Polarization of interacting mesoscopic two-dimensional spintronic systems

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We investigate the effects of the spin-orbit interaction induced by the Rashba coupling on the collective plasma excitations in a two-dimensional spintronic system (2DSS). In addition, we calculate the polarization function of the spin-split 2DSS in the self-consistent-field approximation. With the use of the f-sum rule, we determine the oscillator strengths of the plasmon and electron-hole excitations arising from transitions within and between the spin-split subband branches. The large density of states near the bottom of the band gives rise to a high frequency mode with large oscillator strength.The screened Coulomb potential of a point charge by the density fluctuations within the two-dimensional plane is calculated and we show the role played by spin orbits.

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In recent years, both theoreticians and experimentalists have been investigating the effect of spin-orbit (SO) coupling due to the linear Rashba splitting[1] on the physical properties, both spectral and transport, of low-dimensional semiconductor structures such as the two-dimensional electron system (2DES).[2–12] The SO coupling can be reliably controlled in experiments on these systems and made relatively strong. It has been demonstrated that SO coupling may give rise to interesting behavior in both the noninteracting and interacting properties of low-dimensional semiconductor structures. The mechanism for the Rashba SO coupling is a unique feature of the reduced dimensionality. All the potential profiles that can be produced by various flexible means of band engineering have electrostatic origin and give rise to a local electric field. For some confining potentials, the average electric field within the quantum well is different from zero and electrons in the quantum well experience a finite electric field directed along the normal to the plane of the 2DES.

There have been several works reported in dealing with the Rashba effect on ballistic spin transport, spin-splitting of the energy bands, and the effect on the exchange and many-body effects.[13–25] Sun *et al.*[21] carried out a detailed calculation of the effect of Rashba SO coupling on the electron transport through quantum dots with either ferromagnetic or nonferromagnetic leads. There is also some related research on the properties of the induced electric field spincurrent in which, even though there is no charge current, there is the occurrence of spin polarization in the system.[26–28] Ullrich and Flatté[14] carried out a detailed calculation of the collective intersubband spin-density excitations in quantum wells using a density functional formalism. In the work of Xu,[17] the plasmon excitations were calculated theoretically for a two-dimensional (2D) electron gas in the presence of SO coupling induced by the Rashba effect. In both Refs. 14 and 17, it was shown that plasmon excitations can be achieved as intra- and interband SO transitions. In Ref. 17, the plasmons were calculated in the long wavelength limit, but the Landau damping due to particle-hole mode excitations was not discussed. A related paper has recently been published on 2D plasmons with SO interaction by Yuan *et al.*[29] Although these authors predicted two plasmon mode branches, we disagree with their method of calculation which depends on a 4×4 matrix formalism for the dielectric response function. Oscillator strengths of the plasmons and particle-hole modes are here reported, and we include such calculations which are relevant to their experimental detection. The f-sum rule which is an integral part of this calculation is also obtained. We also calculate the screening of an impurity by the charge density fluctuations arising from the SO coupling.

In this paper, we calculate the polarization function for a longitudinal electric field of a 2D electron gas where the SO coupling is induced by the Rashba effect which lifts the degeneracy of the energy spectrum and produces a linear term in the energy dispersion relation.[30] The Hamiltonian for free electrons in the 2DES is a sum of the kinetic energy and $H_{SO}=\Delta_R/\hbar(\vec{\sigma}\times\mathbf{p})_z$, where $\vec{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ is the vector of Pauli spin matrices. Here, the z component of the momentum $\mathbf{p}$ does not contribute to H_{SO} since in the stationary state there is no transfer of electrons across the interface of confinement. (For a detailed discussion of the Rashba SO coupling parameter Δ_R in an asymmetric quantum well, see, for example, the paper by Moroz and Barnes.[24]) A self-consistent calculation by Wissinger *et al.*[31] of the energy subbands in doped quantum wells has revealed a spin splitting which has been attributed to the asymmetry in the band profile. Since H_{SO} is independent of coordinates, the eigenfunction may be sought in the form of plane waves $\Psi_k(\mathbf{r})=\chi_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{r}}/\sqrt{A}$. Here, $\mathbf{r}$ and $\mathbf{k}$ are in-plane coordinate and wave vectors, A is a normalization area, the spinor χ_k satisfies the equation $H_k\chi_{\mathbf{k}}=\epsilon_k\chi_{\mathbf{k}}$ and the Hamiltonian has the following explicit representation in spin space:

$$H_k=\begin{pmatrix}\hbar^2k^2/2m^* & i\Delta_R k\exp[-i\phi(\mathbf{k})]\\ -i\Delta_R k\exp[i\phi(\mathbf{k})] & \hbar^2k^2/2m^*\end{pmatrix}. \quad (1)$$

Here, m^* is the electron effective mass, $\phi(\mathbf{k})$ is the polar angle of the wave vector $\mathbf{k}$. Upon diagonalizing the matrix in Eq. (1), we obtain the energy eigenvalues $\epsilon_{k\pm}=\hbar^2k^2/2m^*\pm\Delta_R k$ and the corresponding wave functions

$$\chi_{\mathbf{k}}^{+-} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm \exp[i\phi(\mathbf{k})] \end{pmatrix}. \tag{2}$$

The dependence of the spinor (2) on the angle $\phi(\mathbf{k})$ means that the two states, denoted by $+$ and $-$, are polarized along the directions $+(\mathbf{k}\times\hat{\mathbf{z}})$ and $-(\mathbf{k}\times\hat{\mathbf{z}})$, respectively. This lifts the spin degeneracy of the free electron Hamiltonian and the system has a "$+$" and a "$-$" branch. These results show that the effect of the Rashba SO coupling manifests itself through a mutual shift of the spin branches, resulting in an energy gap between the $+$ and $-$ spin branches.

For a total areal electron density n_{2D}, there will be n_+ spins and n_- spins with $n_{2D}=n_-+n_+$. At $T=0$ K, these are determined by

$$\frac{n_\lambda}{n_{2D}} - \frac{1}{2} + \lambda A_R\left[\left(\frac{n_\lambda}{n_{2D}}\right)^{1/2} + \left(1 - \frac{n_\lambda}{n_{2D}}\right)^{1/2}\right] = 0, \tag{3}$$

where $A_R=k_R/k_F$ with $k_R=m^*\Delta_R/\sqrt{2}\hbar^2$ and $k_F=(2\pi n_{2D})^{1/2}$. Clearly, both spin branches are equally occupied when $A_R=0$. When the Rashba parameter is increased for $0<A_R<1/2$, the occupation number of the $-$ spin branch increases while the $+$ spin branch is decreased and for $A_R=1/2$, Eq. (3) is satisfied by $n_-=n_{2D}$ and $n_+=0$. When Δ_R is sufficiently large to make $A_R>1/2$, which is obtained when the Fermi energy $E_F<0$ and $m^*\Delta_R/\hbar^2>2|E_F|$, then Eq. (3) no longer applies for obtaining n_λ. We have in this case, only the $-$ spin branch occupied by electrons. The density of states (DOS) for each subband $\rho_\lambda(E)=\Sigma_{\mathbf{k}}\delta(E-E_{k,\lambda})$ has been calculated and is given by $(\lambda=\pm)$

$$\rho_+(E) = \eta_+(E)\left(\frac{m^*}{2\pi\hbar^2}\right)\left\{1 - \sqrt{\frac{E_\Delta}{E+E_\Delta}}\right\}, \tag{4}$$

$$\rho_-(E) = \left(\frac{m^*}{2\pi\hbar^2}\right)\left\{\eta_+(E)\left(1 + \sqrt{\frac{E_\Delta}{E+E_\Delta}}\right) + 2\eta_+(-E) \times \eta_+(E+E_\Delta)\sqrt{\frac{E_\Delta}{E+E_\Delta}}\right\}, \tag{5}$$

where $\eta_+(E)$ is the unit step function and $E_\Delta=k_R\Delta_R/\sqrt{2}$ is a measure of the spin gap in the DOS. Equations (4) and (5) show that the total DOS $\rho_{2D}=\rho_+(E)+\rho_-(E)$ is $m^*/\pi\hbar^2$ for $E\geqslant 0$ which is equal to the DOS for a spin-degenerate 2D electron system. However, the DOS for $-E_\Delta<E<0$ is increased as E is reduced from zero and even becomes infinite at $E=-E_\Delta$. For each subband, the DOS is not independent of energy and $\rho_-(E)>\rho_+(E)$ for all energies satisfying $-E_\Delta<E<\infty$. Figure 1(a) is a plot of the DOS for the spin-split branches and the total DOS for $E>-E_\Delta$. The nature of the DOS for the two spin branches leads to the results in the collective excitations which we describe below.

The linear screening of an external potential $\varphi_{\rm ext}(\mathbf{r})$ by the two-dimensional spintronic system (2DSS) embedded in a medium with background dielectric constant ϵ_b is given by

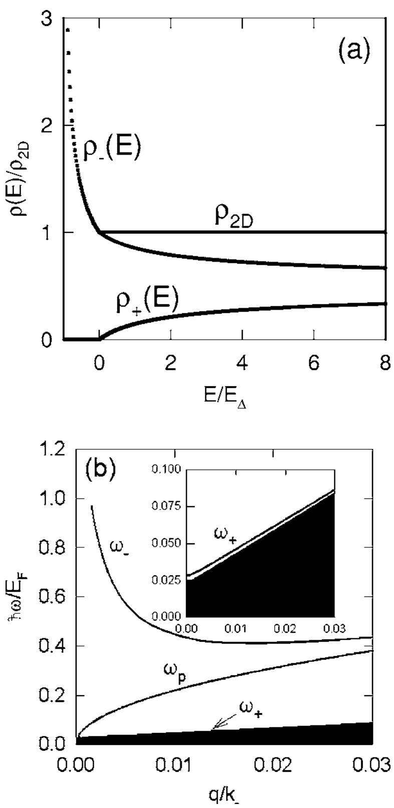

FIG. 1. (a) The density-of-states (DOS) for the $+$ spin $[\rho_+(E)]$ and $-$ spin $[\rho_-(E)]$ branches as well as the total DOS $\rho(E)=\rho_+(E)+\rho_-(E)$. In this notation, $\rho_{2D}=m^*/\pi\hbar^2$ is the DOS for a spin-degenerate 2D system. Also, $E_\Delta=m^*\Delta_R^2/2\hbar^2$. (b) The plasma dispersion for a 2D electron system with spin-orbit interaction. The spin-flip plasmon excitations have frequency ω_- and the intraband plasmons have frequency ω_+. The continuum of single-particle excitations is also shown. The inset shows more details for the particle-hole region and the intraband plasmon excitations. The parameters used in the calculations are $\epsilon_b=13$, $n_e=10^{11}$ cm^{-2} and $m^*=0.065m_e$, where m_e is the free electron mass. Also, $\Delta_R=10$ meV Å was chosen to exhibit the basic features that result from Rashba SO interaction.

$$\Phi_{\rm tot}(\mathbf{r},\omega) = \int_A d\mathbf{r}'\,\epsilon^{-1}(\mathbf{r},\mathbf{r}';\omega)\varphi_{\rm ext}(\mathbf{r}'), \tag{6}$$

where the inverse dielectric function is expressed in terms of the density-density response function through

$$\epsilon^{-1}(\mathbf{r},\mathbf{r}';t) = \delta(\mathbf{r}-\mathbf{r}')\delta(t) + \frac{1}{i\hbar}\int_A d\mathbf{r}''\frac{e^2}{\epsilon_s|\mathbf{r}-\mathbf{r}''|} \times \langle[n(\mathbf{r}'',t),n(\mathbf{r}',0)]_-\rangle. \tag{7}$$

Here, $\epsilon_s=4\pi\varepsilon_0\epsilon_b$. The Fourier transform of the dielectric function must satisfy the f-sum rule expressed as[32]

$$\int_{-\infty}^{\infty} d\omega\,\omega\,{\rm Im}\left(\frac{-1}{\epsilon(q,\omega)}\right) = \pi\Omega_0^2(q), \tag{8}$$

with

$$\Omega_0^2(q) = \omega_p^2 + \frac{2\pi e^2\Delta_R}{\epsilon_s \hbar^2 q}\frac{1}{A}\sum_{\mathbf{k}}[f_0(E_{\mathbf{k}-}) - f_0(E_{\mathbf{k}+})] \times [|\mathbf{k}| + |\mathbf{k}-2\mathbf{q}| - 2|\mathbf{k}-\mathbf{q}|],$$

$$\approx \omega_p^2 + \frac{e^2 q \pi^{1/2}\Delta_R}{\epsilon_s \hbar^2}[n_-^{1/2} - n_+^{1/2}], \tag{9}$$

where $f_0(E)$ is the Fermi-Dirac distribution function. The approximation in Eq. (9) is valid in the long wavelength limit only. Here, $\omega_p = \sqrt{2\pi n_{2D} e^2 q/\epsilon_s m^*}$ is the plasmon frequency of a spin-degenerate 2D electron system in the long wavelength limit. Equation (9) clearly shows that the charge carriers from each subband contribute through their total number density in the first term as well as through the individual number densities for each subband as seen in the second term. We can use these results to define the oscillator strength for plasmons as ($j=\pm$)

$$f_\lambda(q) = \omega_j(q)\left\{\Omega_0^2(q)\left|\frac{\partial\epsilon_R(q,\omega)}{\partial\omega}\right|_{\omega=\omega_j(q)}\right\}^{-1}, \tag{10}$$

where $\epsilon_R(q,\omega)$ is the real part of the dielectric function. The oscillator strength for electron-hole excitations is defined as

$$f_{e-h}(q) = \frac{1}{\pi}\int_{\mathrm{Im}\ \epsilon\neq 0} d\omega \frac{\omega}{\Omega_0^2(q)}\,\mathrm{Im}\left(\frac{-1}{\epsilon(q,\omega)}\right), \tag{11}$$

so that from Eq. (8) the total oscillator strength for plasmons and electron-hole excitations at a chosen q is 1.

In the self-consistent field approximation, the dielectric function is $\epsilon(q,\omega)=1+\alpha(q,\omega)$ where the polarization function is given by

$$\alpha(q,\omega) = \frac{\pi e^2}{2\epsilon_s q A}\sum_{\mathbf{k},\lambda,\lambda'=\pm 1}\frac{f_0(E_{\mathbf{k},\lambda}) - f_0(E_{\mathbf{k}-\mathbf{q},\lambda'})}{\hbar\omega + E_{\mathbf{k}-\mathbf{q},\lambda'} - E_{\mathbf{k},\lambda} + i0^+} \times \left[1 + \lambda\lambda'\frac{|\mathbf{k}| - q\cos\theta}{|\mathbf{k}-\mathbf{q}|}\right], \tag{12}$$

and θ is the angle between the wave vectors $\mathbf{k}$ and $\mathbf{q}$. In the long wavelength limit $q \ll k_F$, we obtain an approximate result for the dielectric function at $T=0$ K given for small but finite wave number by

$$\epsilon(q,\omega) \approx 1 - \frac{1}{\omega^2}\left[\omega_p^2 + \frac{\pi^2 e^2 q}{\epsilon_s \hbar}(\omega_+^{(0)} - \omega_-^{(0)})\right] - \frac{e^2 q}{8\hbar\omega\epsilon_s}\ln\left|\frac{\omega_+^{(0)}\omega_+^{(0)} + (\omega_+^{(0)} - \omega_-^{(0)})\omega - \omega^2}{\omega_+^{(0)}\omega_+^{(0)} - (\omega_+^{(0)} - \omega_-^{(0)})\omega - \omega^2}\right|, \tag{13}$$

where $\omega_\pm^{(0)} = 2\Delta_R(4\pi n_\pm)^{1/2}/\hbar$ and $\omega_-^{(0)} > \omega_+^{(0)}$ follows from Eq. (3). Since $\omega_+^{(0)}$ and $\omega_-^{(0)}$ are intimately related with the electron density in the + and − spin branches and the Rashba parameter, Eq. (13) clearly shows that the SO coupling induces two plasmon branches and each one is a collective oscillation of charge carriers from all occupied subbands. The collective excitations are due to interband (spin-flip) and intraband transitions. Figure 1(b) shows our results for the plasma excitations obtained by numerically solving the dispersion equation $\epsilon(q,\omega)=0$ with the polarization function given by Eq. (12) for arbitrary wave vector. For comparison, we also plot in Fig. 1(b) the plasma frequency ω_p for a spin-degenerate 2D electron system with the same electron density and electron effective mass embedded in a background dielectric medium as the spin-gap system.[33] The frequency ω_+ of the plasmon mode induced by intraband transitions is close to the electron-hole excitation spectrum. We highlight the proximity of the frequency of these modes as a function of wave vector through the inset in Fig. 1(b). As the Rashba parameter Δ_R is increased, the separation between ω_+ and the electron-hole continuum becomes larger. The wave vector dependence of the frequency ω_- of the spin-flip excitations is very different from ω_+. For as q is increased in the long wavelength limit, ω_- initially decreases before increasing at some $q=q_c$ and its frequency is well separated from ω_+. Now, for sufficiently large q, ω_- approaches the plasmon frequency for a spin-degenerate 2D electron system. Therefore, the long wavelength behavior of ω_- must be due to the large DOS for − spins near the bottom of the band $E=-E_\Delta$, which is given in Eq. (5).

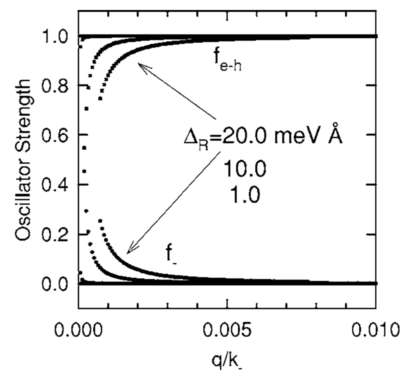

FIG. 2. The oscillator strengths of plasmons and electron-hole excitations for Rashba parameter Δ_R=1, 10, and 50 meV Å. The oscillator strength of plasmons increases with Δ_R. Only the oscillator strength of the ω_- mode and the electron-hole excitations can be seen in the plots. The values for ϵ_b, m^*, and n_{2D} are the same as Fig. 1(b).

In Fig. 2, we plot the calculated oscillator strengths for plasmons, $f_\lambda(q)$, and electron-hole excitations, $f_{e-h}(q)$. The oscillator strength of the mode with higher frequency ω_- is several orders of magnitude larger than the oscillator strength of the ω_+ mode, as shown in Fig. 2. The oscillator strength of the lower branch ω_+ is even weaker than that of the electron-hole excitations. In the long wavelength limit, f_- is appreciable and quickly decreases as the wave vector is increased. However, since most optical measurements are carried out in the long wavelength limit, this mode may be observed.

The static shielded potential within the 2D plane at a distance r from the z axis due to a point charge Ze on the z axis at $(0,0,d)$ is calculated from

$$\Phi_{\mathrm{tot}}(r,\omega=0) = Ze\int_0^\infty dq\frac{1}{\epsilon(q,\omega=0)}J_0(qr)e^{-qd}, \tag{14}$$

where, in the denominator of the integrand, we used $\epsilon(q,\omega)=1+\alpha(q,\omega)$ with the polarization function as given in

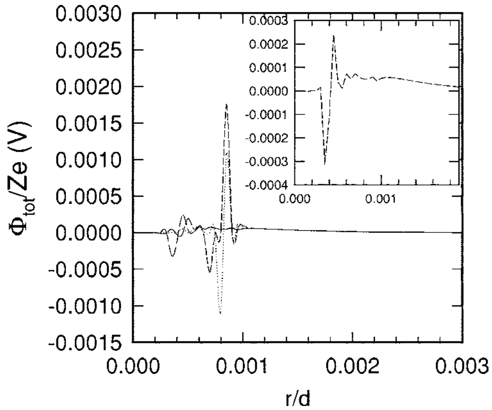

FIG. 3. The screened potential in Eq. (14) as a function of radial distance r within the 2D plane. A point charge is on the z axis at a distance $d=k_F^{-1}$ from the 2D plane. The parameters used in the calculation for ϵ_b, m^*, and n_{2D} are the same as Fig. 1(b). The Rashba parameter is $\Delta=1$ (dashed line), 10 (dot-dashed line), and 50 (dotted line) in units of meV Å. The inset shows only the curve for $\Delta=1.0$ meV Å.

Eq. (12). In Fig. 3, we plot $\Phi_{tot}(r,\omega=0)$ within the 2D plane as a function of the radial distance r from the z axis. Three values of the Rashba parameter were chosen. For each case, we observe Friedel oscillations for large r. in the screened potential. For small Rashba parameter, the largest amplitude of the potential occurs near $r=0$ but as Δ_R is increased, the largest oscillations have an increase in amplitude and the location of these large amplitude oscillations is shifted away from $r=0$. This implies that the impurity potential couples with the spin orbits leading to a depletion of the charge density fluctuations around the z axis and that the radius of this depleted region increases with the Rashba SO coupling. The screening of the impurity is influenced by the coupling of the spin degree of freedom of the electrons to their orbital motion.

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