Enhancing Kerr nonlinearity via spontaneously generated coherence

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A theoretical investigation is carried out into the effect of spontaneously generated coherence on the Kerr nonlinearity of general three-level systems of Λ, ladder, and V-shape types. It is found, with spontaneously generated coherence present, that the Kerr nonlinearity can be clearly enhanced. In the Λ- and ladder-type systems, the maximal Kerr nonlinearity increases and at the same time enters the electromagnetically induced transparency window as the spontaneously generated coherence intensifies. As for the V-type system, the absorption property is significantly modified and therefore enhanced Kerr nonlinearity without absorption occurs for certain probe detunings. We attribute the enhancement of Kerr nonlinearity mainly to the presence of an extra atomic coherence induced by the spontaneously generated coherence.

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I. INTRODUCTION

Spontaneously generated coherence (SGC) refers to the interference of spontaneous emission channels. In the Λ- and V-type systems, spontaneous emissions from a single excited state to two lower closely spaced levels [1] or from two closely spaced upper levels to a common atomic ground state [2] can interfere. In a ladder-type system, SGC can also be created in the equispaced-atomic-level case [3,4]. The existence of such coherence relies on the nonorthogonality of the two transition dipole moments. Experimentally, some efforts have been made in the past to generate SGC. In 1996, Xia *et al.* [5] carried out the first experimental investigation of constructive and destructive spontaneous emission interference using the sodium molecule. However, repetition of this experiment failed [6] and some controversies arose [7,8]. Very recently, there is experimental evidence that SGC plays a role in charged GaAs quantum dots [9], which have been proposed as elements in quantum-information networks. In atomic systems, although it is difficult to realize SGC experimentally, there are several proposals to simulate this effect. Ficek and Swain [10] have put forward a project to obtain SGC from a V-type three-level system with perpendicular dipole moments by a dc field. The effects of SGC on electromagnetically induced transparency (EIT), dark states, short-pulse propagation, resonance fluorescence, transient processes, squeezing spectra, etc., have been extensively studied both in general three-level systems and in multilevel systems [11–21]. All the interesting features due to SGC could have useful applications in laser physics and other areas of quantum optics.

Nevertheless, the effect of SGC on the Kerr nonlinearity has never been investigated to our best knowledge. The Kerr nonlinearity corresponds to the refractive part of the third-order susceptibility in optical media. As is known, Kerr-type nonlinear coefficients play a crucial role in nonlinear optics. The large enhancement of nonlinear susceptibilities with small absorption causes the nonlinear optics to be studied at low light levels [22–25]. In order to enhance the Kerr nonlinearity, several schemes have been proposed [26–29]. Schmidt *et al.* proposed a four-level *N*-configuration system to enhance the Kerr nonlinearity where the ideal EIT regime is disturbed by an additional off-resonant level [26]. Nakajima found that the autoionizing resonance could lead to enhanced third-order susceptibility [28]. In a four-level double-dark resonance system, the interacting double-dark resonance is predicted to be effective in enhancing the Kerr nonlinearity [29]. In the above-mentioned schemes, it is crucial to have additional or autoionizating levels to realize large Kerr nonlinearity. In this paper, we intend to investigate the effect of SGC on the enhancement of Kerr nonlinearity in the popularly discussed three-level systems of Λ, ladder, and V-shaped types. We find that the Kerr nonlinearity depends critically on the presence of SGC. With proper parameters, a large Kerr nonlinearity with vanishing absorption can be realized in general three-level systems. Especially in the V-type system, the Kerr nonlinearity can be manipulated to build up dramatically at the exact point of zero linear absorption. Analytical investigation demonstrates that it is the presence of an extra atomic coherence induced by the SGC that clearly enhances the Kerr nonlinearity.

II. ENHANCEMENT OF KERR NONLINEARITY VIA SGC

A. Λ-type system

We first consider a Λ-type system as shown in Fig. 1(a). A resonant coupling field Ω_c drives the transition between levels $|1\rangle$ and $|2\rangle$ while a probe field Ω_p is applied to the transition $|1\rangle$ and $|3\rangle$. $2\gamma_2$ and $2\gamma_3$ are the spontaneous decay rates of the excited state $|1\rangle$ to the ground states $|2\rangle$ and $|3\rangle$. When the two lower levels $|2\rangle$ and $|3\rangle$ are closely spaced such that the two transitions to the excited state interact with the same vacuum mode, spontaneously generated coherence can be present. Under the rotating-wave approximation, the systematic density matrix in the interaction picture involving the SGC can be written as

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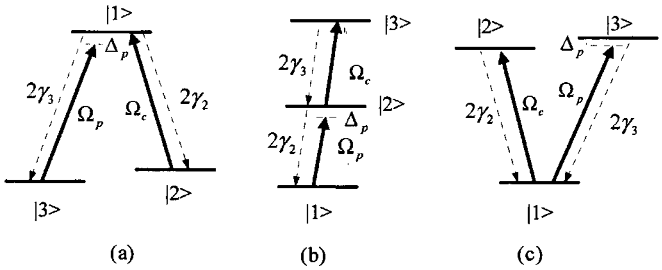

FIG. 1. Schematic energy-level diagrams showing the Λ- (a), ladder- (b), and V-type (c) systems with the coupling field Ω_c, the probe field Ω_p, and the decay rates $2\gamma_2$ and $2\gamma_3$.

$$\dot{\rho}_{11} = -2(\gamma_2+\gamma_3)\rho_{11} + i\Omega_p(\rho_{31}-\rho_{13}) + i\Omega_c(\rho_{21}-\rho_{12}),$$

$$\dot{\rho}_{33} = 2\gamma_3\rho_{11} + i\Omega_p(\rho_{13}-\rho_{31}),$$

$$\dot{\rho}_{23} = -i\Delta_p\rho_{23} + 2p\sqrt{\gamma_2\gamma_3}\rho_{11} + i\Omega_c\rho_{13} - i\Omega_p\rho_{21},$$

$$\dot{\rho}_{13} = -(\gamma_2+\gamma_3+i\Delta_p)\rho_{13} - i\Omega_p(\rho_{11}-\rho_{33}) + i\Omega_c\rho_{23},$$

$$\dot{\rho}_{12} = -(\gamma_2+\gamma_3)\rho_{12} + i\Omega_p\rho_{32} - i\Omega_c(\rho_{11}-\rho_{22}). \quad (1)$$

The above equations are constrained by $\rho_{11}+\rho_{22}+\rho_{33}=1$ and $\rho_{ji}^{*}=\rho_{ij}$. $\Delta_p=\omega_{13}-\omega_p$ means the detuning of the probe field from the optical transition. The effect of SGC is very sensitive to the orientations of the atomic dipole moments $\vec{\mu}_{12}$ and $\vec{\mu}_{13}$. Here, the parameter p denotes the alignment of the two dipole moments and is defined as $p=\vec{\mu}_{12}\cdot\vec{\mu}_{13}/|\vec{\mu}_{12}\cdot\vec{\mu}_{13}| = \cos\theta$ with θ being the angle between the two dipole moments. The terms with $p\sqrt{\gamma_2\gamma_3}$ represent the quantum interference resulting from the cross coupling between spontaneous emission paths $|1\rangle-|2\rangle$ and $|1\rangle-|3\rangle$. With the restriction that each field acts only on one transition, the Rabi frequencies Ω_c and Ω_p are connected to the angle θ and represented by $\Omega_{c(p)}=\Omega_{c(p)}^{0}\sin\theta=\Omega_{c(p)}^{0}\sqrt{1-p^2}$. It should be noted that only for small energy spacing between the two lower levels are the interference terms in Eq. (1) significant; otherwise the oscillatory terms will average out to zero and thereby the SGC effect vanishes.

It is known that the response of the atomic medium to the probe field is governed by its polarization $P=\varepsilon_0(E_p\chi + E_p^{*}\chi^{*})/2$, with χ being the susceptibility of the atomic medium. By performing a quantum average of the dipole moment over an ensemble of N atoms, we find $P=N(\mu_{31}\rho_{13} + \mu_{13}\rho_{31})$. In order to derive the linear and nonlinear susceptibilities, we need to obtain the steady-state solution of the density matrix equations. In the present approach, an iterative method is used and the density matrix elements are expressed as $\rho_{mn}=\rho_{mn}^{(0)}+\rho_{mn}^{(1)}+\rho_{mn}^{(2)}+\rho_{mn}^{(3)}+\cdots$. Assuming that the coupling field is much stronger than the probe field, the zeroth-order solution will be $\rho_{33}^{(0)}=1$ and other elements are equal to zero. Under the weak-probe approximation, we get the matrix element ρ_{13} up to third order:

$$\rho_{13}^{(1)} = \frac{i\Delta_p\Omega_p}{\Delta_p(\gamma_3+\gamma_2+i\Delta_p)-i\Omega_c^2}, \quad (2a)$$

$$\rho_{23}^{(1)} = \frac{\Omega_c}{\Delta_p}\rho_{13}^{(1)}, \quad (2b)$$

$$\rho_{11}^{(2)} = \frac{i\Omega_p}{2\gamma_3}(\rho_{31}^{(1)}-\rho_{13}^{(1)}), \quad (2c)$$

$$\rho_{21}^{(2)} = \frac{\Omega_p}{2i(\gamma_2+\gamma_3)}(\rho_{23}^{(1)}-\rho_{32}^{(1)}) - \frac{\gamma_2\Omega_p}{2\gamma_3\Omega_c}(\rho_{13}^{(1)}-\rho_{31}^{(1)}), \quad (2d)$$

$$\rho_{22}^{(2)} = \frac{(\gamma_2+\gamma_3)(\rho_{12}^{(2)}-\rho_{21}^{(2)})}{2i\Omega_c} - \frac{\Omega_p(\rho_{23}^{(1)}+\rho_{32}^{(1)})}{2\Omega_c} + \rho_{11}^{(2)}, \quad (2e)$$

$$\rho_{23}^{(2)} = \frac{2p\sqrt{\gamma_2\gamma_3}(i\gamma_2+i\gamma_3-\Delta_p)\rho_{11}^{(2)}}{\Delta_p(\gamma_2+\gamma_3+i\Delta_p)-i\Omega_c^2}, \quad (2f)$$

$$\rho_{11}^{(3)} = -\frac{\Omega_p\Omega_c}{2\gamma_3}\left(\frac{\rho_{23}^{(2)}}{\gamma_2+\gamma_3+i\Delta_p} + \frac{\rho_{32}^{(2)}}{\gamma_2+\gamma_3-i\Delta_p}\right), \quad (2g)$$

$$\rho_{13}^{(3)} = \frac{-2p\sqrt{\gamma_2\gamma_3}\Omega_c\rho_{11}^{(3)} + i\Omega_p\Omega_c\rho_{21}^{(2)} + i\Omega_p\Delta_p(2\rho_{11}^{(2)}+\rho_{22}^{(2)})}{-\Delta_p(\gamma_2+\gamma_3+i\Delta_p)+i\Omega_c^2}. \quad (2h)$$

Therefore, the first- and third-order susceptibilities $\chi^{(1)}$ and $\chi^{(3)}$ should be

$$\chi^{(1)} = \frac{-2N|\vec{\mu}_{13}|^2}{\varepsilon_0\hbar\,\Omega_p}\rho_{13}^{(1)} = \frac{-2N|\vec{\mu}_{13}|^2}{\varepsilon_0\hbar}\frac{\Delta_p}{\Omega_c^2+i(\gamma_2+\gamma_3+i\Delta_p)\Delta_p}, \quad (3)$$

$$\begin{aligned}\chi^{(3)} &= \frac{-2N|\vec{\mu}_{13}|^4}{3\varepsilon_0\hbar^3\Omega_p^3}\rho_{13}^{(3)} \\ &= \frac{-2N|\vec{\mu}_{13}|^4}{3\varepsilon_0\hbar^3}(4i\Omega_c^4p^2\gamma_2(\gamma_2+\gamma_3)\Delta_p^2(\Omega_c^2-\Delta_p^2) \\ &\quad + \Delta_p[\Omega_c^2+i(\gamma_2+\gamma_3+i\Delta_p)\Delta_p] \\ &\quad \times[\Omega_c^2-i(\gamma_2+\gamma_3-i\Delta_p)\Delta_p] \\ &\quad \times\{2\Omega_c^4\gamma_3 - i\Omega_c^2\gamma_2(\gamma_2+\gamma_3)\Delta_p \\ &\quad + \Delta_p^2[3\Omega_c^2\gamma_2+\gamma_2\gamma_3^2+\gamma_2^3+2\gamma_3(\Omega_c^2+\gamma_2^2)]\})/\beta, \quad (4)\end{aligned}$$

with

$$\begin{aligned}\beta &= \gamma_3[\Omega_c^3 - i\Omega_c(\gamma_2+\gamma_3-i\Delta_p)\Delta_p]^2 \\ &\quad \times[\Omega_c^2+i(\gamma_2+\gamma_3+i\Delta_p)\Delta_p]^3. \quad (5)\end{aligned}$$

According to the expressions (3) and (4), we show in Fig. 2 the linear absorption (dotted curve) and the refractive part of the third-order susceptibility (solid curve) as a function of the probe detuning. For simplicity, we scale all the parameters by the decay rate γ_2; $\Omega_c^0=4\gamma_2$ and $\gamma_3=\gamma_2$. From this figure, we can see that when $p=0$, which means no interference between spontaneous emission channels, a couple of general linear absorption and Kerr nonlinearity curves occur [30]. With the spontaneous emission interference present, the refractive part of the third-order susceptibility is enhanced

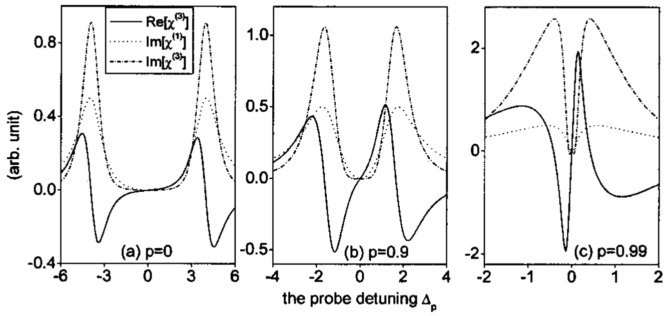

FIG. 2. The Kerr nonlinearity $\mathrm{Re}(\chi^{(3)})$ (solid curve), linear absorption $\mathrm{Im}(\chi^{(1)})$ (dotted curve), and nonlinear absorption $\mathrm{Im}(\chi^{(3)})$ (dash-dotted curve) of the Λ-type system with different SGC. Parameters are $\Omega_c^0=4.0\gamma_2$ and $\gamma_3=1.0\gamma_2$.

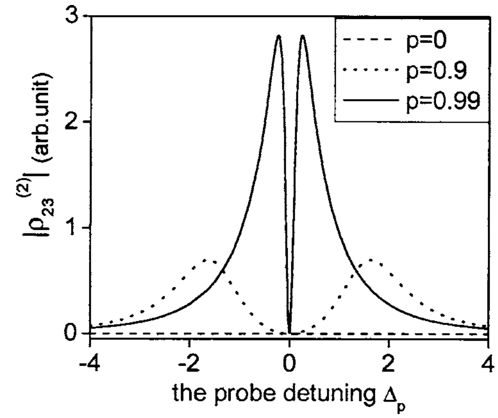

FIG. 3. The extra coherence term $|\rho_{23}^{(2)}|$ as a function of the probe detuning with different SGC. Parameters are the same as in Fig. 2.

and also the EIT window narrows [11]. For small values of p, because the change of the system behavior is small and the results are quite similar to those of $p=0$, we do not show them here. However, for large values of p, which imply strong interference of the spontaneous emission channels, the refractive part of the third-order susceptibility evidently builds up. It is clearly to be seen that the maximal Kerr nonlinearity at $p=0.99$ is about six times that at $p=0$. In addition, although the EIT window narrows, the maximal Kerr nonlinearity enters the EIT window gradually and therefore the corresponding linear absorption becomes negligible. On the other hand, since the linear absorption is very low, the nonlinear absorption given by the imaginary part of $\chi^{(3)}$ should be considered now. According to the analytical expression for $\chi^{(3)}$ shown in Eq. (4), we plot its imaginary part in Fig. 2 (dash-dotted curve). It is noted that the variation of the nonlinear absorption with the SGC is very similar to that of the linear absorption. The enhanced Kerr nonlinearity gradually enters the "nonlinear EIT window" and the ratio of the real and imaginary parts of $\chi^{(3)}$ improves significantly when the SGC intensifies from $p=0$ to 0.99. This means that in the case of optimal SGC, both enhanced Kerr nonlinearity and negligible linear and nonlinear absorption can be realized simultaneously.

Now we provide a qualitative explanation for the above numerical results. It can be seen from Eq. (2f) that the density matrix element $\rho_{23}^{(2)}$ now is an extra term introduced by SGC. This term accordingly makes the third-order susceptibility acquire an additional term associated with SGC. As the analytical expression for $\rho_{13}^{(3)}$ shows, the first term is the product of the coupling field and the additional term. That is to say, in the present Λ-type system, the spontaneous decays from the upper state to the lower closely spaced levels interfere and give rise to an extra coherence between the lower two levels [31,32]. Thereby the coupling field interacts with this extra coherence and consequently the Kerr nonlinearity is clearly enhanced. Figure 3 shows the coherence term $|\rho_{23}^{(2)}|$ as a function of the probe detuning with different SGC. Obviously, the stronger the SGC is, the more remarkable the coherence becomes. Hence, we attribute the enhancement of Kerr nonlinearity to the extra coherence between the lower two levels induced by SGC.

B. Ladder-type system

The ladder-type system considered here is shown in Fig. 1(b). A resonant coupling field Ω_c and a probe field Ω_p with detuning $\Delta_p=\omega_{21}-\omega_p$ are applied to the respective transitions. The higher excited state $|3\rangle$ decays with a rate $2\gamma_3$ to the lower excited state $|2\rangle$ which decays to the ground state $|1\rangle$ with a rate $2\gamma_2$. In the case of nearly equispaced levels, the effect of SGC could occur. The alignment of the two dipole moments $\vec{\mu}_{12}$ and $\vec{\mu}_{23}$ is defined to be $p=\vec{\mu}_{12}\cdot\vec{\mu}_{23}/|\vec{\mu}_{12}\cdot\vec{\mu}_{23}|=\cos\theta$ with θ being the angle between the two dipole moments. Accordingly, the systematic density matrix of the ladder-type system involving the SGC is obtained.

$$\dot{\rho}_{33}=-2\gamma_3\rho_{33}+i\Omega_c(\rho_{23}-\rho_{32}),$$

$$\dot{\rho}_{11}=2\gamma_2\rho_{22}+i\Omega_p(\rho_{21}-\rho_{12}),$$

$$\dot{\rho}_{21}=-\gamma_2\rho_{21}-i\Delta_p\rho_{21}+2p\sqrt{\gamma_2\gamma_3}\rho_{32}+i\Omega_c\rho_{31}+i\Omega_p(\rho_{11}-\rho_{22}),$$

$$\dot{\rho}_{31}=-(\gamma_3+i\Delta_p)\rho_{31}+i\Omega_c\rho_{21}-i\Omega_p\rho_{32},$$

$$\dot{\rho}_{32}=-(\gamma_2+\gamma_3)\rho_{32}-i\Omega_p\rho_{31}+i\Omega_c(\rho_{22}-\rho_{33}). \qquad (6)$$

Using a similar approach to that shown in Sec. II A, the first- and third-order susceptibilities of the probe field can be derived. To be brief, we do not provide $\chi^{(1)}$ and $\chi^{(3)}$ here and just show the numerical results.

Setting $\Omega_{c(p)}=\Omega_{c(p)}^0\sin\theta=\Omega_{c(p)}^0\sqrt{1-p^2}$ and the parameters $\Omega_c^0=4\gamma_2$, $\gamma_3=0.01\gamma_2$, the linear absorption, nonlinear absorption, and Kerr nonlinearity are displayed in Fig. 4. In comparison with the Λ-type system, we find that the variation trend is quite similar. When the optimal SGC effect is taken into account, the refractive part of the third-order susceptibility is enhanced by about two times and also enters the transparency window. In other words, enhancement of the Kerr nonlinearity with reduced linear and nonlinear absorption in ladder-type systems can be achieved via the optimal effect of SGC.

In order to present a qualitative explanation, each order of the matrix elements is investigated:

$$\rho_{21}^{(1)} = \frac{i\Omega_p(\gamma_3 + i\Delta_p)}{(\gamma_3 + i\Delta_p)(\gamma_2 + i\Delta_p) + \Omega_c^2}, \tag{7a}$$

$$\rho_{31}^{(1)} = \frac{-\Omega_p\Omega_c}{(\gamma_3 + i\Delta_p)(\gamma_2 + i\Delta_p) + \Omega_c^2}, \tag{7b}$$

$$\rho_{22}^{(2)} = \frac{i\Omega_p(\rho_{12}^{(1)} - \rho_{21}^{(1)})}{2\gamma_2}, \tag{7c}$$

$$\rho_{32}^{(2)} = \frac{\Omega_p(\rho_{31}^{(1)} - \rho_{13}^{(1)})}{2i(\gamma_2 + \gamma_3)} + \frac{2i\Omega_c\gamma_3\rho_{22}^{(2)} - i\Omega_p\gamma_3(\rho_{13}^{(1)} + \rho_{31}^{(1)})}{2[\Omega_c^2 + \gamma_3(\gamma_2 + \gamma_3)]}, \tag{7d}$$

$$\rho_{33}^{(2)} = \frac{i\Omega_c(\rho_{23}^{(2)} - \rho_{32}^{(2)})}{2\gamma_3}, \tag{7e}$$

$$\rho_{31}^{(2)} = \frac{2i\Omega_c p\sqrt{\gamma_2\gamma_3}\rho_{32}^{(2)}}{(\gamma_3 + i\Delta_p)(\gamma_2 + i\Delta_p) + \Omega_c^2}, \tag{7f}$$

$$\rho_{32}^{(3)} = \Omega_p\left(\frac{\rho_{31}^{(2)} - \rho_{13}^{(2)}}{2i(\gamma_3 + \gamma_2)} - \frac{\gamma_3[\gamma_2(\rho_{31}^{(2)} + \rho_{13}^{(2)}) + (\gamma_3 + i\Delta_p)\rho_{31}^{(2)} + (\gamma_3 - i\Delta_p)\rho_{13}^{(2)}]}{2i\gamma_2[\Omega_c^2 - \gamma_3(\gamma_2 + \gamma_3)]}\right), \tag{7g}$$

$$\rho_{21}^{(3)} = \frac{\Omega_p\Omega_c\rho_{32}^{(2)} - i\Omega_p(i\Delta_p + \gamma_3)(2\rho_{22}^{(2)} + \rho_{33}^{(2)}) + 2p\sqrt{\gamma_2\gamma_3}(i\Delta_p + \gamma_3)\rho_{32}^{(3)}}{(\gamma_3 + i\Delta_p)(\gamma_2 + i\Delta_p) + \Omega_c^2}. \tag{7h}$$

From the analytical expression of $\rho_{21}^{(3)}$, we see that the third term is the contribution from the SGC effect. Close inspection of Eq. (7) tells us that it is the matrix element $\rho_{31}^{(2)}$ that owns the contribution from the SGC term $p\sqrt{\gamma_2\gamma_3}$, which leads to $\rho_{32}^{(3)}$ and hence $\rho_{21}^{(3)}$ being correlated with the SGC effect. The two spontaneous decay paths from the higher excited state $|3\rangle$ and the lower excited state $|2\rangle$ will interfere and therefore cause an extra coherence between the higher excited state and the ground state: $|\rho_{31}^{(2)}|$. This extra atomic coherence will be responsible for the enhancement of Kerr nonlinearity. Figure 5 displays the extra atomic coherence term $|\rho_{31}^{(2)}|$ with different SGC. When the SGC is absent, the extra coherence disappears and so the general absorption and Kerr nonlinearity curves present. As the parameter p increases from 0.90 to 0.99, the extra coherence $\rho_{31}^{(2)}$ intensifies observably. Hence the origin of the enhanced Kerr nonlinearity can be traced to the fact of the extra atomic coherence caused by the SGC.

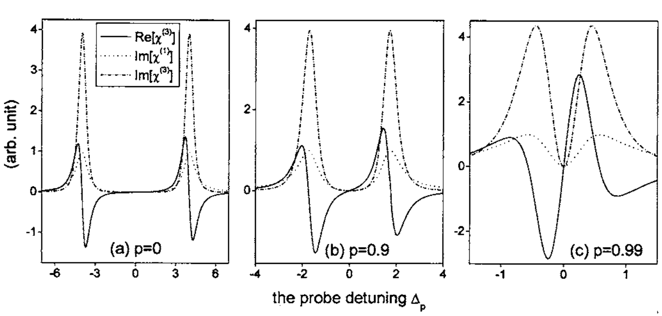

FIG. 4. The Kerr nonlinearity $\mathrm{Re}(\chi^{(3)})$ (solid curve), linear absorption $\mathrm{Im}(\chi^{(1)})$ (dotted curve), and nonlinear absorption $\mathrm{Im}(\chi^{(3)})$ (dash-dotted curve) of the ladder-type system with different SGC. Parameters are $\Omega_c^0=4.0\gamma_2$ and $\gamma_3=0.01\gamma_2$.

C. V-type system

In this part, we will lay stress on the dependence of the third-order susceptibility on the SGC presented in the

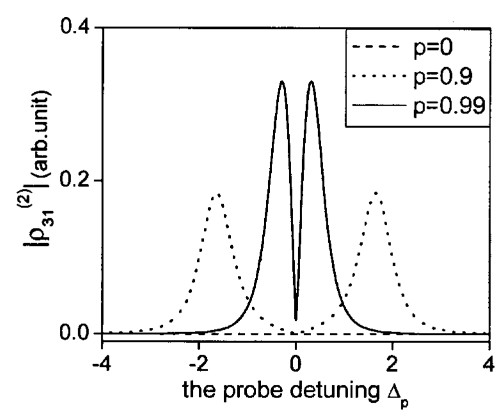

FIG. 5. The extra coherence term $|\rho_{31}^{(2)}|$ as a function of the probe detuning with different SGC. Parameters are the same as in Fig. 4.

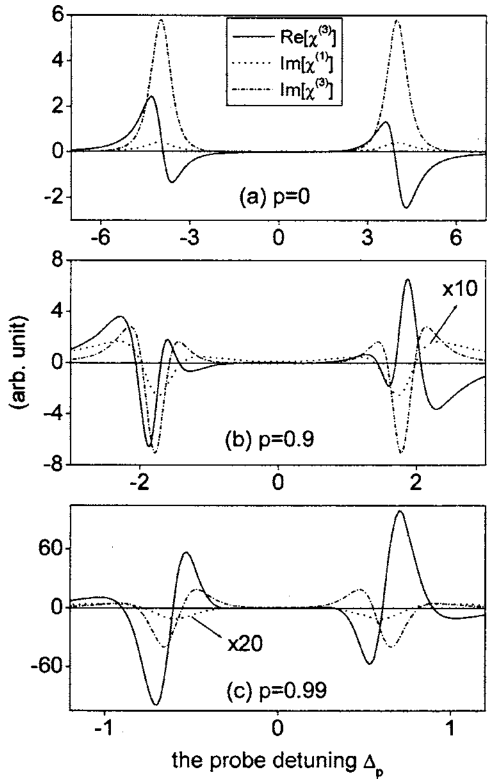

FIG. 6. The Kerr nonlinearity $\mathrm{Re}(\chi^{(3)})$ (solid curve), linear absorption $\mathrm{Im}(\chi^{(1)})$ (dotted curve), and nonlinear absorption $\mathrm{Im}(\chi^{(3)})$ (dash-dotted curve) of the V-type system with different SGC. Parameters are $\Omega_c^0=4.0\gamma_2$, $\gamma_3=0.1\gamma_2$.

V-type system. As displayed in Fig. 1(c), two closely spaced excited states $|2\rangle$ and $|3\rangle$ are coupled to the ground state $|1\rangle$ through a resonant strong drive field Ω_c and a detuned ($\Delta_p=\omega_{31}-\omega_p$) weak probe field Ω_p. The equations of motion for the density matrix involving the SGC read

$$\dot{\rho}_{22}=-2\gamma_2\rho_{22}-p\sqrt{\gamma_2\gamma_3}(\rho_{23}+\rho_{32})+i\Omega_c(\rho_{12}-\rho_{21}),$$

$$\dot{\rho}_{33}=-2\gamma_3\rho_{33}-p\sqrt{\gamma_2\gamma_3}(\rho_{23}+\rho_{32})+i\Omega_p(\rho_{13}-\rho_{31}),$$

$$\dot{\rho}_{23}=-(\gamma_2+\gamma_3+i\Delta_p)\rho_{23}-p\sqrt{\gamma_2\gamma_3}(\rho_{22}+\rho_{33}) + i\Omega_c\rho_{13}-i\Omega_p\rho_{21},$$

$$\dot{\rho}_{21}=-\gamma_2\rho_{21}-p\sqrt{\gamma_2\gamma_3}\rho_{31}+i\Omega_c(\rho_{11}-\rho_{22})-i\Omega_p\rho_{23},$$

$$\dot{\rho}_{31}=-(\gamma_3-i\Delta_p)\rho_{31}-p\sqrt{\gamma_2\gamma_3}\rho_{21}-i\Omega_c\rho_{32}+i\Omega_p(\rho_{11}-\rho_{33}). \tag{8}$$

Here, $2\gamma_2$ and $2\gamma_3$ are the spontaneous decay rates of the excited states $|2\rangle$ and $|3\rangle$ to the ground state $|1\rangle$; $p=\vec{\mu}_{12}\cdot\vec{\mu}_{13}/|\vec{\mu}_{12}\cdot\vec{\mu}_{13}|=\cos\theta$ means the alignment of the two dipole moments. The steady-state solution of the matrix element ρ_{31} is obtained using a mathematical method, leading to the susceptibilities plotted in Fig. 6.

In the case of no incoherent pumping, Paspalakis *et al.* [13] have reported that incomplete EIT is established at resonance while gain can be created in the nonresonant case in two symmetric regions centered around zero probe detuning for large values of p, just as shown in Fig. 6 (dotted curves). We conclude here that zero linear absorption occurs at certain values of the probe detuning and furthermore the points of zero absorption shift as the SGC changes from $p=0.90$ to 0.99. The nonlinear absorption behaves similarly, as shown by the dash-dotted curves. On the other hand, the solid curves of Fig. 6 display the Kerr nonlinearity as a function of the probe detuning. Compared with the case of $p=0$, the maximal value of the Kerr nonlinearity is enhanced by about two times at $p=0.90$ and even 30 times at $p=0.99$. Now, we inspect the absorption and Kerr nonlinearity curves together. First, if the SGC is absent, the maximal Kerr nonlinearity is accompanied by strong linear and nonlinear absorptions, as can be seen from Fig. 6(a). However, when the SGC is taken into account, as at $p=0.90$, the maximal Kerr nonlinearity is located in the gain region. In the case of $p=0.99$, it is interesting to see that the refractive part of the third-order susceptibility has a large value whereas the linear absorption is zero for a certain probe detuning. At the same time, the nonlinear absorption is so small as to be neglected at this exact probe detuning. So in this case, we achieve giant Kerr nonlinearity accompanied by zero linear absorption and vanishing nonlinear absorption via SGC.

Because the analytical expression of $\rho_{31}^{(3)}$ is severely tedious, we do not present it here. Through systematic study, special attention is given to the second-order element $\rho_{23}^{(2)}$. Just as shown in Fig. 7, $\rho_{23}^{(2)}$ appears because of the existence of SGC. When SGC intensifies, $\rho_{23}^{(2)}$ also becomes remarkable, corresponding to the enhancement of Kerr nonlinearity. Hence, we attribute the enhancement of Kerr nonlinearity because of the occurrence of SGC mainly to the extra coherence between the upper two levels. In the present V-type system with equispaced levels, quantum interference is possible because they are damped by the same vacuum modes. The photon decay from one of the upper levels may drive the atom into the other upper level, and vice versa. This type of quantum interference then can induce an extra coherence between the upper two levels. Therefore, the V-type system becomes a proper candidate for large Kerr nonlinearity with near-zero absorption.

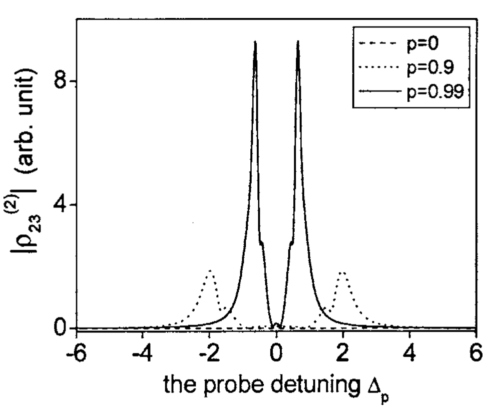

FIG. 7. The extra coherence term $|\rho_{23}^{(2)}|$ as a function of the probe detuning with different SGC. Parameters are the same as in Fig. 6.

III. CONCLUSIONS

We have investigated the effect of SGC on the Kerr nonlinearity in general three-level systems of Λ, ladder, and V-shaped type. It was found that the spontaneously generated coherence is capable of enhancing the Kerr nonlinearity. The spontaneous emission interference induces an extra coherence and therefore gives rise to the enhancement of Kerr nonlinearity. In both the Λ- and ladder-type systems, a large Kerr nonlinearity with negligible absorption can be realized. In the V-type system, we have shown that it is possible to achieve giant Kerr nonlinearity with zero absorption via SGC. Additionally, in the process of getting enhanced Kerr nonlinearity, the second-order susceptibility is found to be nonzero, in contrast to the case without SGC. This feature occurs in all three above-mentioned atomic systems and will be discussed in future work.

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