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## Nonlinear dielectric function of a dusty plasma in the presence of electromagnetic fields

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### Abstract

Using a self-consistent kinetic theory for longitudinal plasma waves and dust grain charging currents, the nonlinear dielectric function of a dusty plasma in the presence of electromagnetic fields has been derived analytically. Resonances with various dusty plasma modes have been investigated.

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There has been much recent progress in the study of charged dust grains in plasmas [1–11] as charged dust particles with sizes ranging between 10 nm and 100  $\mu\text{m}$  are frequently observed in space and laboratories. The dust particles can behave as single particles or as a collective species. Moreover, the charge on the grain surface can vary in response to the ambient plasma density as well as electrostatic potential perturbations. The latter has stimulated a number of studies recently [12–18]. Based on the standard probe theory for the grain charging [19,20], it has been shown that the dust charge fluctuations can significantly modify the linear dispersion relations of the plasma waves [12–18]. However, nonlinear interactions between electromagnetic (EM) waves and plasma slow motions incorporating the effects of dust charge perturbations have not been considered as yet.

The nonlinear dielectric functions of plasmas are well known to be the origin of numerous nonlinear interactions, such as parametric instabilities [21], beam self-focusing and filamentation [22], as well as optical mixing [23], etc. In this Letter, we present the EM wave induced nonlinear dielectric function of a dusty plasma, accounting for both the dust particle dynamics and the grain charge variations. The nonlinearity is assumed to be originating from the ponderomotive force of the EM waves on electrons, the latter are coupled with the ions and the dust grains in order to drive longitudinal plasma waves. Using Vlasov equations for each species and the probe model for the grain charge variations, a general expression for the nonlinear dielectric function is obtained. Four cases with different time scales are considered and the resonance enhancements of the nonlinear dielectrics are investigated when the EM wave frequency differences are close to that of the longitudinal modes of the dusty plasma.

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Let us consider a multi-component dusty plasma in the presence of EM fields

$$\mathbf{E} = \sum_j \frac{1}{2} \mathbf{E}_j \exp[i(\omega_j t - \mathbf{k}_j \cdot \mathbf{r})] + \text{c.c.}, \quad (1)$$

where  $\mathbf{E}_j$ ,  $\omega_j$ , and  $\mathbf{k}_j$  are the amplitude, the angular frequency, and the wavevector of the  $j$ th wave, c.c. stands for the complex conjugate. Because of the large inertia of the ions and the dust grains, only the electrons can respond to the high-frequency EM fields. The total dielectric function of the plasma in response to the  $j$ th EM wave is

$$\varepsilon(\omega_j) = 1 - n_e/n_c(\omega_j) = \varepsilon_0 - \tilde{n}_e/n_c(\omega_j), \quad (2)$$

where  $n_e$  is the total electron number density,  $n_c(\omega_j) = m_e \omega_j^2 / 4\pi e^2$  is the critical density of the  $j$ th wave,  $e$  and  $m_e$  are the magnitude of the electron charge and the mass, respectively,  $\varepsilon_0 = 1 - n_{e0}/n_c$  is the linear dielectric function of the plasma,  $n_{e0}$  is the equilibrium electron number density, and  $\tilde{n}_e$  is the electron density perturbation which determines the nonlinear part of the dielectric function and is associated with the longitudinal plasma waves driven by the ponderomotive force of the EM waves. The ponderomotive force can be written as  $\mathbf{F}_{pd} = -\nabla \phi_{pd}$ , where the ponderomotive potential is given by

$$\phi_{pd} = \sum_{j < k} \frac{e^2}{4m_e \omega_j \omega_k} \mathbf{E}_j \cdot \mathbf{E}_k^* \exp(i\psi) + \text{c.c.} \quad (3)$$

Here,  $\psi = \Delta \omega_{jk} t - \Delta \mathbf{k}_{jk} \cdot \mathbf{r}$ ,  $\Delta \omega_{jk} = \omega_j - \omega_k \ll \omega_{j,k}$  and  $\Delta \mathbf{k}_{jk} = \mathbf{k}_j - \mathbf{k}_k$  are the frequency and the wavevector differences between the  $j$ th and the  $k$ th EM wave, respectively. In Eq. (3), we have neglected the terms containing sum frequencies, since we are considering the situation in which the EM wave frequencies are much higher than that of the plasma waves.

The ponderomotive force causes the electrons to move away from the ions and the dust grains so as to induce longitudinal plasma oscillations. The driven longitudinal plasma perturbations are described by the linearized Vlasov equation

$$\partial_t \tilde{f}_\alpha + \mathbf{v} \cdot \nabla \tilde{f}_\alpha - (q_\alpha/m_\alpha) \nabla (\tilde{\phi} - \delta_\alpha \phi_{pd}/e) \cdot \partial_v f_{\alpha 0} = 0, \quad (4)$$

where the subscript  $\alpha$  is e for the electrons, i for the ions, and d for the dust particles,  $\delta_\alpha = 1$  for the electrons and  $\delta_\alpha = 0$  for the ions and the dust particles,  $m_\alpha$  is the mass of the species  $\alpha$ ,  $f_{\alpha 0}$  and  $\tilde{f}_\alpha$  are the equilibrium and perturbation distribution functions of the corresponding species, respectively, and  $\tilde{\phi}$  is the electrostatic potential perturbation associated with longitudinal plasma waves. The charge  $q_\alpha$  is  $-e$  for the electrons,  $e$  for the ions, and  $q_{d0}$  (the average equilibrium charge on the grain surface) for the dust particles.

The dust grains are assumed to be charged by plasma currents collected at the grain surface. Furthermore, the charge is subject to fluctuations because of the plasma density and electrostatic potential perturbations. In general, the grains have various sizes and consist of metallic as well as dielectric materials. In the following, we shall assume that the characteristic wavelengths of the longitudinal plasma modes are much larger than the grain diameter so that the finite-size effect can be neglected. Moreover, we shall suppose the grains to be spherical conductors, as this case often occurs in many laboratory plasmas. By using the standard probe theory [19,20] for the grain charging, it is shown that the grain charge fluctuation  $\tilde{q}_d$  is governed by

$$\partial_t \tilde{q}_d + \eta \tilde{q}_d = \sum_{\alpha=i,e} q_\alpha \int_{|v| > v_{\alpha m}} \tilde{f}_\alpha v \sigma_{\alpha d}(v, q_{d0}) dv, \quad (5)$$

where  $v_{im} = 0$  and  $v_{em} = (-2eq_{d0}/m_e C)^{1/2}$  is the minimum speed an electron must have in order to arrive at the grain surface, and

$$\sigma_{\alpha d}(v, q_{d0}) = \pi \alpha^2 (1 - 2q_\alpha q_{d0}/m_i C v^2) \quad (\alpha = e, i) \quad (6)$$

is the effective collision cross sections of an electron and ion with a dust grain of radius  $a$  [14], and  $C$  is the capacitance of the grain. The charge relaxation rate  $\eta$ , originating from the variations in the effective collision cross section due to the charge perturbations at the grain surface as experienced by the unperturbed particles, can be written as [13]

$$\eta = e|I_{e0}| \left( \frac{1}{CT_e} + \frac{1}{CT_i - eq_{d0}} \right), \tag{7}$$

where  $T_e$  ( $T_i$ ) is the electron (ion) temperature, and

$$I_{e0} = -\pi a^2 e (8T_e/\pi m_e)^{1/2} n_{e0} \exp(eq_{d0}/CT_e), \tag{8}$$

is the equilibrium electron-grain charging current [13].

The system is closed with the help of Poisson’s equation

$$\nabla^2 \tilde{\phi} = -4\pi \sum_{\alpha=e,i,d} q_\alpha \int \tilde{f}_\alpha d\mathbf{v} - 4\pi n_{d0} \tilde{q}_d, \tag{9}$$

where  $n_{d0}$  is the unperturbed dust number density, and we have assumed overall charge neutrality in equilibrium, i.e.,  $en_{i0} - en_{e0} + q_{d0}n_{d0} = 0$  ( $n_{i0}$  is the unperturbed ion number density). The last term on the right-hand-side of (9) arises from the contribution of the dust charge fluctuations.

The system of equations (4), (5) and (9) is driven by the ponderomotive potential  $\phi_{pd}$  which consists of Fourier components at difference frequencies of the EM waves. Using the Fourier representation  $\exp[i(\Delta\omega_{jk}t - \Delta\mathbf{k}_{jk} \cdot \mathbf{r})]$ , where the component indices  $j$  and  $k$  will be dropped in the following for convenience, we readily obtain

$$\tilde{f}_\alpha = -(q_\alpha \Delta k / m_\alpha) (\tilde{\phi} - \delta_\alpha \phi_{pd} / e) \partial_{v_x} f_{\alpha 0} / (\Delta\omega - \Delta k v_x), \tag{10}$$

where  $x$  is a coordinate along the direction of  $\Delta\mathbf{k}$ . Substituting (10) into (5) and (9), we obtain

$$\epsilon(\Delta\omega, \Delta k) \tilde{\phi} = (\chi_e + \chi_{qe}) \phi_{pd} / e. \tag{11}$$

Here,

$$\epsilon(\Delta\omega, \Delta k) = 1 + \sum_{\alpha=e,i,d} \chi_\alpha + \chi_{qe} + \chi_{qi}, \tag{12}$$

and

$$\chi_\alpha = \frac{\omega_{p\alpha}^2}{(\Delta k)^2} \frac{1}{n_{\alpha 0}} \int \frac{\partial_{v_x} f_{\alpha 0}}{v_\phi - v_x} d\mathbf{v} \tag{13}$$

is the linear susceptibility of the  $\alpha$  species,  $\omega_{p\alpha} = (4\pi q_\alpha^2 n_{\alpha 0} / m_\alpha)^{1/2}$  is the plasma frequency, and  $v_\phi = \Delta\omega / \Delta k$  is the phase velocity of the beat-wave. The linear susceptibility  $\chi_{q\alpha}$  ( $\alpha = e, i$ ) arising from the dust charge fluctuations caused by the electrostatic perturbations is given by

$$\chi_{q\alpha} = -\frac{i}{\Delta\omega - i\eta} \frac{n_{d0}}{n_{\alpha 0}} \int_{|v| > v_{m\alpha}} \frac{\partial_{v_x} f_{\alpha 0}}{v_\phi - v_x} v \sigma_{\alpha d} d\mathbf{v}. \tag{14}$$

Eq. (11) shows that the electrostatic potential is driven by the ponderomotive potential. We note that in the absence of the ponderomotive force, Eq. (11) reduces to the usual dispersion relation  $\epsilon(\omega, k) = 0$  of a dusty plasma.

The electron density perturbation  $\tilde{n}_e = \int \tilde{f}_e d\nu$  can be obtained by substituting (11) into (10). Thus, we have

$$\tilde{n}_e = \frac{n_{e0}}{m_e} \frac{(\Delta k)^2}{\omega_{pe}^2} \chi_e \left( \frac{\chi_e + \chi_{qae}}{\epsilon} - 1 \right) \phi_{pd}. \quad (15)$$

The electron density perturbation is responsible for a nonlinear current, which, in turn, results in a nonlinear dielectric function defined by

$$\epsilon_{NL} = \sum_{j,k} \epsilon_{jk} (\mathbf{E}_j \cdot \mathbf{E}_k^*), \quad (16)$$

where  $\epsilon_{jk}$  can be considered as the nonlinear dielectric coefficient of the corresponding Fourier component. Comparing (2) and (16) and using (3) and (15), we obtain

$$\epsilon_{jk} = \epsilon_{jk}^0 (\Delta k)^2 \lambda_e^2 \chi_e [1 - (\chi_e + \chi_{qae})/\epsilon], \quad (17)$$

where

$$\epsilon_{jk}^0 = \frac{e^2}{4m_e \omega_j \omega_k T_e} \frac{n_{e0}}{n_c(\omega)}$$

is the non-resonant nonlinear dielectric coefficient, and  $\lambda_e = (T_e/4\pi e^2 n_{e0})^{1/2}$  is the electron Debye length. Eq. (17) is a general form of the nonlinear dielectric coefficient in a dusty plasma, which contains both the resonant and non-resonant cases. It is readily applicable for studying various quasilinear processes such as stimulated Raman and Brillouin scatterings as well as EM wave mixing [24] in a dusty plasma. In what follows, we shall examine separately the resonances with various plasma modes and the enhancement of the  $\epsilon_{jk}$ .

We first consider the quasi-steady state case in which all the EM waves are of the same frequency, i.e.,  $\Delta\omega = 0$ . This happens on a very long time scale during which all the species can reach thermal equilibrium [12]. The linear susceptibilities are  $\chi_\alpha = 1/(\Delta k)^2 \lambda_\alpha^2$ , where  $\lambda_\alpha = u_\alpha/\omega_{p\alpha}$  is the Debye length, and  $u_\alpha = (T_\alpha/m_\alpha)^{1/2}$  is the thermal speed of the species  $\alpha$ . The susceptibilities arising from the dust charge fluctuations are given by  $\chi_{qae} = (\beta/\eta)/(\Delta k)^2 \lambda_e^2$  and  $\chi_{qai} = \chi_{qae} T_e/T_i$ , where  $\beta = |I_{e0}/e| n_{d0}/n_{e0}$ . Thus, the nonlinear dielectric coefficient reads

$$\epsilon_{jk} = \epsilon_{jk}^0 \frac{1 + (\Delta k \Lambda)^2 - (\Lambda/\lambda_e)^2 [1 - (\beta/\eta) T_e/T_i]}{1 + (\Delta k \Lambda)^2 + (\Lambda/\lambda_e)^2 (\beta/\eta) (1 + T_e/T_i)}, \quad (18)$$

where  $\Lambda = (\lambda_e^{-2} + \lambda_i^{-2} + \lambda_d^{-2})^{-1/2}$  is the effective Debye length of the dusty plasma. Since usually  $\lambda_d \ll \lambda_i \ll \lambda_e$ ,  $\Lambda \ll \lambda_e$ . Thus,  $\epsilon_{jk} \approx \epsilon_{jk}^0$ , which is exactly the non-resonant case.

When  $\Delta k = 0$ , the situation corresponds to a single EM wave propagating in the dusty plasma. In this case, the nonlinearity arises from the nonuniform field envelope of the EM wave. Thus, the nonlinear dielectric function can also be used to study the modulational and filamentation instabilities of the EM wave. We note that in the absence of the dust particles,  $\beta = 0$  and the terms containing  $\lambda_d^{-2}$  can be omitted. Thus, we have

$$\epsilon_{jk} = \frac{\epsilon_{jk}^0 T_e}{T_e + T_i} = \frac{e^2}{4\pi m_e \omega^2 (T_e + T_i)} \frac{n_{e0}}{n_c},$$

which is exactly the cubic nonlinearity in a usual two-component plasma [25].

Second, we consider the dust-acoustic resonance in which  $\Delta\omega$  is close to the frequency of the slow dust-acoustic wave [9]. On the time scale of dust dynamics,  $u_d \ll v_\phi \ll u_{i,e}$ , the electrons and ions follow the Boltzmann distributions, but the dust particles are inertial. In this case,  $\chi_\alpha$  as well as  $\chi_{q\alpha}$  are the same as in the previous case, except that  $\chi_d$  is now given by  $\chi_d = -\omega_{pd}^2/(\Delta\omega)^2$ . Note that the dust charge relaxation

rate  $\eta$  is comparable to the ion-acoustic wave frequency which is much higher than the slow dust-acoustic frequency, as  $\eta \gg \Delta\omega$ . Hence, we can write the nonlinear dielectric coefficient in the form

$$\epsilon_{jk} = \epsilon_{jk}^0 \left[ 1 - \frac{1}{R_{jk}} \left( 1 + \frac{\beta}{\eta} - i \frac{\beta \Delta\omega}{\eta^2} \right) \right], \tag{19}$$

where

$$R_{jk} = \frac{\lambda_e^2}{\lambda^2} \left( 1 - \frac{(\Delta k)^2 c_{ds}^2}{(\Delta\omega)^2} \right) + \mu - i\mu \frac{\Delta\omega}{\eta}$$

is a resonance factor,  $\mu = (1 + T_e/T_i)\beta/\eta$  can be considered as a damping factor,  $\lambda = (\lambda_e^{-2} + \lambda_i^{-2})^{-1/2}$ , and  $c_{ds} = \omega_{pd}\lambda/[1 + (\Delta k)^2\lambda^2]^{1/2} \approx \omega_{pd}\lambda$  is the dust-acoustic velocity. It is seen that the resonance occurs at the frequency  $\Delta\omega_{rd} = \Delta kc_{ds}/(1 + \mu\lambda^2/\lambda_e^2)^{1/2}$  which is reduced relative to the dust-acoustic frequency  $\Delta kc_{ds}$ . Since

$$\frac{\mu\lambda^2}{\lambda_e^2} = \frac{\alpha}{\eta} \frac{1 + T_e/T_i}{1 + (T_e/T_i)n_{i0}/n_{e0}},$$

and  $n_{e0}$  and  $n_{i0}$  are of the same order of magnitude, we have  $\mu\lambda^2/\lambda_e^2 \approx \alpha/\eta$ , which is of the order of unity. Thus, the frequency reduction is considerable. The resonant enhancement of the nonlinear dielectric coefficient is about  $|\epsilon_{jk}/\epsilon_{jk}^0| \approx (\alpha + \eta)/\mu\Delta\omega$ . Since  $\eta \gg \Delta\omega$  in this case, there is a significant enhancement in the nonlinear dielectrics.

Next, we consider the ion-acoustic resonance in which  $\Delta\omega$  is close to the frequency of the fast dust ion-acoustic wave [26]. On this time scale,  $u_d \ll u_i \ll v_\phi \ll u_{i,e}$ , the electrons obey the Boltzmann distribution but the ions are inertial. The dust particles, on the other hand, can be regarded as stationary because of their large inertia as compared to the ions. The dust charge, because its relaxation rate is comparable with the ion-wave frequency, is subject to variations. Thus,  $\chi_e$  and  $\chi_{qde}$  remain unchanged, but  $\chi_d = 0$ . The ion susceptibility is given by  $\chi_i = -\omega_{pi}^2/(\Delta\omega)^2$ , where the ion Landau damping is neglected for long wavelength (in comparison with  $\lambda$ ) perturbations. On the other hand,  $\chi_{qdi}$  can be obtained [18] by the kinetic integration of (14) in the limit of  $v_\phi \gg u_i$ . The result is

$$\chi_{qdi} = \frac{i\beta}{\Delta\omega - i\eta} \frac{2}{3} \frac{\omega_{pi}^2}{(\Delta\omega)^2} \frac{n_{e0}}{n_{i0}} A,$$

where  $A = [1 + (1 - eq_{d0}/CT_i)^{-1}]$ . Substituting the expressions for the  $\chi$  into (17), we obtain

$$\epsilon_{jk} = \epsilon_{jk}^0 \left[ 1 - \frac{1}{R_{jk}} \left( 1 - \frac{i\beta}{\Delta\omega - i\eta} \right) \right], \tag{20}$$

where the resonance factor is denoted by

$$R_{jk} = 1 - \frac{(\Delta k)^2 c_s^2}{(\Delta\omega)^2} - \frac{i\gamma_i}{\Delta\omega - i\eta}.$$

Here  $c_s \approx \omega_{pi}\lambda_e \equiv (n_{i0}/n_{e0})^{1/2}c_{s0}$  is the fast dust ion-acoustic velocity [26],  $c_{s0} = (T_e/m_i)^{1/2}$  is the sound speed in a dust-free plasma.  $\gamma_i = \beta[1 - \frac{2}{3}A(\Delta kc_{s0}/\Delta\omega)^2]$  is the damping rate caused by the dust grain charging. Thus, the resonant frequency for this case is

$$\Delta\omega_{ri} = \Delta kc_s \left( 1 - \frac{1}{2} \frac{\gamma_i \eta}{(\Delta k)^2 c_s^2 + \eta^2} \right),$$

which shows a slight frequency down-shift at the resonance. The enhancement of the nonlinear dielectric coefficient is roughly  $[(\Delta k)^2 + \eta^2]/\gamma_i \Delta kc_s$ , which is quite large for small damping rate  $\gamma_i$ . The resonant

feature can be used to determine the ion-acoustic velocity in the dusty plasma, which is proportional to  $(n_{i0}/n_{e0})^{1/2}$ . Thus, it may have applications for diagnosing the ion-to-electron number density ratio or the dust particle concentration.

Finally, we consider high-frequency Langmuir wave resonance in which  $\Delta\omega \sim \omega_{pe}$ . On this fast time scale, the ions and dust particles cannot respond to the high-frequency perturbations and form stationary neutralizing backgrounds. Thus,  $\chi_{i,d} = 0$  as well as  $\chi_{qai} = 0$ . The warm electron susceptibility is given by [27]

$$\chi_e = -\frac{\omega_{pe}^2}{(\Delta\omega)^2} \left( 1 + \frac{3(\Delta k)^2 u_e^2}{(\Delta\omega)^2} \right) - i\chi_L,$$

where

$$\chi_L = \frac{\sqrt{\pi}}{\sqrt{2}(\Delta k)^3 \lambda_e^3} \exp\left(-\frac{3}{2} - \frac{1}{2(\Delta k)^2 \lambda_e^2}\right)$$

arises from the electron Landau damping. The dust charge fluctuation can have a significant effect on the Langmuir wave. In order to demonstrate this, we calculate  $\chi_{qae}$  by the kinetic integration of (14) in the limit of  $v_\phi \gg u_e$  (the Landau effect can be neglected in the integration since  $\omega_{pe} \gg \eta$ ). The result is

$$\chi_{qae} = \frac{i\beta}{\Delta\omega - i\eta} \frac{4}{3} \left( 1 - \frac{eq_{d0}}{2CT_e} \right) \frac{\omega_{pe}^2}{(\Delta\omega)^2}.$$

Hence, the nonlinear dielectric coefficient can be readily obtained as

$$\varepsilon_{jk} = -\varepsilon_{jk}^0 \frac{(\Delta k)^2 u_e^2 [1 + 3(\Delta k)^2 \lambda_e^2 + i\chi_L]}{(\Delta\omega)^2 - \omega_L^2 - \frac{4}{3}\alpha\eta G + i\gamma_e \omega_{pe}^2}, \quad (21)$$

where  $\omega_L^2 = \omega_{pe}^2 + 3(\Delta k)^2 u_e^2$  is the Bohm–Gross frequency squared,  $\gamma_e = \chi_L - \frac{4}{3}(\beta/\omega_{pe})G$  is the damping coefficient, and  $G = 1 - eq_{d0}/2CT_e$ . The resonance occurs when  $(\Delta\omega)^2 = \omega_L^2 + \frac{4}{3}\alpha\eta G$ , which is exactly the electron plasma frequency except for a negligibly small frequency upshift arising from the dust grain charge fluctuation. At the resonance, the enhancement of  $\varepsilon_{jk}$  is approximately  $(\Delta k)^2 \lambda_e^2 / \gamma_e$ , which is directly proportional to the square of the beat wavenumber. Thus, the nonlinearity is more pronounced for short wavelengths of the beat EM waves. Note that since  $q_{d0} < 0$ , we have  $G > 0$ . Thus, the damping  $\gamma_e$  is reduced. Hence, when the Landau damping effect is compensated by the dust charging effect, the enhancement becomes infinity. In this case, other dissipation processes, such as inter-particle collisions, should be taken into consideration and will eventually limit the enhancement.

In conclusion, we have derived a general expression for the nonlinear dielectric function of a dusty plasma in the presence of EM fields, and have shown that the dust charge fluctuations can have a significant influence on the nonlinear dielectrics. It is found that when the frequency difference of the EM waves is close to one of the characteristic frequencies of the dusty plasma system, the nonlinear dielectric function is resonantly enhanced. In addition, it is also observed that there is a large down-shift of the resonance frequency in the slow dust-acoustic wave and a slight down-shift in the fast dust ion-acoustic wave. On the time scale of the electron plasma period, the resonant enhancement can be rather considerable when the Landau damping effect is balanced by the dust charging process. The present results are readily applicable for studying parametric instabilities, viz. stimulated Raman and Brillouin scatterings, the modulational and filamentation instabilities, etc., of electromagnetic waves in dusty plasmas. Finally, we mention that in the present study we have neglected Joule heating [28] and relativistic [29] nonlinearities because the frequency and scale size of the plasma slow motion are assumed to be much larger than the electron collision frequency and the electron skin depth, respectively. The inclusion of these nonlinear effects in our investigation is under consideration.

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