Interlevel electromagnetic response of quantum dots of shapes with uniaxial rotation symmetry

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Within the density response and density matrix formalisms, employing the self-consistent field approach in the quasistatic limit it is considered the interlevel infrared electromagnetic response of individual infinite deep quantum dots (QDs) as well as of two-dimensional infinite square lattices of the identical QDs of different shapes with uniaxial rotation symmetry: semispherical, lens, slightly ellipsoidal, and cylindrical. It is shown that the QD shape can critically affect the interlevel optical properties of the considered QD systems, in particular, the dependence of the QD polarizability upon the polarization of the incident radiation. To facilitate consideration of the dipole coupled interlevel transitions and of the formation of (interacting) modes of the electron interlevel collective excitation, and calculation of the absorption spectra for the QDs, illustrative maps of the transitions are utilized and the concept of the "*m* family" for the eigenstates is found to be fruitful, where *m* is the magnetic quantum number labeling the eigenstates of the QDs. It is shown that the shape and size of QDs can impact the depolarization effect considerably. Numerical calculations show that the dipole-dipole interaction approximation very well describes the depolarization shift caused by the interdot electron-electron interaction for each of the considered QD shapes and for any reasonable values of the size parameters of the lattices.

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I. INTRODUCTION

Quantum dots (QDs) have attracted much theoretical, experimental, and practical interest as promising objects for various aspects of infrared optoelectronics.[1] Electromagnetic response of systems of QDs has been investigated theoretically and experimentally. We restrict our interest to the effect of the spatial confinement of the electrons in QDs on the interlevel electromagnetic response of the QDs systems. The shape and size parameters of a QD are the important factors shaping the QD features. Determination of the shape of manufactured QDs is an important and difficult task (see, e.g., Refs. 2 and 3). Although in earlier theoretical works the confining potential was mainly considered as parabolic, which was appropriate for the QDs with many electrons and simplified the calculations, later much attention was paid to the confining potential determined by the QD shape. As examples, the spherical QDs (SQDs),[4–8] semispherical QDs (SSQDs),[4,9] ellipsoidal QDs (EQDs),[8,10,11] lens QDs (LQDs),[8,12,13] conical QDs,[14] and[8] piramidal QDs were considered in recent literature. It is interesting that QDs of even the same shape were treated by means of different approaches. One can see that to simplify calculations the symmetric shapes of QDs are frequently used, though even in such cases the three dimensionality of QDs creates serious difficulty for calculations. On the other hand, the symmetric shapes give rise to additional specific properties of QDs which can be attractive for applications in nanooptoelectronics. It is important to mention the parabolic confining potential. It was shown that the far-infrared response of the quantum system with the parabolic confining potential is not affected by the electron-electron (*e-e*) interaction (see, e.g., Refs. 15–17), which is known as the generalized Kohn theorem. Thus, to observe the nonzero effect of the *e-e* interaction on the response one should work beyond the framework of the parabolic confining potential in QDs.

This work is an extension to different QD systems of our previous work[7] where we investigated the interlevel electromagnetic response of an individual spherical QD as well as of a SQD located in a periodic two-dimensional lattice of interacting SQDs within the framework of the density response and density matrix formalisms, and self-consistent field approach.[18] In the calculations the lattice was taken as infinite. We assumed the electromagnetic field to be uniformed in the QD system and interaction of electrons with incident radiation was considered in the dipole approximation, while the retardation effects were omitted. The full attention was paid to the electronic structure of the QD systems. Static and dynamic intradot and interdot *e-e* interaction were considered. A many-electron QD and a lattice of QDs were treated as many-electron systems where the *e-e* interaction caused formation of the collective interlevel electron excitation when electrons transited between energy levels in QDs due to absorption and emission of photons. Note that it is the interlevel collective excitation caused by the dynamic *e-e* interaction that constitutes the depolarization effect (DE) in a many particle system, and it is the energy of the mode(s) of the interlevel collective excitation that determines the depolarization shift of the peak(s) on absorption spectra. In Ref. 7 the DE was considered in detail and the depolarization shift was calculated as for many-electron single SQD as for the SQD lattices. It was shown that the size parameters, namely the SQD radius and the lattice period, the number of electrons per dot, and the polarization of the incident radiation affect considerably the absorption spectra, in part, the depolarization shift.

In Ref. 7 it was shown that the absorption spectra in the many-electron SQD systems could become multipeak which represented the interacting modes of the electron interlevel

collective excitations depending upon the number of electrons occupying a few states in the SQD. The modified oscillator strength (MOS) approach was shown to drastically simplify the calculations and enhance analysis of such complicated spectra. The MOS approach becomes very useful for QDs of symmetrical and near symmetrical shapes when a few types of interlevel transitions with close interlevel energies can be provided to occur. Thus, knowledge of the eigenstates and of the allowed dipole coupled electron interlevel transitions for a QD system is of good help for applying the MOS approach. It is worth noting a special case of strongly interacting modes of the electron collective excitation which could take place for (near) symmetric shape of QDs. Then the shape of the absorption spectra is not obvious but can be quite unexpected, like only one actual peak for 8 electrons per SQD as it is in Fig. 6 in Ref. 7. Thus, a detail analysis is required and the MOS approach becomes very helpful.

Also in Ref. 7 it was found that the contributions to the depolarization shift from the interdot and intradot dynamic *e-e* interaction in the infinite lattice of SQDs can be treated completely separately within the dipole-dipole interaction approximation (DDA). It was shown that the contribution of the interdot dynamic *e-e* interaction can be very simply and accurately calculated within the DDA when the SQDs were considered as point dipoles. It should be noted that the dipoles were found to be the hard bare dipoles calculated for the isolated SQD. This result is very important if the *e-e* interaction in the QD systems is of interest. It should be noted that the DDA has been used to treat the optical phenomena, in particular nonlinear ones, in arrays of interacting atoms and metallic nanoparticles (see, e.g., Refs. 19–21). We show that systems of interacting QDs can also be treated within the DDA. As was shown in Ref. 7 it is the problem of the electron self-interaction whose solution let us prove the validity of the DDA for the QDs. In this work we show numerically that the way of solving the problem of the electron self-interaction proposed in Ref. 7 works well for all the considered shapes of QDs, and we use the self-interaction problem as an advantage.

The general analytical and numerical results of Ref. 7 allow and inspire us to apply them to other systems of QDs. In this work we are going to investigate the interlevel electronagnetic response of systems of quantum dots of different shapes with uniaxial rotation symmetry. Namely, we shall consider optically active interlevel electron transitions and the effect of the direct dynamic *e-e* interaction on the optical spectra in the SSQDs, LQDs, slightly ellipsoidal QDs (sEQDs), and cylindrical QDs (CQDs). The QDs are considered to be infinite deep with one electron per dot within the effective mass approximation in the one-band limit. It is the dynamic *e-e* interaction whose effect on the response of systems of QDs of different shapes is in the focus of the present paper. To our best knowledge this problem has not been discussed in the literature yet. In particular, we want to answer the question: Does the DDA represent the interdot dynamic *e-e* interaction for different QD shapes as good as it does for the SQDs?

II. INTERLEVEL OPTICAL RESPONSE OF QUANTUM DOT SYSTEMS

The goal of this work is to calculate the linear interlevel electron response of the QD systems to the external incident electromagnetic field which is assumed to be homogeneous on the scale of the considered QD systems and taken in the form $\mathbf{E}^{\mathrm{ext}}(t)=\mathbf{e}_j\tilde{E}_j(\omega)e^{-i\omega t}$, where ω is the frequency. Here $\mathbf{e}_j$ $(j=1,2,3)$ form the set of basis vectors of the Cartesian coordinate system, with $\mathbf{e}_3$ is chosen along the axis of the rotation symmetry of the QD, and $\mathbf{e}_1$ and $\mathbf{e}_2$ lie in the plane of the QD basis. The origin of the coordinate systems is chosen at the center of the basis of the QDs. The three-dimensional radius-vector is $\mathbf{r}=(\mathbf{r}_\parallel,r_3)$, with $\mathbf{r}_\parallel=(r_1,r_2)$ being a two-dimensional vector in the QD basis. The angle between $\mathbf{r}_\parallel$ and $\mathbf{e}_1$ in the plane $\mathbf{e}_1$-$\mathbf{e}_2$ of the QD basis is denoted as φ, with $0<\varphi<2\pi$. Due to the uniaxial rotation symmetry the QD eigenstates are labeled by the magnetic quantum number m, with $m=0,\pm1,\pm2,\ldots$. The radius of the QD basis (which is a circle) is denoted as a. The incident radiation is considered to be polarized in the plane of the QD basis (the in-plane polarization), and in the direction normal to the basis (the normal polarization). The dielectric constant is taken as $\epsilon=13.18$ (GaAs). The effective mass of the electrons in the QDs is m^*.

In the following an eigenstate of the whole QD system is denoted by ν. Within the density-matrix formalism the linear electromagnetic response of each considered QD system is described by the complex tensor of the linear polarizability given by

$$\bar{\alpha}_{jj}(\omega)=\frac{-e}{\tilde{E}_j(\omega)}\sum_{\nu,\nu'}\rho^{(1,j)}_{\nu,\nu'}(\omega)(r_j)_{\nu',\nu}, \tag{1}$$

where $\rho^{(1,j)}_{\nu,\nu'}(\omega)$ is the matrix element of the jth component of the first harmonic of the density matrix operator, $(r_j)_{\nu',\nu}$ is the matrix element of the jth component of the radius-vector $\mathbf{r}$, and $-e$ is the electron charge.

The density matrix elements $\rho^{(1,j)}_{\nu,\nu'}(\omega)$ are calculated by self-consistent solving the master equation for $\rho^{(j)}_{\nu,\nu'}(t)$

$$\frac{\partial\rho^{(j)}_{\nu,\nu'}(t)}{\partial t}=\frac{1}{i\hbar}[H_0+V^{(j)}(t),\rho(t)]_{\nu,\nu'}-\frac{[\rho^{(j)}(t)-\rho^{(0)}]_{\nu,\nu'}}{\tau_{\nu,\nu'}}, \tag{2}$$

where H_0 is the Hamiltonian describing the electrons in the system in absence of the external field, $V^{(j)}(t)$ $[=V^{(j)}(\omega)e^{-i\omega t}]$ is the perturbing potential depending upon the density matrix $\rho^{(1,j)}_{\nu,\nu'}(\omega)$, $\rho^{(0)}$ is the equilibrium value of the density matrix operator, and $\tau_{\nu,\nu'}=\hbar/\Gamma_{\nu,\nu'}$ stands for phenomenological relaxation times. We do not consider scattering processes of one electron in a QD, but only assume a rather small damping that verifies using the phenomenological relaxation time (see, e.g., Refs. 22 and 23). For simplicity we assume that the broadening parameters $\Gamma_{\nu\nu'}$ are independent of a state index, that is $\Gamma_{\nu\nu'}=\Gamma$. The spatial dependence in Eq. (2) is omitted.

An eigenstate of an individual QD is labeled by a composite index $\mathbf{a}$, which contains the whole set of the quantum numbers describing the eigenstate, so that $E_{\mathbf{a}}$ stands for the eigenenergy, $\Psi_{\mathbf{a}}$ stands for the eigenfunction, and $n_{\mathbf{a}}$ stands for the number of the electrons at the state per QD at the thermal equilibrium. As it was shown in Ref. 7 the dynamic intradot *e-e* interaction in an individual QD for allowed transitions $\mathbf{a} \rightleftarrows \mathbf{a}'$ and $\mathbf{b} \rightleftarrows \mathbf{b}'$ is represented by [see Eq. (11) in Ref. 7]

$$L^{(j)}(\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}') = \frac{e^2}{8\pi^2\epsilon\epsilon_0}\int d\mathbf{Q}_{\parallel}\frac{1}{Q_{\parallel}}\int dr_3 dr_3' e^{-Q_{\parallel}|r_3-r_3'|} \times A^{-}_{\mathbf{a},\mathbf{a}'}(\mathbf{Q}_{\parallel},r_3)A^{+}_{\mathbf{b}',\mathbf{b}}(\mathbf{Q}_{\parallel},r_3'), \quad (3)$$

where

$$A^{\pm}_{\mathbf{c},\mathbf{c}'}(\mathbf{Q}_{\parallel},r_3) = \int d\mathbf{r}_{\parallel}\Psi^{*}_{\mathbf{c}}(\mathbf{r}_{\parallel},r_3)e^{\pm i\mathbf{Q}_{\parallel}\mathbf{r}_{\parallel}}\Psi_{\mathbf{c}'}(\mathbf{r}_{\parallel},r_3), \quad (4)$$

with $\mathbf{Q}_{\parallel}=(Q_1,Q_2)$ is a two-dimensional vector in the $\mathbf{e}_1$-$\mathbf{e}_2$ plane. To take into account the anisotropy of QD shape, the superscript (j) is introduced into Eq. (3) compared to Eq. (11) in Ref. 7. ϵ is the dielectric constants of the QD. Note that we neglect any effects associated with difference between the dielectric constants of a QD and its surrounding.

An eigenstate of the two-dimensional lattice of identical nontunneling QDs is as $\nu=(\mathbf{a},\mathbf{k}_{\parallel}^{\mathbf{a}})$ where $\mathbf{k}_{\parallel}^{\mathbf{a}}=(k_1^{\mathbf{a}},k_2^{\mathbf{a}})$ is a two-dimensional wave vector of an electron at state ν. The square lattice in the $\mathbf{e}_1$-$\mathbf{e}_2$ plane with period d is described by the two-dimensional reciprocal lattice vector $\mathbf{G}_{\mathbf{m}_{\parallel}}[=(m_1 2\pi/d, m_2 2\pi/d)]$ where $\mathbf{m}_{\parallel}=(m_1,m_2)$, with m_1 and m_2 take values $0,\pm 1,\pm 2,\ldots$. The dynamic *e-e* interaction in the square lattice of QDs for allowed transitions $\mathbf{a} \rightleftarrows \mathbf{a}'$ and $\mathbf{b} \rightleftarrows \mathbf{b}'$ is represented by [Eq. (26) in Ref. 7]

$$\beta^{(j)}(\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}') = \frac{e^2}{2\epsilon\epsilon_0 d^2}\sum_{\mathbf{G}_{\mathbf{m}_{\parallel}}}\frac{1}{G_{\mathbf{m}_{\parallel}}}\int dr_3 dr_3' e^{-G_{m_{\parallel}}|r_3-r_3'|} \times F^{(j)-}_{\mathbf{a},\mathbf{a}'}(\mathbf{G}_{\mathbf{m}_{\parallel}},r_3)F^{(j)+}_{\mathbf{b}',\mathbf{b}}(\mathbf{G}_{\mathbf{m}_{\parallel}},r_3'), \quad (5)$$

where $G_{\mathbf{m}_{\parallel}}=|\mathbf{G}_{\mathbf{m}_{\parallel}}|$, and

$$F^{(j)\pm}_{\mathbf{c}',\mathbf{c}}(\mathbf{G}_{\mathbf{m}_{\parallel}},r_3) = \int d\mathbf{r}_{\parallel}\Psi^{*}_{\mathbf{c}'}(\mathbf{r}_{\parallel},r_3)e^{\pm i\mathbf{G}_{\mathbf{m}_{\parallel}}\mathbf{r}_{\parallel}}\Psi_{\mathbf{c}}(\mathbf{r}_{\parallel},r_3). \quad (6)$$

Importantly to note that $L^{(j)}(\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}')$ is the limit of $\beta^{(j)}(\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}')$ at $d\to\infty$ which let us solve the problem of the electron self-interaction in QDs.

The depolarization shift is represented by $\bar{L}^{(j)}_{\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}'}$ and $\bar{\beta}^{(j)}_{\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}'}$ which are obtained from $L^{(j)}(\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}')$ and $\beta^{(j)}(\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}')$ by taking into account the map of interstate transitions, the number of electrons in QDs, and the self-interaction problem. For the two-state and four-state electron systems in SQDs the relationships for $\bar{L}^{(j)}_{\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}'}$ and $\bar{\beta}^{(j)}_{\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}'}$ are presented by Eqs. (13) and (28), respectively, in Ref. 7. However, principally different multistate electron systems considered in this work require adequate relationships as one will see further [see Eqs. (8), (9), and (10)].

We would like to especially emphasize an important point. In this paper every QD is considered to have only *one* electron. It sounds as if we restrict ourselves to the *interdot* dynamic *e-e* interaction only. However, it is the inevitable problem of the electron self-interaction that helps us make valuable estimation of the *intradot* dynamic *e-e* interaction in QDs. As a matter of fact, a *calculated* value of the depolarization shift caused by the dynamic *e-e* interaction in a QD lattice with *one* electron per QD consists of two contributions: (i) from the pure *interdot* interaction, and (ii) from pure *intradot* interaction of one electron with itself. The latter contribution from the electron self-interaction should be excluded to obtain the correct value of the depolarization shift caused by the interdot interaction. However, it is the self-interaction contribution that let us evaluate the magnitude of the depolarization shift caused by the intradot interaction of two real electrons in a QD even without accurate consideration of much more complicated cases of QD lattices with two electrons per QD. Thus, we actually use this advantage of the problem of self-interaction in this paper.

Also it is important to note that we restrict ourselves to situations when one electron in a QD occupies the lowest energy level in the QD at the thermal equilibrium and the electron can transit from the lowest level to the first excited level only due to photon absorption. Importantly that we do not consider any mechanisms of loading electrons into QDs. Otherwise the situation would become specific for methods of manufacturing and exploiting QD systems when QD environment would be necessary to define the Fermi level of the whole system. We just assume that each QD is loaded with one electron. It is also important to emphasize that it is the number of electrons per QD, n, that is shown[7] to be the only *independent* parameter representing electron population in the whole QD system in the analytical relationships describing the electromagnetic response of the QD systems. For example, the sheet density of electrons in a two-dimensional system of QDs cannot itself represent the effect of the electron population on the response since the effect of the *e-e* interaction depends crucially upon size parameters of the system, shape of QDs, and n itself, at the same sheet density. Note that although n is obviously an integer (0,1, 2,…) it could be a slip of calculations when instead of n some other quantities are used, but the value of n, being an intermediate quantity, is not checked out, so that noninteger values of n could be used in calculations unintentionally.

To calculate $\bar{L}^{(j)}_{\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}'}$ and $\bar{\beta}^{(j)}_{\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}'}$ and all the matrix elements we need to know the QD eigenstates. Determining the eigenstates is our starting point in considering QDs of different shapes.

A. Semispherical quantum dots

An eigenstate of a single SSQD is labeled as $\mathbf{a}=(n,l,m)$, where n, l, and m are the main, azimuthal, and magnetic quantum numbers, respectively. The complete orthonormal set of the eigenfunctions of the infinitely deep SSQD with one electron takes the form[9]

$$\Psi^{\mathrm{SSQD}}_{n,l,m}(r,\theta,\varphi) = \frac{J_{l+1/2}(\alpha_n^{(l)}r/a)}{D_{n,l}\sqrt{r}}\frac{P_l^{|m|}(\cos\theta)}{C_{l,m}}\frac{e^{im\varphi}}{\sqrt{2\pi}}, \quad (7)$$

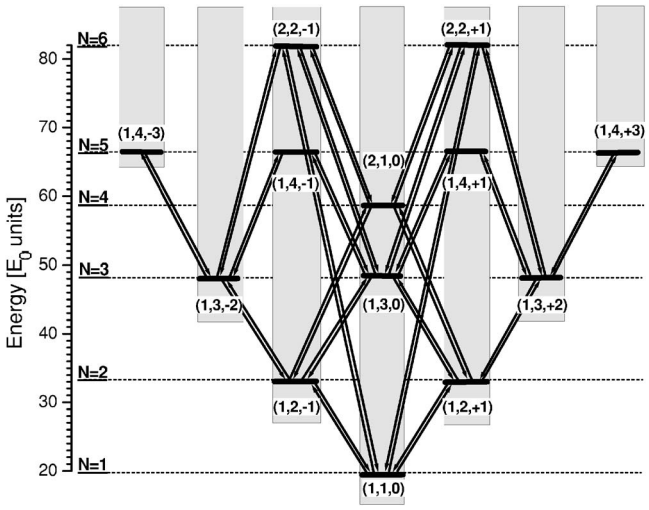

FIG. 1. Energy levels calculated for the infinite deep semispherical QD with one electron. The eigenstates are labeled by (n,l,m). The levels are numbered by N. The allowed dipole coupled interlevel transitions for the in-plane radiation polarization are shown by lines with arrows. The m families of the SSQD eigenstates are emphasized by shadowed columns.

with $[l-|m|]$ is *odd*. We want to emphasize that this link between l and m determines many features of the SSQD and distinguishes the semispherical shape among other QD shapes. Here $r=|\mathbf{r}|$, with $\mathbf{r}$ is the three-dimensional radius-vector and $0\leqslant r\leqslant a$. θ is the angle between $\mathbf{r}$ and the rotation axis, and $0\leqslant\theta\leqslant\pi/2$. $J_{l+1/2}(x)$ is the spherical Bessel function, which has the nth root $\alpha_n^{(l)}$ and is normalized by $D_{n,l}$. $P_l^{|m|}(\cos\theta)$ is the associate Legendre function normalized by $C_{l,m}$.

The eigenenergy of one electron in the infinite deep SSQD is determined by $E_{n,l}=(\alpha_n^{(l)})^2E_0$, where $E_0=\hbar^2/2m^*a^2$. We shall use the term "energy level" for all l states which share the same energy $E_{n,l}$ (whose degeneracy is, thus, l).

Figure 1 presents six lowest values of $E_{n,l}/E_0$ which are numbered in increasing order by an integer N $(=1,2,\ldots,6)$, so that there is the relation $N=N_n^l$. (Note also that the values of $E_{n,l}$ were presented in Fig. 2 in Ref. 12 at $b/a=1$.) It is important to emphasize that the introduced above integer N becomes the main quantum number which the authors of Ref. 12 used to label the eigenenergies of the LQD, see Fig. 2 there. It is convenient for further discussion to assign all the eigenstates with the same m to the "m family" of the eigenstates. In Fig. 1 the "m families" are shown by the shadowed columns.

Direct calculation shows that the dipole matrix element for the electron interlevel transition in the SSQD equals zero for the normal polarization of the incident radiation. The reason for that is that to absorb a photon of the normal polarization an electron would be supposed to transit from an existing, initial, state to another, final, state so that l would change by 1, while m would be unchanged. However, due to the requirement on $(l-|m|)$ to be odd, there is no such final state. Thus, there are no dipole coupled states in the infinite deep SSQD to absorb the normal polarized incident radiation. In the following physical quantities calculated for the in-plane light polarization will bear the index "$\|$."

For the in-plane radiation polarization the matrix element of r_1 is as $(r_1)_{(n,l,m)(n',l',m')}=aS_{n,l,m}^{n',l',m'}\delta_{l',l\pm1}\delta_{m',m\pm1}$. (An explicit expression for $S_{n,l,m}^{n',l',m'}$ can be found in Ref. 9.) That means that in Fig. 1 the transitions are allowed only between the dipole coupled states from the neighboring m families only, while no transition is allowed between states from the same m family or remote m families. This is a consequence of $\delta_{m',m\pm1}$ for the SSQD.

In Fig. 1 we present schematically the map of the allowed dipole coupled interlevel transitions of an electron absorbing a photon of the in-plane polarization in the infinite deep SSQD. In the figure one can see that even for one electron transiting between two lowest energy levels there is a three-state electron system with a variety of possible transitions. If electrons occupy the first excited level $(E_{1,2})$ the situation becomes much more complicated. This makes the calculations of the response of the SSQD more complicated than that of the SQD even for one electron per SSQD. Note that for the complicated electron systems the application of the DDA and the MOS approximation becomes very profitable.

The calculations show that the dynamic e-e interaction in the two-dimensional square lattice of the infinite deep SSQDs with one electron per dot is represented by such $\beta^{\|}(\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}')$ that if $\Delta m=+1(-1)$ for $\mathbf{a}\to\mathbf{a}'$ then $\Delta m=-1(+1)$ for $\mathbf{b}\to\mathbf{b}'$. Further $\beta^{\|}(\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}')$ will be labeled by two superscripts (out of "+" and "−") which represent the sign of Δm for $\mathbf{a}\to\mathbf{a}'$ (the first superscript) and for $\mathbf{b}\to\mathbf{b}'$ (the second superscript), e.g., $\beta^{\|-+}(\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}')$. Numerical calculations show that $\beta^{\|--}(\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}')=\beta^{\|++}(\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}')\equiv0$ which results in the following relations: $\beta^{\|}(\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}')=\beta^{\|}(\mathbf{a}',\mathbf{a};\mathbf{b}',\mathbf{b})$ and if $\beta^{\|}(\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}')\neq0$ then $\beta^{\|}(\mathbf{a},\mathbf{a}';\mathbf{b}',\mathbf{b})=0$ and $\beta^{\|}(\mathbf{a}',\mathbf{a};\mathbf{b},\mathbf{b}')=0$. Figure 1 shows that for one electron at the ground state per SSQD the depolarization shift is determined by $\beta^{\|-+}(=\beta^{\|+-})$, which is $\beta^{\|}(\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}')$ for $\mathbf{a}=(1,1,0)$, $\mathbf{a}'=(1,2,+1)$, $\mathbf{b}=(1,1,0)$, and $\mathbf{b}'=(1,2,-1)$ [or $\mathbf{a}=(1,1,0)$, $\mathbf{a}'=(1,2,+1)$, $\mathbf{b}=(1,2,+1)$, and $\mathbf{b}'=(1,1,0)$]. The same relations are valid for $L^{\|}(\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}')$ which will be denoted as $L^{\|-+}$. Note that the above relations are valid because the considered three-state system is symmetric and $|(r_1)_{\mathbf{a},\mathbf{a}'}|=|(r_1)_{\mathbf{b},\mathbf{b}'}|$.

1. Relations for $\bar{L}^{\|-+}$ and $\bar{\beta}^{\|-+}$ for the symmetric three-state system

The above consideration allows us to write down the relationships for $\bar{L}^{\|}_{\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}'}$ and $\bar{\beta}^{\|}_{\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}'}$, which will be denoted as $\bar{L}^{\|-+}$ and $\bar{\beta}^{\|-+}$, respectively, for the symmetric *three-state* electron system. First, we assume the ground state to be $\mathbf{a}$ which is the same as $\mathbf{b}$ (i.e., $\mathbf{a}=\mathbf{b}$), so that $n_{\mathbf{a}}(>0)$ electrons occupy the ground state at the thermal equilibrium. Note that in general $n_{\mathbf{a}}=1$ or $n_{\mathbf{a}}=2$. An electron at the ground state $\mathbf{a}$ can transit to the excited state $\mathbf{a}'$ or $\mathbf{b}'$, and, thus, there is a three-state system: $\mathbf{a}(=\mathbf{b})$, $\mathbf{a}'$, and $\mathbf{b}'$. Let $n_{\mathbf{cc}'}$ be the number of electrons per dot participating in $\mathbf{c}\to\mathbf{c}'$ transitions. For the symmetric three-state system $n_{\mathbf{a},\mathbf{a}'}=n_{\mathbf{b},\mathbf{b}'}=n_{\mathbf{a}}/2$. Al-

though in this work we perform calculations for $n_{\mathbf{a}}=1$ only, it is possible and important to present the general relationships for $\bar{L}^{\parallel-+}$ and $\bar{\beta}^{\parallel-+}$ for the three-state system with $n_{\mathbf{a}}=1,2$. However, it should be noted that for $n_{\mathbf{a}}=2$ the eigenstates and, thus, values of $L^{\parallel-+}$ and $\beta^{\parallel-+}$ are different from those for $n_{\mathbf{a}}=1$ (when $\mathbf{a}(=\mathbf{b})$, $\mathbf{a}'$, and $\mathbf{b}'$ are the eigenstates of one electron in the QD). The relationships can be written as follows:

$$\bar{L}^{\parallel-+}=L^{\parallel-+}[n_{\mathbf{a}}-1], \tag{8}$$

and

$$\bar{\beta}^{\parallel-+}=[\beta^{\parallel-+}-L^{\parallel-+}]n_{\mathbf{a}}/2. \tag{9}$$

When $n_{\mathbf{a}}=1$, then $n_{\mathbf{a},\mathbf{a}'}=1/2$, $\bar{\beta}^{\parallel-+}$ represents the depolarization shift due to merely interdot dynamic *e-e* interaction of the electrons in a lattice of QDs with one electron per dot, and $\bar{L}^{\parallel-+}=0$. However, $L^{\parallel-+}\neq 0$ represents the *calculated* depolarization shift due to intradot interaction of one electron with itself. The value of $L^{\parallel-+}$ is twofold. From the one side, $L^{\parallel-+}$ serves to set the correct zero values for $\bar{L}^{\parallel-+}$ and $\bar{\beta}^{\parallel-+}$ in Eqs. (8) and (9). On the other side, $L^{\parallel-+}$ equals $\bar{L}^{\parallel-+}$ for $n_{\mathbf{a}}=2$, with the eigenstates are different for $n_{\mathbf{a}}=2$ and $n_{\mathbf{a}}=1$. Thus, the value of $L^{\parallel-+}$, calculated on the eigenstates for $n_{\mathbf{a}}=1$, can serve as an estimate value for $\bar{L}^{\parallel-+}$for real two interacting electrons in the dot (which should be calculated on the eigenstates for $n_{\mathbf{a}}=2$). It is worth mentioning the presentation of the electron occupation in Eqs. (8) and (9): for the symmetric three-state system it is sufficient to use $n_{\mathbf{a}}$ only.

Finally we come to the relationship for $\bar{\beta}^{\parallel-+}$ within the DDA for the symmetric three-state system

$$\bar{\beta}^{\parallel-+}=\bar{L}^{\parallel-+}-|\mu^{\parallel}_{\mathbf{a},\mathbf{a}'}|^2S^{\parallel}n_{\mathbf{a}}/2, \tag{10}$$

where $\mu^{\parallel}_{\mathbf{a}',\mathbf{a}}=-e(r_{\parallel})_{\mathbf{a}',\mathbf{a}}$ is the matrix element of the electron dipole moment. For the infinite square lattice the configuration factor $S^{\parallel}=-\xi_0/(2d^3)$, with $\xi_0=-9.0336$. Note that the anisotropy of $\bar{\beta}^{\parallel-+}$ in Eq. (10) is taken into account not only by the configuration factor $S^{\parallel}$, as in Eqs. (48) and (49) in Ref. 7, but also by $\bar{L}^{\parallel-+}$ and $\mu^{\parallel}_{\mathbf{a},\mathbf{a}'}$ representing the anisotropy of the QDs shape. It should be noted that Eqs. (8)–(10) cover the case of symmetric three-state system, which is realized in the infinite deep SSQD. Asymmetric three-state systems with $(r_j)_{\mathbf{a},\mathbf{a}'}\neq(r_j)_{\mathbf{b},\mathbf{b}'}$ and $n_{\mathbf{a},\mathbf{a}'}\neq n_{\mathbf{b},\mathbf{b}'}$ require special considerations.

For the infinite deep SSQDs with one electron per dot we have the following relationships:

$$\bar{\beta}^{\parallel-+}=\frac{\tilde{\beta}^{\parallel-+}(a/d)}{d}, \tag{11}$$

where $\tilde{\beta}^{\parallel-+}(a/d)$ is a function of the ratio a/d, which, within the DDA by Eq. (10), becomes proportional to $(a/d)^2$, and

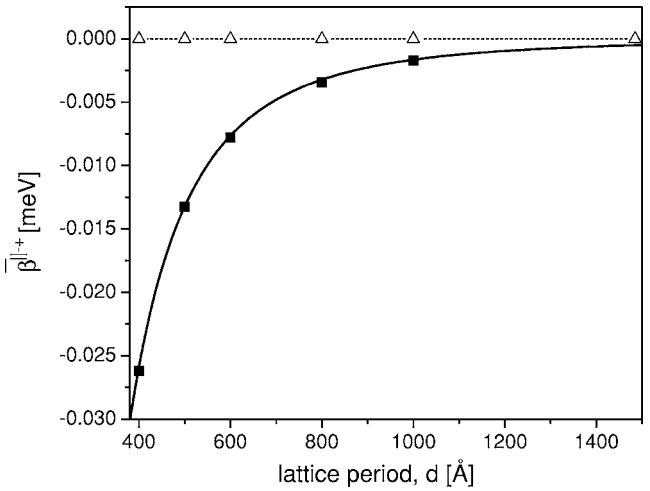

FIG. 2. Dependence of $\bar{\beta}^{\parallel-+}$ upon the period of the square two-dimensional infinite lattice of the semispherical QDs calculated for one electron per dot transiting between the ground and the first excited levels. $a=95$ Å. The zero level is labeled by the symbols in order to emphasize that the zero is the calculated reference level $L^{\parallel-+}=5.7779$ meV.

$$L^{\parallel-+}=\frac{\tilde{L}^{\parallel-+}}{a}, \tag{12}$$

where $\tilde{L}^{\parallel-+}=$const. The values of $\tilde{\beta}^{\parallel-+}(a/d)$ and $\tilde{L}^{\parallel-+}$, multiplied by ϵ, are valid for any infinite deep SSQDs.

In Fig. 2 by the symbols the values of $\bar{\beta}^{\parallel-+}$ for the two-dimensional square lattice of the infinite deep SSQDs with one electron per dot at the ground level (within the three-state system) are plotted as a function of the lattice period. Note that the limit of $\beta^{\parallel-+}$ at $d\to\infty$ is $L^{\parallel-+}=5.7779$ meV. The value of $L^{\parallel-+}$ represents the self-interaction of one electron with itself in the SSQD, and is the "calculated zero" in Fig. 2. The solid lines in Fig. 2 represent the DDA. One can see that the DDA very well represents the interdot dynamic *e-e* interaction for any lattice period. Note that the radius $a=95$ Å for the SSQDs, and the LQDs in the following section, is chosen just to make easy the comparison of the results for the SSQDs, the LQDs, and the results for 95-Å-SQDs presented in Ref. 7, with ϵ being also the same. For the two-state system in infinite deep 95 Å SQD with one electron at the ground state the value of $L(\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}')$ due to the self-interaction is 4.1295 meV. Thus, one can expect the depolarization shift due to the intradot dynamic *e-e* interaction in the SSQD to be bigger than that in the SQD with the same a.

2. Polarizability of the symmetric three-state system

For the in-plane light polarization the complex polarizability describing the symmetric three-state electron system in QDs, when all possible transitions are taken into account, is given by

$$\alpha_{\parallel}(\omega)=\frac{2|\mu^{\parallel}_{\mathbf{a}',\mathbf{a}}|(2n_{\mathbf{a},\mathbf{a}'})E_{\mathbf{a}',\mathbf{a}}}{(\tilde{E}^{\parallel}_{\mathbf{a}',\mathbf{a}})^2-(\hbar\omega)^2-i2\hbar\omega\Gamma}, \tag{13}$$

where $E_{\mathbf{a}',\mathbf{a}}=E_{\mathbf{a}'}-E_{\mathbf{a}}$ and

$$(\tilde{E}^{\parallel}_{\mathbf{a}',\mathbf{a}})^2 = (E_{\mathbf{a}',\mathbf{a}})^2 + 2E_{\mathbf{a}',\mathbf{a}}\bar{\beta}^{\parallel+-}. \tag{14}$$

In Eq. (13) Γ^2 is omitted in the denominator since $\Gamma^2 \ll (\tilde{E}^{\parallel}_{\mathbf{a}',\mathbf{a}})^2$. Note that Eqs. (13) and (14) are applied to a single QD as well as to a QD in the lattice of interacting QDs.

The imaginary part of $\alpha_{\parallel}(\omega)$ can be written in the form

$$\alpha''_{\parallel}(\omega) = \frac{4|\mu^{\parallel}_{\mathbf{a}',\mathbf{a}}|^2(2n_{\mathbf{a},\mathbf{a}'})E_{\mathbf{a}',\mathbf{a}}\hbar\omega\Gamma}{[(\tilde{E}^{\parallel}_{\mathbf{a}',\mathbf{a}})^2 - (\hbar\omega)^2]^2 + [2\hbar\omega\Gamma]^2}. \tag{15}$$

When only the resonant transitions are taken into account (the resonant approximation) and $|\bar{\beta}^{\parallel+-}| \ll E_{\mathbf{a}',\mathbf{a}}$ the above equation simplifies to the form

$$\alpha''_{\parallel}(\omega) = \frac{|\mu^{\parallel}_{\mathbf{a}',\mathbf{a}}|^2(2n_{\mathbf{a},\mathbf{a}'})\Gamma}{[\bar{E}^{\parallel}_{\mathbf{a}',\mathbf{a}} - \hbar\omega]^2 + \Gamma^2}, \tag{16}$$

where $\bar{E}^{\parallel}_{\mathbf{a}',\mathbf{a}} = E_{\mathbf{a}',\mathbf{a}} + \bar{\beta}^{\parallel+-}$.

It should be noted that $\tilde{E}^{\parallel}_{\mathbf{a}',\mathbf{a}} < E_{\mathbf{a}',\mathbf{a}}$ and $\bar{E}^{\parallel}_{\mathbf{a}',\mathbf{a}} < E_{\mathbf{a}',\mathbf{a}}$ for one electron at the ground state per dot, i.e., when $n_{\mathbf{a},\mathbf{a}'} = n_{\mathbf{a}}/2 = 1/2$. In this case the DE is determined by the inter-dot dynamic *e-e* interaction in the lattice only, so that the depolarization shift is negative and relatively small. However, $\tilde{E}^{(j)}_{\mathbf{a}',\mathbf{a}} > E_{\mathbf{a}',\mathbf{a}}$ and $\bar{E}^{(j)}_{\mathbf{a}',\mathbf{a}} > E_{\mathbf{a}',\mathbf{a}}$ in case of two electrons per dot with normal occupation of the levels ($n_{\mathbf{a},\mathbf{a}'} > 0$). In this case the intradot dynamic *e-e* interaction makes the major contribution to the DE so that the depolarization shift becomes positive and relatively big.

It is interesting to compare the obtained expressions for the polarizability of the three-state electron system with those for the two-state electron system [see Eqs. (30)–(33) in Ref. 7]. If we take into account that $2n_{\mathbf{a},\mathbf{a}'} = n_{\mathbf{a}}$ for the three-state system and $n_{\mathbf{a},\mathbf{a}'} = n_{\mathbf{a}}$ for the two-state system then the polarizability takes the same form for the both cases if $n_{\mathbf{a}}$ is used instead of $n_{\mathbf{a},\mathbf{a}'}$. However, the difference between the two cases is that the depolarization shift value is proportional to $n_{\mathbf{a}}/2$ for the three-state system and to $n_{\mathbf{a}}$ for the two-state system.

B. Lens quantum dots

We use the eigenstates of the infinite deep LQD which were obtained in Ref. 12, where the lens shape was formed by deformation of the semisphere shape, which allowed the authors to find the eigenstates of the LQD from the eigenstates of the SSQD by applying the perturbation theory.

Let the height of the LQD be b. Then the ratio b/a (<1) defines the deviation of the lens shape from the semisphere shape where $b/a=1$. To represent the deviation in Ref. 12 a small parameter, $\lambda = \arctan(b/a)/(\pi/4) - 1$, was used, with $\lambda = 0$ for the semisphere. It is supposed that the lens shape differs from the semisphere shape not too much.

There are only two quantum numbers labeling the eigenstates of the infinite deep LQD: the main quantum number N (due to the boundaries restricting the QD), and the magnetic quantum number m (due to the uniaxial rotation symmetry of the boundaries). To facilitate the following consideration of the interlevel transitions in the LQD it is convenient to keep in mind the relation $N = N^l_n$ mentioned in the previous section, see Fig. 1.

When constructing the eigenfunctions of the LQD the semispherical domain is "restored" from the lens domain due to the conformal mapping which is described by the Jacobian $J(\lambda)$. The new coordinates for the restored semispherical domain are (ρ, θ, φ), with $0 \leqslant \rho \leqslant a$, $0 \leqslant \theta \leqslant \pi/2$, and $0 \leqslant \varphi \leqslant 2\pi$. Integration of a function $f(\rho, \theta, \varphi)$ is performed as $\int d\rho d\theta d\varphi \rho J(\lambda) f(\rho, \theta, \varphi)$. Then, following Ref. 12, we present the LQD eigenfunctions as

$$\Psi^{\mathrm{LQD}}_{N,m}(\rho,\theta,\varphi) = \Bigg[(1 + \lambda A_{n,l,m})\Psi^{\mathrm{SSQD}}_{n,l,m}(\rho,\theta,\varphi) + \lambda \sum_{(n',l') \neq (n,l)} B^{n',l',m'}_{n,l,m} \Psi^{\mathrm{SSQD}}_{n',l',m'}(\rho,\theta,\varphi) \times \delta_{l',l\pm 2p}\delta_{m',m} \Bigg] \sqrt{\rho \sin\theta}, \tag{17}$$

where $p = 1, 2, 3, \ldots$. In Eq. (17) $\delta_{l',l\pm 2p}$ represents the link between l and m for the SSQD, so that $\Psi^{\mathrm{LQD}}_{N,m}$ is expressed by a series of $\Psi^{\mathrm{SSQD}}_{n',l',m}$ where l' has the same parity (even or odd) as l has. As a result, there is a link between the quantum numbers N for the LQD and l for the corresponding SSQD. The relationships for $A_{n,l,m}$ and $B^{n',l',m}_{n,l,m}$ can be found in Ref. 12. Note that in Eq. (17) we have introduced additionally the φ dependence of the wave functions, which was not presented in Ref. 12. It is just to unify denotation of the wave functions for QDs of different shapes in this paper.

Equation (17) is seen to have the SSQD case as its limiting case at $\lambda = 0$. It should be noted that the contribution of $\Psi^{\mathrm{SSQD}}_{n,l,m}$ to $\Psi^{\mathrm{LQD}}_{N,m}$ is major at $\lambda \neq 0$, which establishes the link between N and n. Within the perturbation approach the factors $B^{n',l',m}_{n,l,m}$ are inversely proportional to the gap between $E_{n,l}$ and $E_{n',l'}$ for the SSQD, so that the contribution from the remote (in the energy scale) states $\Psi^{\mathrm{SSQD}}_{n',l',m}$ is vanishing.

We would like to make some remarks about the notation. First of all, $\Psi^{\mathrm{LQD}}_{N,m}$ keeps memory of the magnetic quantum number m, so that $\Psi^{\mathrm{LQD}}_{N,m}$ is represented by a series of $\Psi^{\mathrm{SSQD}}_{n',l',m}$ with the same m, the m family (see Fig. 1). As mentioned in the previous section, the interlevel transitions in the SSQDs for the in-plane light polarization are allowed only between the states of the neighboring m families because of $\delta_{m',m\pm 1}$. It is obvious that, because of $\delta_{m',m\pm 1}$, the states of the LQD also form their m families as it is shown in Fig. 3 by shadowed columns, so that the interlevel transitions in the LQDs are allowed only between the states of the neighboring m families.

Figure 3 resembles Fig. 1. However, there is a principal difference between the figures. Any transitions between the states of the neighboring m families in the LQD are allowed, while there is the restriction $\Delta l = \pm 1$ for the transitions in the SSQD. Note, however, that the strongest coupling happens between the LQD states (N^l_n, m) and $(N^{l\pm 1}_{n'}, m \pm 1)$. Thus, keeping in mind that $N = N^l_n$ is very helpful for searching the eigenstates and the interlevel electron transitions in the

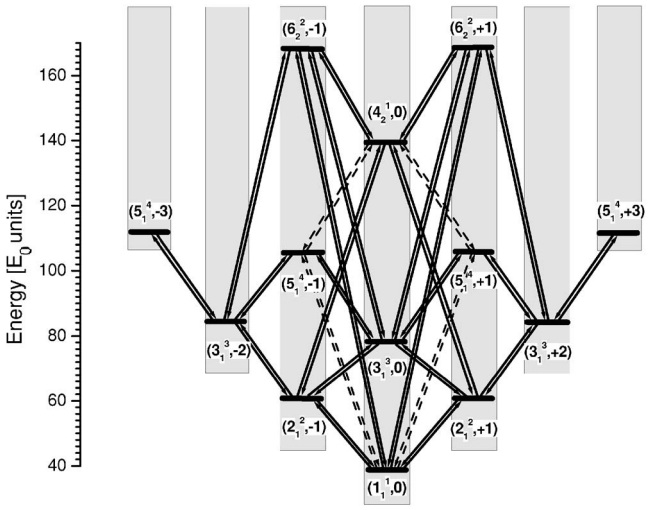

FIG. 3. Energy levels calculated for the infinite deep lens QD with one electron. The eigenstates are labeled by (N_n^l,m) where $N_n^l=N$, with N, n, l, and m are the same as in Fig. 1. $b/a=0.5$. The allowed dipole coupled interlevel transitions for the in-plane radiation polarization are shown by lines with arrows. The transitions allowed for LQDs but forbidden for SSQDs are shown by dashed lines with arrows. The m families of the LQD eigenstates are emphasized by shadowed columns.

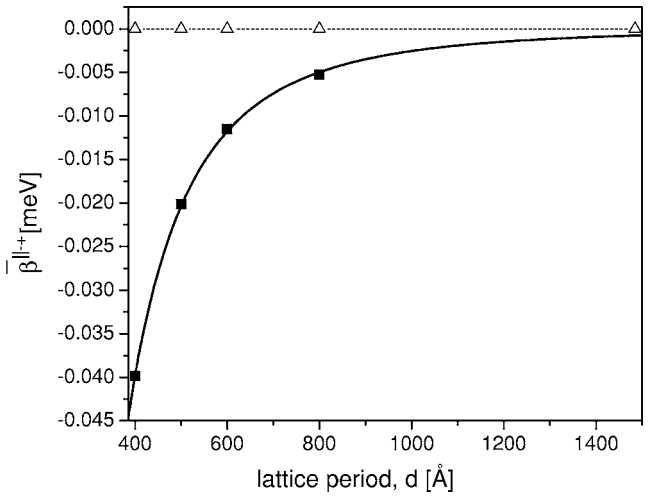

FIG. 4. Dependence of $\bar{\beta}^{\|-+}$ upon the period of the square two-dimensional infinite lattice of the lens QDs calculated for one electron per dot transiting between the ground and the first excited levels. $a=95$ Å and $b/a=0.5$. The square symbol represents Eq. (9) and the solid line represents Eq. (10). The zero level is labeled by the symbols in order to emphasize that the zero is the calculated reference level $L^{\|-+}=7.1539$ meV.

LQDs, and determining the most effective of them. In addition, the eigenenergy $E_{N,|m|}$ of the LQD in Fig. 3 is modified from the corresponding "source" eigenenergy $E_{n,l}$ of the SSQD in Fig. 1. The relationships for the eigenenergies, $E_{N,|m|}$, in the LQD one can find in Ref. 12 (see also Fig. 2 there).

As it is for the SSQD, we are interested in the case of one electron at the ground state within the three-state system in the LQD, see Fig. 3. Numerical calculations show that $\Psi_{N,m}^{\mathrm{LQD}}(=\Psi_{N_n^l,m}^{\mathrm{LQD}})$ is practically determined by $\Psi_{n,l,m}^{\mathrm{SSQD}}$ for $b/a>0.5$, especially for bigger b/a, while the other states make little contribution. Calculations give $A_{1,1,0}=-1.1897$, $B_{1,1,0}^{1,3,0}=0.54498$, $B_{1,1,0}^{2,1,0}=0.10635$, $A_{1,2,\pm1}=-0.9592$, $B_{1,2,\pm1}^{2,2,\pm1}=0.007282$.

It is obvious that, within the dipole approximation, the incident radiation of the normal polarization is not absorbed by the LQDs. The calculation of $\bar{\beta}^{\|-+}$ and of the polarizability for the LQD is done in the same fashion as it is for the SSQD, so that the notation $\bar{\beta}^{\|-+}$ works for the LQD too. The polarizability of the LQD for the in-plane light polarization is given by Eqs. (13)–(16).

Figure 4 presents the d dependence of $\bar{\beta}^{\|-+}$ for the square lattice of the LQDs. It is seen that the depolarization shift caused by the interdot dynamic e-e interaction is very well represented by the DDA. This depolarization shift, as well as the value of $L^{\|-+}$, for the LQDs is bigger (in the absolute value) than those for the SSQDs with the same basis size a (see Fig. 2). Figure 5(a) shows that the reason for that is the bigger dipole matrix element for the lens shape than for the semisphere shape of QDs. The value of $L^{\|-+}$, which represents the electron self-interaction, let us estimate the depolarization shift caused by the intradot direct dynamic interaction between a pair of electrons. Since the value of $L^{\|-+}$ for the LQD with $b/a=0.5$ is considerably bigger than that for the SSQD, see Fig. 5(b), we can conclude that the depolarization shift caused by the intradot e-e interaction is bigger in the LQD than that in the SSQD. The reason is the smaller room in the LQD compared with the SSQD which makes the stronger intradot e-e interaction in the LQD.

One can see that qualitatively the LQDs with one electron have the same considered features as the SSQDs do. However, there can be an essential quantitative difference in values of the depolarization shift for them if b/a deviates from 1. One can reasonably suggest that such a conclusion is held also for the QDs with a few electrons per dot.

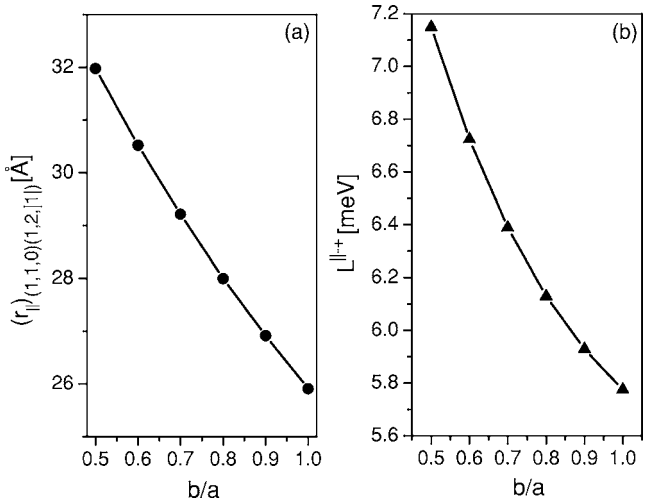

FIG. 5. b/a dependence of the matrix element of the in-plane radius-vector (a) and the value of $L^{\|-+}$, which represents the self-interaction, (b) calculated for one electron per LQD transiting between the ground and first excited levels. $a=95$ Å and $d=400$ Å.

C. Slightly ellipsoidal quantum dots

Here we consider the case of the sEQDs in the one-electron limit. To get the eigenstates of such sEQDs the perturbation approach can be used. It appears to be very profitable for analysis to assume that a sEQD is formed from a spherical QD by small deformation without change of the QD volume. Note that the Jacobian of such transformation equals 1. If a is the radius of the SQD then the semiaxes of the sEQD can be taken as $b=a(1-\gamma/3)$ and $c=a(1+2\gamma/3)$, where γ is a small parameter representing the measure of the deformation (and the eccentricity of the cross-section ellipse), with $\gamma=0$ is for the SQD. Reciprocating, one can say that a sEQD with semiaxes b and c has its source SQD of the same volume with radius $a=(b^2c)^{1/3}$. The small deformation allows us to write the Hamiltonian of the sEQD as a sum of the source SQD Hamiltonian and some perturbation,[24] $V^{\mathrm{def}}=(\gamma/3m^*)(\mathbf{p}^2-3p_z^2)$, where $\mathbf{p}$ is the momentum operator, and p_z is the operator of the z projection of the momentum, with the z-axis being the axis of the deformation. It is seen that the angular momentum operator does not commute with V^{def} which means that the angular quantum number l cannot be used for labeling the eigenstates of the EQD. Likewise for the LQD, the eigenstates of the sEQD can be labeled by two quantum numbers: the main quantum number, N, and the magnetic quantum number, m.

In the first order of the perturbation the eigenfunctions of the sEQD take a general form as

$$\Psi^{\mathrm{sEQD}}_{N,m}(\mathbf{r})=\Psi^{\mathrm{SQD}}_{n,l,m}(\mathbf{r})+\gamma\sum_{(n',l')\neq(n,l)}D^{n',l',m'}_{n,l,m}\Psi^{\mathrm{SQD}}_{n',l',m'}(\mathbf{r})\times\delta_{l',l\pm2p}\delta_{m',m},\tag{18}$$

where $p=1,2,3,\ldots$, and $D^{n',l',m}_{n,l,m}$ is a factor built by V^{def} with respect to the eigenstates of the source SQD. The relationship for the eigenenergies, $E_{N,|m|}$, in the sEQD can be found in Ref. 24.

Comparing Eqs. (17) and (18) one can see that an eigenstate of the sEQD is represented by the m family of the source SQD, similarly as it is for the LQD and the SSQD. In addition, the relation $N=N^l_n$ is useful for the sEQD also, but here (n,l,m) stands for a state of the source SQD. However, there is a principal difference between the sEQD and LQD: the dipole coupled electron transitions in the sEQD are allowed only between the states with the same magnetic quantum number, as it is for the SQD, i.e., within the same m family (see Fig. 6).

Our numerical calculations show that for small γ (less than 0.15), when V^{def} can be reasonably presented as a linear function of γ, the effect of the deformation on the eigenstates of the sEQD is negligible, and $\Psi^{\mathrm{sEQD}}_{N,m}$ $(=\Psi^{\mathrm{sEQD}}_{N^l_n,m})$ can be taken as $\Psi^{\mathrm{SQD}}_{n,l,m}$. As a result, the depolarization effect due to the dynamic intradot and interdot e-e interaction in the sEQD is practically the same as in the source SQD with radius $a=(b^2c)^{1/3}$. Note, however, that the modification of the eigenenergies due to the shape deformation can be considerable. In part, the degenerate states of the SQD with eigenenergies $E_{n,l}$ and $m\neq0$ split into a few eigenstates of the sEQD with the eigenenergies $E_{N,|m|}$ $(=E_{N^l_n,|m|})$.

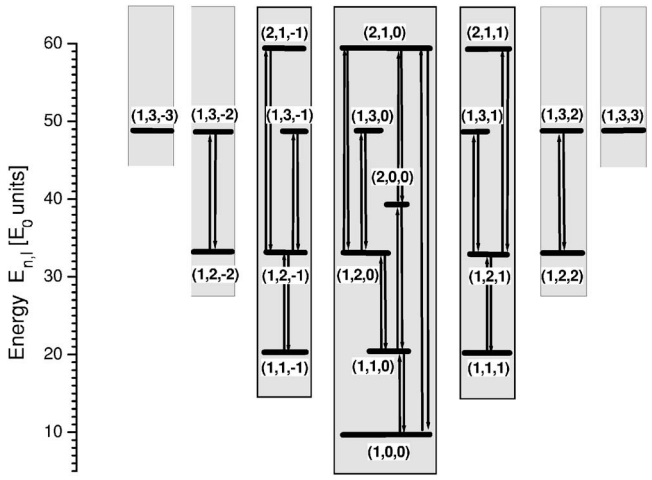

FIG. 6. Energy levels calculated for the infinite deep spherical QD with one electron. The eigenstates are labeled by (n,l,m). The allowed dipole coupled interlevel transitions are shown by lines with arrows. The m families of the SQD eigenstates are emphasized by shadowed columns.

In this section without using complicated calculations we obtained some results on the DE in the limit of the sEQDs when b/c is close to 1. This consideration is also helpful in determining the dipole coupled interlevel electron transitions and absorption spectra in the sEQDs. In the next section we consider cylindrical QDs by which a highly prolate $(b/c\gg1)$ or highly oblate $(b/c\ll1)$ ellipsoidal QDs can be approximated depending upon the CQD size parameters. Thus, again, without complicated calculations we are able to get some important results on the DE in another size limit of the ellipsoidal QDs.

D. Cylindrical quantum dots

We consider here the CQDs because of an important feature of the depolarization effect in CQDs which does not manifest itself in the SQDs, SSQDs, LQDs, or sEQDs. This is associated with the fact that among the mentioned QDs it is only the infinite deep CQD whose Hamiltonian allows separation of motions in the in-plane (in the QD basis) and normal (along the rotation axis) directions.

The eigenstate of the CQD is labeled by the main (n), magnetic (m) quantum numbers for the in-plane motion as well as the (main) quantum number in the z-direction, normal to the basis plane, $l_z=1,2,\ldots$. The wave functions of the CQD with the infinite barrier height is presented as

$$\Psi^{\mathrm{CQD}}_{n,m,l_z}(r_\parallel,\varphi,z)=\frac{J_m(\beta^{(m)}_n r_\parallel/a)}{A_{n,m}}\frac{\sin(\pi l_z z/H)}{\sqrt{H/2}}\frac{e^{im\varphi}}{\sqrt{2\pi}}.\tag{19}$$

Here $0\leqslant r_\parallel\leqslant a$ and $0\leqslant z\leqslant H$ where H is the CQD height. $J_m(x)$ is the cylindrical Bessel function of the first type, with $\beta^{(m)}_n$ is its nth root and A_{nm} is its normalization factor. We emphasize that the motion in the z direction is completely independent of the in-plane motion.

The eigenenergies of the CQD are determined by $E_{n,|m|,l_z}=E^{\parallel}_{n,|m|}+E^{\perp}_{l_z}$, where $E^{\parallel}_{n,|m|}=(\beta^{(m)}_n)^2E_0$ is the eigenenergy for

(n,m) state of the in-plane motion, and $E_{l_z}^{\perp}=\pi^2 l_z^2(a/H)^2E_0$ is the eigenenergy for (l_z) state of the normal motion.

The matrix elements of r_1 become as $(r_1)_{(n,m,l_z)(n',m',l_z')}=aC_{n,m}^{n',m'}\delta_{m',m\pm1}\delta_{l_z',l_z}$ (for the in-plane light polarization) and $z_{(n,m,l_z)(n',m',l_z')}=HD_{l_z}^{l_z'}\delta_{n',n}\delta_{m',m}$ (for the normal polarization), where $C_{n,m}^{n',m'}$ and $D_{l_z}^{l_z'}$ are constants. Thus, $\mu_{\mathbf{aa}'}^{\parallel}\sim a$ and $\mu_{\mathbf{aa}'}^{\perp}\sim H$. According to Eq. (10) one can see that within the DDA the contribution to the depolarization shift from the interdot dynamic *e-e* interaction is proportional to a^2 and H^2 for the in-plane and normal light polarization, respectively.

There are the following relationships for the infinite deep CQDs with one electron per dot for the in-plane $(j=\parallel)$ and normal $(j=\perp)$ polarization of the incident radiation

$$\bar{\beta}_{\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}'}^{(j)}=\frac{\tilde{\beta}^{(j)}(h^{(j)}/d)}{d}, \tag{20}$$

where $h^{\parallel}=a$ and $h^{\perp}=H$, and $\tilde{\beta}^{(j)}(h^{(j)}/d)$ is a function of the ratio $h^{(j)}/d$, which, within the DDA by Eq. (10), becomes proportional to $(h^{(j)}/d)^2$, and

$$L^{(j)}(\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}')=\frac{\tilde{L}^{(j)}(H/a)}{h^{(j)}}, \tag{21}$$

where $\tilde{L}^{(j)}(H/a)$ is a function of H/a. Further we assume one electron occupying the ground state. Note that $\bar{L}_{\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}'}^{\parallel}(=\bar{L}^{\parallel-+})$ is calculated by Eq. (8) for the symmetric three-state system, while $\bar{L}_{\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}'}^{\perp}$ is calculated by Eq. (13) in Ref. 7 for two-state system. For brevity, $\bar{\beta}_{\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}'}^{(j)}=\bar{\beta}^{(j)}$, $L^{(j)}(\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}')=L^{(j)}$, and $\bar{L}_{\mathbf{a},\mathbf{a}';\mathbf{b},\mathbf{b}'}^{(j)}=\bar{L}^{(j)}$.

Let us consider in detail the cases of the normal and in-plane light polarization separately. For the normal polarization an electron can transit between the neighbor levels which form a simple ladder in the energy $(E_{l_z}^{\perp})$ scale. For an electron at the ground state transiting to the first excited state it is a simple two-state system [as the $(1,0,0)-(1,1,0)$ system in Fig. 6], which was described in detail for the SQD in Ref. 7.

Now we are interested in the depolarization shift caused by the direct *intradot* dynamic *e-e* interaction for the normal polarization of the incident radiation. Here again we take the advantage of the electron self-interaction problem. As an approximate value for the depolarization shift caused by the intradot interaction between two real electrons we use the value of $L^{\perp}$, calculated by Eqs. (3), with $z=r_3$, and (19) for one electron in the CQD. Figure 7 presents the dependence of $L^{\perp}$ upon the CQD height, H. The value of $L^{\perp}$ is seen to grow with the growing H till $H\lesssim 1000$ Å. Only for bigger H, the value of $L^{\perp}$ decreases with increasing H. From the first look it is quite unexpected: for the SQD, SSQD, and LQD we have seen that the bigger room for electrons is the weaker intradot *e-e* interaction becomes. The reason for such an unexpected behavior of $L^{\perp}$ in the CQD lies in the nature of the direct *dynamic e-e* interaction. The dynamic *e-e* interaction describes the interaction between the electrons transiting between different levels in a quantum system, namely between different *orthogonal* eigenstates. If there is one direction in which the motion can be separated from the motion in other direction(s), then the orthogonality of the eigenstates becomes crucially important for the DE for the radiation polarization in that direction. The situation is as follows. A CQD is a three-dimensional system. Then for rather small QD height the z coordinate has less effect on the distance between two three-dimensional radius-vectors within the CQD, i.e., the distance looks like rather a two-dimensional one. As a result, the integration over the z coordinate for $L^{\perp}$ [see Eq. (3)] becomes closer to the overlapping integration for two orthogonal eigenstates for the motion in the z direction, which is zero. That is why, for rather small H the value of $L^{\perp}$ grows with growing H since we go away from that zero case. On the other side, for rather big H the z coordinate becomes as important as the other coordinates for determining the three-dimensional distance within the CQD. Then $L^{\perp}$ decreases with growing size of the QD, because the bigger room makes weaker the intradot *e-e* interaction (as it is found to be true for the SQDs, SSQDs, and LQDs). Thus, the dependence of $L^{\perp}$ upon such a QD size experiences a maximum value. Note that such behavior of $L^{(j)}$ upon the QD size, as $L^{\perp}$ has (i.e., having a maximum), takes place for each of the three spatial directions of the light polarization for the rectangular shape of infinite deep QD.

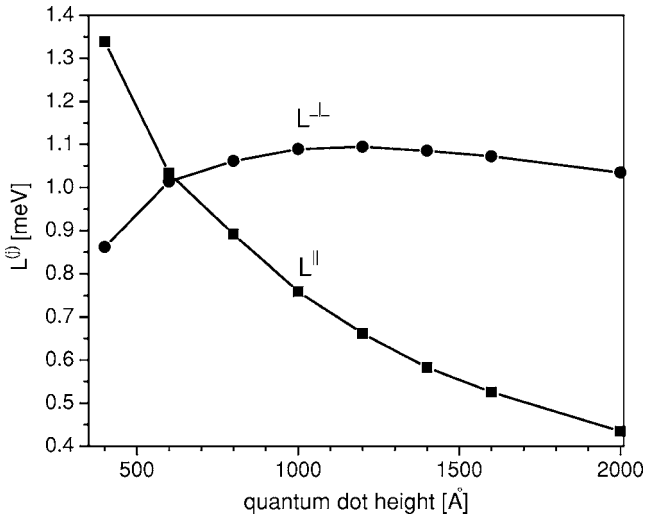

FIG. 7. Dependence of $L^{(j)}$, which represents the self-interaction, upon the height of the cylindrical QD calculated for one electron per dot transiting between the ground and first excited levels for the in-plane (square symbols) and normal (circle symbols) polarization of the incident radiation. $a=350$ Å.

Figure 7 shows that the value of $L^{\parallel}$ is always decreasing with increasing any of the CQD sizes, either a or H, since the orthogonality in the z direction, i.e., δ_{l_z',l_z}, does not play any role for the in-plane polarization of the incident radiation. Figure 8 shows that both $L^{\parallel}$ and $L^{\perp}$ decrease considerably, especially $L^{\perp}$, with growing QD radius at a fixed H.

For the in-plane motion the schematic map of a few lowest eigenstates, $E_{n,|m|}^{\parallel}$, and the electron transitions between allowed dipole coupled states for the in-plane light polarization are presented in Fig. 9. For simplicity we assume that

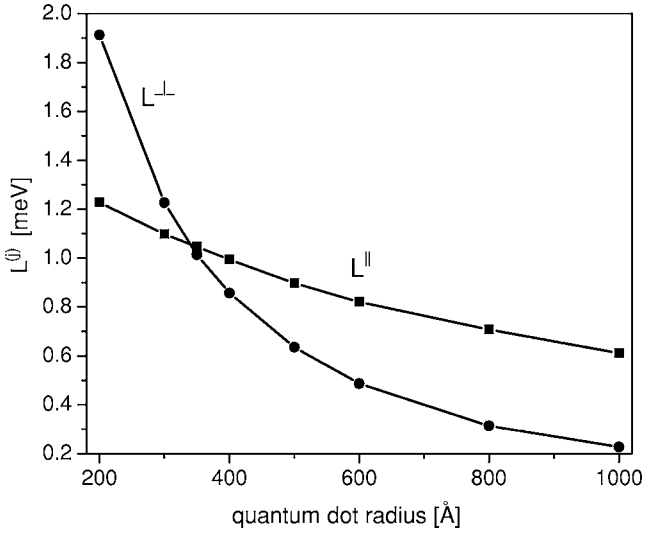

FIG. 8. Dependence of $L^{(j)}$, which represents the self-interaction, upon the radius of the cylindrical QD calculated for one electron per dot transiting between the ground and first excited levels for the in-plane (square symbols) and normal (circle symbols) polarization of the incident radiation. $H=640$ Å.

$E_1^{\perp}$ is big enough (than the highest $E_{n,|m|}^{\parallel}$ considered), so that $E_1^{\perp}$ just lifts up the origin of the scale in Fig. 9 and does not affect the interlevel transitions displayed. That requires H to be small enough. Under this condition Fig. 9 resembles Figs. 1 and 3 with their m families and features of the interlevel transitions. It is again because of $\delta_{m',m\pm1}$.

For calculation of the depolarization shift and polarizability we consider one electron in the two-state system (for the normal polarization) and in the three-state system (for the in-plane polarization) as mentioned earlier. Figure 10 presents the d dependence of $\bar{\beta}^{(j)}$ which represents the depolarization shift caused by the interdot dynamic e-e interaction. Figure 10 shows that the DDA very well represents the depolarization shift.

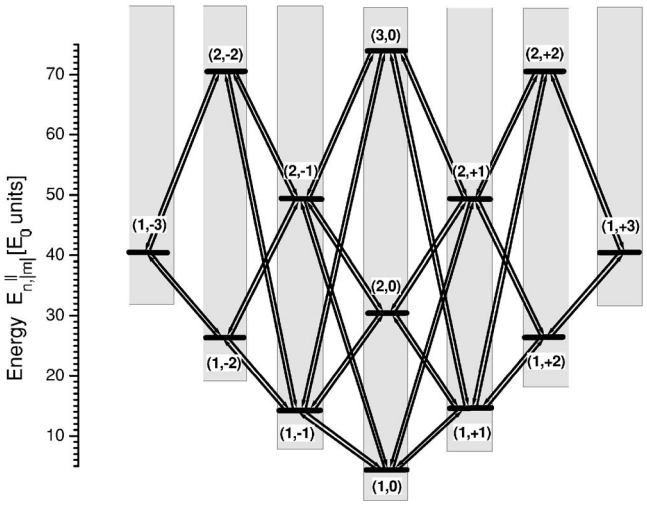

FIG. 9. Eigenenergies for the in-plane motion, $E_{n,|m|}^{\parallel}$, calculated for the infinite deep cylindrical QD with one electron. The eigenstates are labeled by (n,m) shortened from (n,m,l_z). The allowed dipole coupled interlevel transitions are shown by arrows. The m families of the CQD eigenstates are emphasized by shadowed columns.

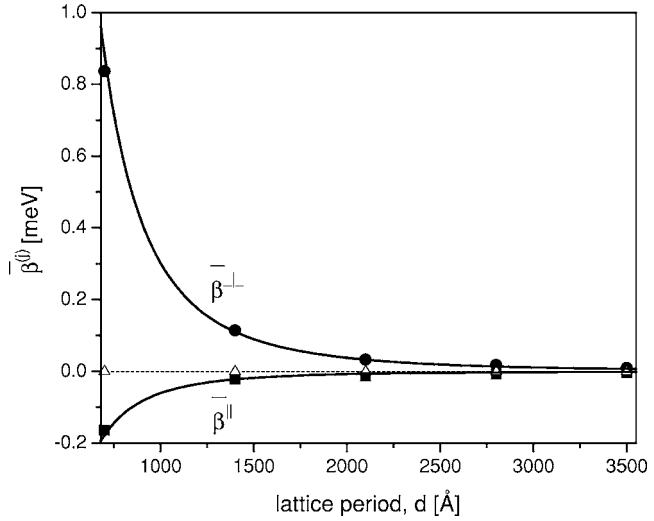

FIG. 10. Dependence of $\bar{\beta}^{\parallel}$ (square symbols) and $\bar{\beta}^{\perp}$ (circle symbols) upon the period of the square two-dimensional infinite lattice of the cylindrical QDs calculated for one electron per dot transiting between the ground and the first excited levels. The solid lines represent the DDA. $a=350$ Å and $H=1000$ Å. The zero level is labeled by the symbols in order to emphasize that the zero is the calculated reference level being different for the in-plane and normal light polarization as $L^{\parallel}=0.76065$ meV and $L^{\perp}=1.08975$ meV, respectively.

Analysis of Figs. 7, 8, and 10 shows that at certain size parameters of the CQD lattice the contributions into the depolarization shift from the interdot and intradot dynamic e-e interaction can be comparable to each other and reach near 1 meV for two electrons per dot for very dense lattices. Such situation can be reasonably suggested also for rectangular shape of QDs and, as is mentioned above, can be considered as a limiting case for ellipsoidal QDs whose ratio b/c is far from 1. On the other hand, our numerical results for the SSQDs, LQDs, sEQDs, and for the SQDs in Ref. 7 show that for those shapes of QDs the contribution from the intradot e-e interaction is about one or more orders of magnitude bigger than the contribution from the interdot interaction, with the latter one being of a rather small fraction of 1 meV for one electron per QD even for very dense lattices. Thus, shape of QDs, like cylindrical or rectangular one, which allows separation of the electron motion in one or more directions, let the depolarization effect reveal its fundamental qualitative properties as well as quantitative properties.

III. CONCLUSION

In this paper we consider the infrared dipole coupled interlevel electron transitions, the depolarization effect, the absorption spectra of an isolated QD as well as of the lattice of QDs of different shapes with uniaxial rotation symmetry. All the QDs are assumed to be infinite deep, with one electron per dot. This allows us to get the values for the depolariza-

tion shift which, if multiplied by the dielectric constant of the QD material, can be used for estimation of the shift for any sizes of the QD systems independently of the material parameters.

It is shown that the QD shape can crucially affect the optical properties of the QD systems, in particular, the dependence upon the polarization of the incident radiation. The semispherical and lens QDs do not absorb the normal polarized radiation within the dipole approximation. To facilitate consideration of the dipole coupled interlevel transitions in the QDs simple and illustrative maps of the transitions are utilized, and the concept of the *m* family is found to be fruitful for the QDs with the uniaxial rotation symmetry. The maps allow us to see how the interacting modes of the electron collective excitation are formed in the considered many-electron QD systems, and, thus, how to apply the MOS approach for description of the absorption spectra. The maps also let us suggest that the degenerate levels in a perfectly symmetric QD become split in a QD whose shape is distorted from the perfect one so that different modes of the collective excitation can be formed and the MOS approximation can be helpful.

The considered here maps also do another service. Sometimes to explain experimental results on QDs the map of states of atoms with the meaningful state notation "*s*," "*p*," "*d*,"… is used without any concern about the QD shape and polarization of the incident radiation. Probably it comes from literal using the name "artificial atoms" given to QDs. The considered in this paper maps for the QDs of different shapes show that careless applying of the map of the atom states for QDs is not only an oversimplification, but it can seriously mislead.

It is also worth mentioning that there is no simple and unambiguous relation between the number of peaks on the absorption spectra and number of electrons (carriers) in a QD, as it could be expected for atoms. For example, our results show that there is one actual peak in the spectra of absorption of linear polarized incident radiation for a single SQD with eight electrons per dot. On the other hand, for a two-dimensional lattice of such SQDs the position of that peak depends upon the polarization (normal or in-plane) of the incident radiation, so that the absorption spectra can have two actual peaks if the radiation has the both components polarization direction. The latter is true for even CQDs with one electron per dot: a spectrum can have two peaks each of them representing absorption of the normal and in-plane polarized component of the incident radiation, with the position of the peaks is determined by the size parameters of the CQD, so that the peaks can have the same position or be distant from each other.

It is shown that the shape and size of QDs can impact the depolarization effect considerably. It is found that the depolarization shift in slightly ellipsoidal QDs is practically the same as that in the spherical QDs, provided the volume of the sEQD and the SQD is the same. The cylindrical QDs are shown to be very illustrative in order to manifest fundamental properties of the depolarization effect which is the result of the dynamic *e-e* interaction in the multilevel electron systems: in the CQDs the contribution of the intradot dynamic *e-e* interaction into the depolarization shift can decrease with decreasing CQD volume. In addition, the CQDs have qualitatively different maps of the dipole coupled interlevel electron transitions and, consequently, different absorption spectra depending upon the polarization of the incident radiation (and the number of electrons per QD). Our numerical results show that for the CQDs the contributions to the depolarization shift from the interdot and intradot dynamic *e-e* interactions can be comparable to each other, with being of the order of 1 meV for two electrons per dot for very dense lattices. Finally, we have found the answer on the question about the applicability of the DDA: Numerical calculations show that the DDA very well represents the depolarization shift caused by the interdot dynamic *e-e* interaction for all kinds of the considered shapes of QDs for any reasonable interdot distances in the infinite lattices. The obtained results reveal some important optical properties of the QD systems, in particular those associated with the *e-e* interaction, which can be useful for designing, manufacturing, and exploiting QD-based devices in nanooptoelectronics.

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[1] E. H. Sargent, Adv. Mater. (Weinheim, Ger.) **17**, 575 (2005).
[2] H. Lee, R. Lowe-Webb, W. Yang, and P. Sercel, Appl. Phys. Lett. **72**, 812 (1998).
[3] V. M. Ustinov, Semiconductors **38**, 923 (2004).
[4] S. J. Prado, C. Trallero-Giner, A. M. Alcalde, V. López-Richard, and G. E. Marques, Phys. Rev. B **69**, 201310(R) (2004).
[5] D. L. Ferreira and J. L. A. Alves, Nanotechnology **15**, 975 (2004).
[6] C. L. N. Oliveira, J. A. K. Freire, V. N. Freire, and G. A. Farias, Appl. Surf. Sci. **237**, 266 (2004).
[7] V. Bondarenko, M. Załużny, and Y. Zhao, Phys. Rev. B **71**, 115304 (2005).
[8] F. M. Alvesa, F. Aristone, and G. E. Marquesb, Superlattices Microstruct. **37**, 248 (2005).
[9] J. L. Gondar and F. Comas, Physica B **322**, 413 (2002).
[10] G. Cantele, G. Piacente, D. Ninno, and G. Iadonisi, Phys. Rev. B **66**, 113308 (2002).
[11] G. G. Zergya, O. V. Konstantinov, and A. V. Matveentsev, Semiconductors **37**, 317 (2003).
[12] A. H. Rodríguez, C. Trallero-Giner, S. E. Ulloa, and J. Marín-Antuña, Phys. Rev. B **63**, 125319 (2001).
[13] L. C. Lew, Y. Voon, and M. Willatzen, J. Phys.: Condens. Matter **14**, 13667 (2002).

[14] R. V. N. Melnic and M. Willatzen, Nanotechnology **15**, 1 (2004).
[15] L. Brey, N. F. Johnson, and B. I. Halperin, Phys. Rev. B **40**, R10647 (1989).
[16] M. Taut, Phys. Rev. B **63**, 115319 (2001).
[17] L. Wendler and R. Haupt, Phys. Rev. B **52**, 9031 (1995).
[18] H. Ehrenreich and M. H. Cohen, Phys. Rev. **115**, 786 (1959).
[19] C. M. Bowden and M. E. Crenshaw, Opt. Commun. **179**, 63 (2000).
[20] H. R. Stuart and D. G. Hall, Phys. Rev. Lett. **80**, 5663 (1998).
[21] S. Zhou, N. Janel, and G. C. Schatz, J. Chem. Phys. **120**, 10871 (2004).
[22] N. D. Mermin, Phys. Rev. B **1**, 2362 (1970).
[23] L. Wendler and V. G. Grigoryan, Physica B **245**, 127 (1998).
[24] See the Migdal's problem in Chapter 6 in: L. D. Landau and E. M. Lifshitz, *Quantum Mechanics: Non-relativistic theory* (Pergamon Press, New York, 1965).