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A stringy correspondence principle in cosmology

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Abstract

We study a d-dimensional FRW universe, containing a perfect fluid with $p = w\rho$ and $\frac{1}{d-1} \leqslant w \leqslant 1$, and find a correspondence principle similar to that of Horowitz and Polchinski in the black hole case. This principle follows quite generally from thermodynamics and the conservation of energy–momentum tensor, and can be stated along similar lines as in the black hole case: "When the temperature T of the universe becomes of order string scale the universe state becomes a highly excited string state. At the transition, the entropies and energies of the universe and strings differ by factors of $\mathcal{O}(1)$". Such a matching is absent for $w \neq 1$ if the transition is assumed to be when the curvature or the horizon length is of order string scale.

1. Consider a Schwarzschild black hole. As it evaporates through Hawking radiation, its mass and radius r_{Sch} decrease and its temperature T_{bh} increases, all eventually reaching Planckian scales at which Einstein's equations are no longer valid.

In string theory, gravity arises as a low-energy mode and Einstein's equations are obtained as a low-energy approximation. Planck length l_{pl} is related to the string length l_s and the string coupling constant g_s, taken to be $\leqslant 1$. Given this origin of gravity, it is natural to expect that when $r_{\text{Sch}} \simeq l_s$ a stringy description will take over.

This idea was proposed by Susskind in [1] and later incorporated by Horowitz and Polchinski in their correspondence principle for black holes and strings [2]. This principle states that when $r_{\text{Sch}} \simeq l_s$ a black hole state becomes a string state. At this transition, masses and entropies of black hole and strings differ by numerical factors of $\mathcal{O}(1)$ and not by, say, g_s-dependent factors which can be $\gg 1$ or $\ll 1$. This is true for a variety of black holes [2].

In cosmology also, as time decreases in the past, the temperature generically increases eventually reaching Planckian scale. It is then natural to wonder whether a similar correspondence principle exists in cosmology. In the FRW universe we study here, we find that such a principle indeed exists and follows quite generally from thermodynamics and the conservation of energy–momentum tensor. It is applicable near big bang/crunch singularities and is likely to be applicable in more general cases also wherever the temperature is expected to reach string scale.

This correspondence principle in cosmology can be stated along similar lines as in the black hole case: When the temperature T of the universe becomes of the order of string scale the universe state becomes a string state containing highly excited strings. At the transition, the entropies and energies of the universe and strings differ by factors of $\mathcal{O}(1)$. These factors presumably depend on when the transition takes place and cannot be calculated within the present approach.

The criterion $T \simeq 1/l_s$ is equivalent to $S \simeq S_s$ where S is the entropy of the universe contained within its horizon and S_s is the entropy of the strings with the same amount of energy as in the universe within its horizon. It turns out that this is the correct criterion for the transition and not, for example, $L_H \simeq l_s$ or $\mathcal{R} \simeq 1/l_s^2$ where L_H is the physical size of the horizon and $\mathcal{R}$ is the curvature scale. If the later criteria are used then, at the transition, the entropies and energies differ by g_s-dependent factors. Also, the criterion $T \simeq 1/l_s$ or, equivalently, $S \simeq S_s$ is more general since, applied to black holes using suitably red shifted black hole temperatures, it yields the same results as in [2].

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In this Letter, we consider the evolution of a d-dimensional spatially flat homogeneous isotropic universe containing a perfect fluid with density ρ and pressure $p = w\rho$. We assume that $d \geqslant 4$ and, since high temperatures are involved, that $\frac{1}{d-1} \leqslant w \leqslant 1$. We also assume that the universe saturates the Fischler–Susskind holographic bound [3] at Planckian times in the beginning. This ensures that numerical coefficients in various expressions are all of $\mathcal{O}(1)$.

In Section 2 we show that for the above FRW universe, a transition to string state is likely to occur at $T \simeq 1/l_s$. In Section 3 we show that the main results follow quite generally from thermodynamics and the conservation of energy–momentum tensor. In Section 4 we explain this transition intuitively in a few different ways, similar to the black hole case. At the end of this section and in Section 5 we mention some issues that may be studied further.

2. Consider the evolution of a d-dimensional spatially flat homogeneous isotropic universe containing a perfect fluid with density ρ and pressure $p = w\rho$. We assume that $d \geqslant 4$ and, since high temperatures are involved, that $\frac{1}{d-1} \leqslant w \leqslant 1$. The relevant line element ds is given by

$$ds^2 = -dt^2 + a^2\left(dr^2 + r^2\, d\Omega_{d-2}^2\right), \tag{1}$$

where $a(t)$ is the scale factor and $d\Omega_{d-2}$ is the line element on an unit $(d-2)$-dimensional sphere. Einstein's equations are

$$(d-1)(d-2)h^2 = 2\kappa^2\rho,$$
$$\rho_t + (d-1)(1+w)h\rho = 0, \tag{2}$$

where the suffix t denotes time derivative. In Planck units defined by

$$\hbar = c = 1, \qquad 2\kappa^2 = l_{\rm pl}^{d-2} = t_{\rm pl}^{d-2}, \tag{3}$$

the solution to the above equations can be written as

$$a(t) = a_{\rm pl}\left(\frac{t}{t_{\rm pl}}\right)^{\alpha},$$
$$\rho(t) = \rho_{\rm pl}\left(\frac{t}{t_{\rm pl}}\right)^{-2} = \frac{\alpha^2(d-1)(d-2)}{l_{\rm pl}^{d-2}t^2}, \tag{4}$$

where $\alpha = \frac{2}{(d-1)(1+w)}$. Equivalently, $a(t)$ and $\rho(t)$ can be expressed in terms of $a(t_0)$ and $\rho(t_0)$ where t_0 is some conveniently chosen initial time. The relations between $(a(t_0), \rho(t_0))$ and $(a_{\rm pl}, \rho_{\rm pl})$ are easy to obtain.

During the cosmological evolution the comoving entropy density σ, given by

$$\sigma = \frac{(\rho + p)}{T}a^{d-1},$$

where T is the temperature, remains constant. We assume that this constant is such that, at $t = t_{\rm pl}$, it saturates the holographic bound [3]

$$\sigma V_{d-1} r_H^{d-1} \leqslant \frac{C\omega_{d-2}L_H^{d-2}}{l_{\rm pl}^{d-2}}, \tag{5}$$

where C is a constant of $\mathcal{O}(1)$, V_n is the volume of an unit n-dimensional ball, $\omega_{n-1} = nV_n = \frac{2\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2})}$ is the area of an unit $(n-1)$-dimensional sphere, and

$$r_H = \int_0^t \frac{dt}{a} = \frac{t_{\rm pl}}{(1-\alpha)a_{\rm pl}}\left(\frac{t}{t_{\rm pl}}\right)^{1-\alpha},$$
$$L_H = r_H a = \frac{t}{1-\alpha}$$

are, respectively, the coordinate and the physical size of the horizon. For more details on and a precise formulation of the holographic bound, see [3,4]. It then follows that

$$\sigma = C(d-1)(1-\alpha)\left(\frac{a_{\rm pl}}{l_{\rm pl}}\right)^{d-1}, \tag{6}$$

$$T = \frac{\alpha^2(d-2)(1+w)}{C(1-\alpha)l_{\rm pl}}\left(\frac{t}{t_{\rm pl}}\right)^{-\frac{2w}{1+w}}. \tag{7}$$

The entropy S and the energy E, contained within a region whose comoving coordinate is r and the physical size is $L = ra$, are

$$S = \sigma V_{d-1}r^{d-1}, \qquad E = \rho V_{d-1}L^{d-1}. \tag{8}$$

In the past the time decreases and the temperature increases and eventually reaches Planckian scale at which Einstein's equations are no longer valid. In string theory, gravity arises as a low energy mode and Einstein's equations are obtained as a low energy approximation. Planck length $l_{\rm pl}$ can be taken to be related to the string length l_s and the string coupling constant g_s, taken to be $\leqslant 1$, as follows:

$$l_{\rm pl}^{d-2} = g_s^2 l_s^{d-2} \leftrightarrow l_{\rm pl} = g_s^X l_s, \quad X = \frac{2}{d-2}. \tag{9}$$

In the past, as time decreases, the temperature will increase and first reach the string scale $\simeq 1/l_s$. At such a scale higher modes of strings will be excited copiously, because high energies are involved, and the universe state becomes a string state containing highly excited strings. The FRW description of the universe, given above, is then replaced by a stringy description.

That such a transition is likely can be seen from comparing the entropy S of the universe with the entropy S_s of the strings with the same amount of energy. The entropy S and the energy E of the universe depend on its size. It is natural to take this size to be given by the horizon with comoving coordinate r_H and the physical size $L_H = r_H a$ as given above. This is the maximum size within which causal contact is possible. In the following, we do so and consequently S and E will refer to the entropy and the energy of the universe contained within its horizon unless mentioned otherwise. From the equations given above, it then follows that

$$\rho = \frac{C_\rho}{l_{\rm pl}^{d-2}t^2}, \qquad T = \frac{C_T}{l_{\rm pl}}\left(\frac{t}{t_{\rm pl}}\right)^{-\frac{2w}{1+w}},$$
$$S = C_S\left(\frac{t}{t_{\rm pl}}\right)^{(d-1)(1-\alpha)}, \qquad E = \frac{C_E}{l_{\rm pl}}\left(\frac{t}{t_{\rm pl}}\right)^{d-3}, \tag{10}$$

where the constants C_ρ, C_T, C_S, and C_E are all of $\mathcal{O}(1)$ and are given by

$$C_\rho = \alpha^2(d-1)(d-2),$$
$$C_T = \frac{\alpha^2(d-2)(1+w)}{C(1-\alpha)},$$
$$C_E = C_\rho V_{d-1}(1-\alpha)^{-(d-1)},$$
$$C_S = C\omega_{d-2}(1-\alpha)^{-(d-2)}. \tag{11}$$

The above expressions for (ρ, T, S, E) can also be expressed in terms of g_s and l_s using $l_{\rm pl} = g^X l_s$, see Eq. (9).

The entropy S_s of highly excited strings having energy E is given by

$$S_s(E) = C_s l_s E, \tag{12}$$

where C_s is a numerical constant of $\mathcal{O}(1)$. The ratio of the entropy S of the universe to the entropy S_s of highly excited strings with the same energy can now be calculated and is given by

$$\frac{S}{S_s} = \frac{1+w}{C_s l_s T} = \frac{T_*}{T}, \quad T_* = \frac{1+w}{C_s l_s}, \tag{13}$$

where the temperature T_* is of the order of string scale. This shows that if the temperature of the universe $T > T_*$ then its entropy $S < S_s$ and, hence, the stringy phase is entropically favourable and the state becomes a string state. At lower temperatures $S > S_s$ and, hence, the FRW phase is entropically favourable and the state becomes a FRW state. At the transition, we have

$$T \simeq \frac{1}{l_s}, \qquad S \simeq \left(g_s^X l_s E\right)^{\frac{(d-1)(1-\alpha)}{d-3}} \simeq S_s \simeq l_s E \simeq \sqrt{N},$$

where $N \gg 1$ is the string excitation number and, here and in the following, $A \simeq B$ means that A and B are equal upto numerical factors of $\mathcal{O}(1)$. It also follows that the string coupling constant[1]

$$g_s \simeq N^{-\gamma} \ll 1, \quad \gamma = \frac{(d-2)w}{2(d-3+(d-1)w)} \tag{14}$$

since $\frac{1}{2(d-1)} \leqslant \gamma \leqslant \frac{1}{4}$ when w lies in the range $\frac{1}{d-1} \leqslant w \leqslant 1$ as assumed here. Since $g_s \ll 1$ it is consistent to use the weak coupling formulas for strings. Also, note that the transition temperature in Planck units $l_{\rm pl} T_* \simeq g_s^X \ll 1$ so that the universe is well away from Planckian regime.

This suggests the following correspondence between a FRW universe state and a highly excited string state: When the temperature is lower than string scale, the universe state evolves as in FRW cosmology. When the temperature becomes of the order of string scale the universe state becomes a string state containing highly excited strings. At the transition, the entropies and energies of the universe and strings differ by factors of $\mathcal{O}(1)$. These factors presumably depend on when the transition takes place and cannot be calculated within the present approach.

3. The main results above follow quite generally and are valid for spatially curved homogeneous isotropic universes also. We have, from thermodynamics, that $p_T = \frac{\rho+p}{T}$ [5] which, together with energy–momentum conservation equation $\rho_t + (d-1)h(\rho+p) = 0$, implies that the comoving entropy density $\sigma = (\frac{\rho+p}{T})a^{d-1} =$ constant. Let S and E be the entropy and the energy of the universe contained within a region whose comoving coordinate is r and the physical size is $L = ra$. Then, it follows that

$$S = \sigma V_{d-1} r^{d-1}, \qquad E = \rho V_{d-1} L^{d-1},$$
$$\frac{S}{E} = \frac{\rho+p}{\rho T}.$$

If one assumes that $p = w\rho$ then it follows that $S = \frac{(1+w)E}{T}$.

The ratio of the entropy S of the universe to the entropy S_s, given in Eq. (12), of the highly excited strings with the same energy is thus given by

$$\frac{S}{S_s} = \frac{1+w}{C_s l_s T} = \frac{T_*}{T}, \quad T_* = \frac{1+w}{C_s l_s}, \tag{15}$$

where the temperature T_* is of the order of string scale. This shows, as before and more generally, that if the temperature of the universe $T > T_*$ then its entropy $S < S_s$ and, hence, the stringy phase is entropically favourable and the state becomes a string state. At lower temperatures $S > S_s$ and, hence, the FRW phase is entropically favourable and the state becomes a FRW state. At the transition, we have

$$T \simeq \frac{1}{l_s}, \qquad S \simeq l_s E \simeq S_s \simeq \sqrt{N},$$

where $N \gg 1$ is the string excitation number.

In particular, the above results are valid near a big crunch singularity also which occurs in future in a closed universe and where the temperature is expected to reach string scale. One then infers that a FRW universe state becomes a string state near a big crunch singularity also when the temperature $T \simeq 1/l_s$.

For $p = w\rho$ it follows that $T = \frac{C_T}{l_{\rm pl}}(\frac{a}{a_{\rm pl}})^{-(d-1)w}$. The density ρ and the curvature scale $\mathcal{R}$[2] can now be expressed as

$$\rho = \frac{C_\rho}{l_{\rm pl}^d}\left(\frac{l_{\rm pl}T}{C_T}\right)^{\frac{1+w}{w}},$$
$$\mathcal{R} = \frac{C_R}{l_{\rm pl}^2}\left(\frac{l_{\rm pl}T}{C_T}\right)^{\frac{1+w}{w}} \simeq l_{\rm pl}^{d-2}\rho,$$

where the C's are numerical constants of $\mathcal{O}(1)$ as will follow from the holographic bound. The explicit time dependence of the various quantities can only be obtained by solving Einstein's equations. One can then express the entropy $S(T)$ within the horizon of the universe as a function of its temperature T, and thereby obtain the N-dependence of g_s at the transition using $S(1/l_s) \simeq S_s \simeq \sqrt{N}$.

[1] We used $X = \frac{2}{d-2}$ and the identities $\frac{(d-1)(1-\alpha)}{d-3} = \frac{d-3+(d-1)w}{(d-3)(1+w)} = 1 + \frac{2w}{(d-3)(1+w)}$.

[2] The curvature scale $\mathcal{R}$ is given, for example, by the Ricci scalar R_μ^μ (which vanishes if $T_{\mu\nu}$ is traceless) or $\sqrt{R^{\mu\nu}R_{\mu\nu}}$. Einstein's equations imply that $\mathcal{R} \simeq \kappa^2\rho$.

At the transition, $T \simeq \frac{1}{l_s}$ and we have

$$\rho_* \simeq \frac{1}{g_s^b l_s^d}, \qquad \mathcal{R}_* \simeq \frac{g_s^{b_R}}{l_s^2},$$

where $b = (d-1-\frac{1}{w})X$, $b_R = 2-b = (\frac{1-w}{w})X$, and $X = \frac{2}{d-2}$. For the range $\frac{1}{d-1} \leqslant w \leqslant 1$ for w assumed here, we have

$$\frac{1}{l_s^d} \leqslant \rho_* \leqslant \frac{1}{g_s^2 l_s^d}, \qquad \frac{g_s^2}{l_s^2} \leqslant \mathcal{R}_* \leqslant \frac{1}{l_s^2}.$$

For weak coupling, $g_s \ll 1$. Then, at the transition, the density is of order string scale or higher whereas the curvature is much smaller than the string scale for $w < 1$ and is of string scale only for $w = 1$. Still, however, the transition to highly excited string state is likely to occur at $T \simeq 1/l_s$ because: at the transition, the temperature is of order string scale; the energy $E \gg 1/l_s$, so higher stringy modes will be excited copiously; the stringy phase is entropically more favourable and the state becomes a string state.

It is unlikely, for $w < 1$, that 'curvature, or the physical horizon size, be of order string scale' is the correct criterion for a FRW universe state to become a string state. To see this, assume that the transition occurs when $\mathcal{R} \simeq 1/l_s^2$. Then $g_s^{(w-1)X} \simeq (l_s T)^{1+w}$ at the transition and the ratio $\frac{S}{S_s} \simeq \frac{1}{l_s T} \simeq g^{(\frac{1-w}{1+w})X}$ would be $\ll 1$ at weak coupling, and not of $\mathcal{O}(1)$ as above. For the spatially flat case, note that $\mathcal{R} \simeq 1/L_H^2$ and, hence, the two criteria $\mathcal{R} \simeq 1/l_s^2$ and $L_H \simeq l_s$ are equivalent to each other.

4. Horowitz and Polchinski have formulated a correspondence principle for black holes where there is a transition from a black hole state to a string state containing highly excited strings [2] which happens when the horizon radius of the black hole $r_{\rm bh} \simeq l_s$. This transition can be explained intuitively in a few different ways [1,2,6,7]. The transition from a FRW universe state to a string state can also be explained intuitively in similar ways. For example:

(i) At the transition one would like to equate the energy E of the universe within its horizon

$$E \simeq \frac{1}{l_{\rm pl}} (l_{\rm pl} T)^{-\frac{(d-3)(1+w)}{2w}}$$

to the string energy $E_s \simeq \sqrt{N}/l_s$. This cannot be true for all values of g_s since E_s is independent of g_s (for weak coupling) and E depends on g_s and l_s through the combination $l_{\rm pl} = g_s^X l_s$. Therefore, the equality may hold only at one value of g_s which we take to be that where $T \simeq 1/l_s$. Equating E and E_s at this value of T gives $g_s \simeq N^{-\gamma}$ with γ given in (14). It then follows that for this value of g_s, the entropy S of the universe within its horizon

$$S \simeq (l_{\rm pl} T)^{-\frac{(d-1)(1-\alpha)(1+w)}{2w}} \simeq \sqrt{N}$$

becomes of the order of string entropy S_s. It is easy to show that if the transition criterion had been $L_H \simeq l_s$ or $\mathcal{R} \simeq 1/l_s^2$ then S and S_s would differ by a g_s-dependent factor, which would not be of $\mathcal{O}(1)$ and would not match as above.

(ii) Or, consider following the state of the FRW universe as g_s decreases. This means that the entropy S of the universe within its horizon remains constant. Then at the transition, taken to occur when $T \simeq 1/l_s$, we have

$$g_s^X \simeq S^{-\frac{2w}{(d-1)(1-\alpha)(1+w)}}.$$

Using this value for g_s and the identity $\frac{d-3}{(d-1)(1-\alpha)} + \frac{2w}{(d-1)(1-\alpha)(1+w)} = 1$, it follows that the energy E of the universe within its horizon

$$E \simeq \frac{1}{l_{\rm pl}} S^{\frac{d-3}{(d-1)(1-\alpha)}} \simeq \frac{S}{l_s}$$

becomes of the order of string mass E_s. It is again easy to show that if the transition criterion had been $L_H \simeq l_s$ or $\mathcal{R} \simeq 1/l_s^2$ then E and E_s would differ by a g_s-dependent factor, which would not be of $\mathcal{O}(1)$ and would not match as above.

(iii) Or, consider the transition in Planck units with $l_{\rm pl}$ fixed. Consider the universe at a temperature T, for example, $\simeq 1$ GeV. Its relation to entropy S and energy E of the universe can then be read off from the expressions given in Section 2. Now, as g_s decreases with $l_{\rm pl}$ held fixed, l_s increases and the string temperature $\simeq 1/l_s$ decreases. For a Schwarzschild black hole, the string length eventually approaches $r_{\rm Sch} \simeq (l_{\rm pl}^{d-2} M_{\rm bh})^{\frac{1}{d-3}}$ after which a string state description must take over. Similarly, for the universe, the string temperature eventually approaches $T \simeq \frac{1}{l_{\rm pl}} (l_{\rm pl} E)^{-\frac{2w}{(d-3)(1+w)}} \simeq 1$ GeV, after which a string state description must take over.

In fact, the transitions from a black hole state or a FRW universe state to a corresponding string state can both be explained quite similarly if one makes the following identifications:

$$(M_{\rm bh}, S_{\rm bh}, r_{\rm bh}) \leftrightarrow (E, S, T),$$

$$r_{\rm bh} \simeq l_s \leftrightarrow T \simeq \frac{1}{l_s} \quad \text{or} \quad S(E) \simeq S_s(E),$$

$$r_{\rm bh} > l_s \leftrightarrow T < \frac{1}{l_s}.$$

The meaning of various symbols above are clear. The criterion for transition in the black hole case is $r_{\rm bh} \simeq l_s$; in the FRW case it is $T \simeq 1/l_s$ or, equivalently, $S(E) \simeq S_s(E)$. A black hole state description is valid for $r_{\rm bh} > l_s$ and similarly a FRW state description is valid for $T < 1/l_s$; a string state description takes over after the transition.

The criterion $T \simeq 1/l_s$ or, equivalently, $S(E) \simeq S_s(E)$ is more general since, applied to black holes using suitably red shifted black hole temperatures, it yields the same results as in [2]. This can be seen easily for Schwarzschild black holes and can be inferred for other cases studied in [2] because the suitably red shifted temperatures at the transitions all turn out to be $\simeq 1/l_s$, as shown in [2].

Note that in the FRW case, for weak coupling, the curvature scale at the transition is typically less than the string scale. One would therefore expect no higher derivative corrections to the low energy effective action of the string theory and, hence, to the FRW evolution dictated by it. But, at the transition, the temperature $T \simeq 1/l_s$ and stringy higher modes are excited copiously because the energy at the transition $E_* \simeq \frac{1}{l_s} g_s^{-\frac{1}{2\gamma}} \gg 1$ for weak coupling. This means that the original low energy effective action is not valid because the energy is not low any more and must include a huge number, of the order of $N \simeq l_s^2 E_*^2 \gg 1$, of fields corresponding to the higher excitations.

It is quite interesting to ask how the reverse transition proceeds, namely what happens when one starts from a highly excited string state at weak coupling and increases the coupling constant g_s. The string to black hole transition is studied in detail in [8], see also [9]. The picture that emerges is that as g_s increases the gravitational effects become more important and eventually the excited string state collapses to a black hole state.

But the present results suggest the possibility that increased effects of gravitation may instead make the excited string state expand into a FRW universe state. It is not clear to us what aspects of the excited strings will decide the course of evolution. We note that the density ρ_* at the FRW universe to string transition has a range $\frac{1}{l_s^d} \leqslant \rho_* \leqslant \frac{1}{g_s^2 l_s^d}$ for $\frac{1}{d-1} \leqslant w \leqslant 1$; whereas the density $\rho_{\mathrm{bh}*}$ at the black hole to string transition is $\rho_{\mathrm{bh}*} \simeq (1/(l_{\mathrm{pl}}^{d-2} r_{\mathrm{Sch}}^2))_{r_{\mathrm{Sch}} \simeq l_s} \simeq \frac{1}{g_s^2 l_s^d}$. This suggests that the density of the excited strings will perhaps play a crucial role in deciding whether, as g_s increases, the excited string state will eventually collapse to a black hole state or expand into a FRW universe state.

Also note that at the transition $g_s \simeq N^{-\frac{1}{4}}$ for $w = 1$ case (and also in the black hole case) whereas $g_s \simeq N^{-\frac{1}{2(d-1)}} > N^{-\frac{1}{4}}$ for $w = \frac{1}{d-1}$ case. This suggests that, as g_s is increased from 0_+, a highly excited string state will first expand into a $w = 1$ FRW universe (or collapse to a black hole). As g_s is increased further, it will expand into a $w = \frac{1}{d-1}$ FRW universe.

In this context, note that there are a lot of similarities between the $w = 1$ case and the black hole case. Banks and Fischler have studied them extensively [10] in connection with, among others, black holes and early universe cosmology. They have presented an interesting scenario where the universe starts off with $w = 1$ matter; then black holes are nucleated which percolate the space and eventually evaporate through Hawking radiation; the universe thus becomes dominated by radiation ($w = \frac{1}{d-1}$). This sequence of evolution is quite similar to that indicated above where one starts from a highly excited string state and increases the string coupling. Although the connections between these two situations are not clear, the ideas in [10] seem very relevant to understanding the string to black hole and/or FRW universe transition as g_s is increased.

5. We studied here the correspondence between a FRW universe state to a string state for the case of isotropic homogeneous universe containing a perfect fluid with density ρ and pressure $p = w\rho$. We assumed that $\frac{1}{d-1} \leqslant w \leqslant 1$ since high temperatures are involved. The results can also be seen to follow from general considerations involving thermodynamics and the conservation of energy–momentum tensor.

An important consequence of this correspondence between a FRW universe state to a string state is that the problem of big bang/crunch singularity is obviated. This is because such singularities are expected to occur at Planckian temperature T_{pl}, whereas the transition to a string state description takes place at a temperature which is $\ll T_{\mathrm{pl}}$ in the weak coupling limit.

Therefore, it is of interest to establish the generality of this correspondence. For example, one may study this correspondence for anisotropic cases, for an universe containing more than one perfect fluid, or containing matter whose equation of state $p(\rho)$ is more general than $p = w\rho$ with the constant w in the range $\frac{1}{d-1} \leqslant w \leqslant 1$ as assumed here.

Another interesting study is that of the transition from a string state to black hole/FRW universe state, mentioned in more detail in Section 4. Also, in the case of black holes, extremal and near extremal black holes of various kinds have provided an ideal setting to understand the black hole $\leftrightarrow$ string correspondence. Similar examples in the context of cosmology will help us better understand the FRW universe $\leftrightarrow$ string correspondence.

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