $V_{\mathbf{k}\mathbf{k}'} = V \eta_{\mathbf{k}} \eta_{\mathbf{k}'}$ to generate a $d_{x^2-y^2}$ OP and $\beta_c = (k_B T_c)^{-1}$. Because of the nontrivial $\xi_{\mathbf{k}}$, the normal state ($\Delta = 0$) self-energy entering (4) becomes frequency dependent. Hence, T_c cannot be expressed in the standard AG form, and the Matsubara sum has to be evaluated numerically with a cutoff, typically $\omega_n \leq 50 ~(\approx 7.5 \text{ eV})$. For the pure case, this results in an accuracy better than 10^{-4} compared to the value without cutoff. The coupling constant was chosen as V = 2.2, which corresponds to a near neighbor attraction of $V/8 = 0.275 \approx 41 \text{ meV}$ (see [18]). Without impurities, this leads to $T_c = (111 \text{ K}, 92 \text{ K}, 65 \text{ K})$ for $\delta = (0.25, 0.18, 0.13)$.

Figures 3(a) and 3(b) show the n_i dependence of T_c at $\delta = 0.25, 0.18$ for the values of 1/u assigned to Zn and Ni. The shape of the curves is essentially identical to the AG form (see, for example, [20]). The T_c suppression rate is about 2 times as fast for 1/u = -0.1 (Zn) than for 1/u = 0.7 (Ni). Furthermore, it strongly increases



FIG. 2. The ω dependence of the DOS for small $n_i = 0.001$ near resonance (a),(b), and for larger n_i far from resonance (c),(d).

for lower δ , in qualitative agreement with experiments on YBa₂Cu₃O_{7-y} [21]. Figure 3(c) illustrates the substantial increase in dt_c/dn_i [$t_c = T_c/T_{c0}, T_{c0} = T_c(n_i = 0)$] upon lowering δ . In analogy to N_{res} [Fig. 1(a)], the peak shifts from negative to positive 1/u as δ is lowered. Figure 3(d) shows the frequency dependence of the self-energy Σ_0 on the imaginary axis for various values of 1/u [Re{ $\Sigma_0(T_c)$ } = 0 and Im{ $\Sigma_3(T_c)$ } = 0]. The large variation of dt_c/dn_i between 1/u = -0.1 and 0.7 originates in the big differences of Σ_0 particularly at intermediate ω_n .

Finally, we need to check whether the *u* values assigned to Zn and Ni can produce a quasiparticle damping Im Σ_n , which could explain the similarity in the residual resistivity caused by the two. The normal state dc conductivity is evaluated using the Kubo formula (vertex



FIG. 3. (a),(b) The T_c suppression for potentials near and far from resonance, for $\delta = 0.18, 0.25$. (c) 1/u dependence of the T_c suppression rate for $\delta = 0.13, 0.18, 0.25$. (d) Frequency dependence of Im Σ_0 on the imaginary axis at T_c for various 1/u.

corrections vanish for a contact impurity potential [22])

$$\sigma_{\rm imp}^{xx} = \frac{2\pi e^2}{\Lambda} \int d\omega \left(-\frac{\partial f}{\partial\omega}\right) \sum_{\mathbf{k}} \left[\frac{\partial \xi_{\mathbf{k}}}{\partial k_x} A(\mathbf{k},\omega)\right]^2,$$
(5)

where the spectral function is defined as

$$A(\mathbf{k}, \boldsymbol{\omega}) = -\frac{\operatorname{sgn} \boldsymbol{\omega}}{\pi} \frac{\operatorname{Im} \boldsymbol{\Sigma}_n}{(\boldsymbol{\omega} - \boldsymbol{\xi}_{\mathbf{k}} - \operatorname{Re} \boldsymbol{\Sigma}_n)^2 + (\operatorname{Im} \boldsymbol{\Sigma}_n)^2},$$
(6)

and $f(\omega)$ the Fermi function. At low T, $\partial f/\partial \omega$ converges to a delta function, and only the self-energy around ε_F is important. We assume that the 3D coupling in the cuprates is sufficient, so that localization effects can be neglected in the low density limit under consideration.

Figure 4(a) shows the ω dependence of Im Σ_n for $\delta = 0.25, n_i = 0.005$, and various values of u. The surprising observation is that, even though the overall shape of the curves is different, in the vicinity of ε_F , they are very similar. In particular, for 1/u = -0.1, 0.7the values at ε_F are essentially the same. The VHS creates some type of an inversion point for the frequency dependent scattering rate. This similarity reflects itself in the impurity induced dc resistivity which we plot as a function of 1/u for $n_i = 0.01, T \approx 100$ K, and $\delta = 0.25, 0.18$ in Fig. 4(b). The values between 1/u =-0.5, 0.7 vary by less than 15%, even though the T_c suppression within this range changes by a factor of 2 [see Fig. 3(c)]. The magnitude also agrees fairly well with reported values for Zn and Ni ($\approx 20-30 \ \mu\Omega \ cm/at\%$) [6]. This shows that the proximity to the VHS causing the strong ω dependence of Σ_n is capable to severely violate the conventional proportionality of residual resistivity and T_c suppression found for a constant DOS. The large resistivity caused by Ni together with its mild effect on T_c as compared to Zn might be explainable by this effect.

In conclusion, we have shown that a simple BCS model with a $d_{x^2-y^2}$ OP, supplemented by the experimental quasiparticle dispersion and nonmagnetic impurity scattering, provides a consistent description of the experimentally ob-



FIG. 4. (a) Frequency dependence of the normal state selfenergy Im Σ_n for various of 1/u. (b) Impurity induced resistivity at T = 100 K, as a function of 1/u for $\delta = 0.18, 0.25$.

served effects of Zn and Ni. The presence of the VHS slightly below ε_F is necessary to resolve the puzzle given by the violation of the proportionality between residual resistivity and T_c suppression for the two types of impurities.

The author would like to thank M. R. Norman, B. Farid, and R. Zeyher for useful comments, and acknowledges financial support by the NSF (DMR-91-20000) through the Science and Technology Center for Superconductivity. Part of this work was done at the Materials Science Division, Argonne National Laboratory.

- [1] P.W. Anderson, J. Phys. Chem. Solids 11, 26 (1959).
- [2] L. P. Gorkov, Pis'ma Zh. Eksp. Teor. Fiz. 40, 351 (1984)
 [Sov. Phys. JETP Lett. 40, 1155 (1985)]; K. Ueda and T. M. Rice, in *Theory of Heavy Fermions and Valence Fluctuations*, edited by T. Kasuya and T. Saso (Springer, Berlin, 1985), p. 267.
- [3] R. Villeneuve *et al.*, Physica (Amsterdam) 235C-240C, 1597 (1994).
- [4] K. Ishida et al., J. Phys. Soc. Jpn. 62, 2803 (1993).
- [5] T. Hotta, J. Phys. Soc. Jpn. 62, 274 (1993); P.J. Hirschfeld and N. Goldenfeld, Phys. Rev. B 48, 4219 (1993).
- [6] T. R. Chien, Z. Z. Wang, and N. P. Ong, Phys. Rev. Lett. 67, 2088 (1991); D. J. C. Walker, A. P. Mackenzie, and J. R. Cooper, Phys. Rev. B 51, 15653 (1995); J.-T. Kim *et al.*, Phys. Rev. B 49, 15970 (1994); J. T. Kim, J. Giapintzakis, and D. M. Ginsberg, Phys. Rev. B 53, 5922 (1996).
- [7] A. Abrikosov and L.P. Gorkov, Zh. Eksp. Teor. Fiz. 39, 1781 (1960) [Sov. Phys. JETP 10, 593 (1960)].
- [8] P.J. Hirschfeld, P. Wölfle, and D. Einzel, Phys. Rev. B 37, 83 (1988); H. Monien, K. Scharnberg, and D. Walker, Solid State Commun. 63, 263 (1987).
- [9] A. V. Mahajan et al., Phys. Rev. Lett. 72, 3100 (1994).
- [10] G. V. M. Williams, J. L. Tallon, and R. Meinhold, Phys. Rev. B 52, R7034 (1995).
- [11] Y. Kitaoka, K. Ishida, and K. Asayama, J. Phys. Soc. Jpn. 63, 2052 (1994).
- [12] D. Poilblanc, D. J. Scalapino, and W. Hanke, Phys. Rev. B 50, 13020 (1994); J. Riera *et al.*, Report No. cond-mat/9510028 (to be published).
- [13] P. Monthoux and D. Pines, Phys. Rev. B 49, 4261 (1994).
- [14] D.S. Dessau *et al.*, Phys. Rev. Lett. **71**, 2781 (1993);
 H. Ding *et al.*, Phys. Rev. Lett. **74**, 2784 (1995).
- [15] K. Gofron et al., J. Phys. Chem. Solids 54, 1193 (1993).
- [16] C. J. Pethick and D. Pines, Phys. Rev. Lett. 57, 118 (1986);
 S. Schmitt-Rink, K. Myake, and C. M. Varma, Phys. Rev. Lett. 57, 2575 (1986).
- [17] T. Xiang and J.M. Wheatley, Phys. Rev. B 51, 11721 (1995).
- [18] R. Fehrenbacher and M. R. Norman, Phys. Rev. Lett. 74, 3884 (1995).
- [19] M.R. Norman et al., Phys. Rev. B 52, 615 (1995).
- [20] R. Fehrenbacher and M.R. Norman, Phys. Rev. B 50, 3495 (1994).
- [21] P. Mendels *et al.*, Physica (Amsterdam) **235C-240C**, 1595 (1994).
- [22] R. Gerhardts and J. Hajdu, Z. Phys. 245, 126 (1971).