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Hollow-core resonator based on out-of-plane two-dimensional photonic band-gap crystal cladding at microwave frequencies

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We report on the demonstration of a resonator based on electromagnetic field confinement in a hollow-core by implementing an out-of-plane two-dimensional (2D) photonic band-gap (PBG) crystal cladding. In contrast with in-plane 2D PBG crystal devices, the PBG crystal studied here is perpendicular to the propagation axis. A resonator was constructed with silica rods to prove the concept at frequencies around 30 GHz. We show that the technique has the potential to reach quality factors (Q) of 5×10^5. © *2010 American Institute of Physics*. [doi:10.1063/1.3303857]

Microwave and millimeter wave frequency devices often depend on the quality of a resonator. For example, low noise oscillators require high-Q resonators for low phase noise and high stability. The Q-factor of a standard dielectric resonator is usually limited by the dielectric loss tangent of the material. One way of beating the loss-tangent limit is by confining the electromagnetic field in a hollow-core with the help of a PBG crystal that forbids field extension in the plane of the crystal. Previously work in this area includes the conception and fabrication of hollow-core resonators based on in-plane photonic crystal (PhC) composed by a hollow-core surrounded by a spatially two-dimensional (2D) periodic arrangement of metallic[1–4] or dielectric rods.[5] Recently, high Q-factors (3×10^5) have been obtained by a hollow-core resonator with one-layer structure [one-dimensional (1D) crystal] that confines the mode mainly in the central air region through Bragg reflection.[6,7] In contrast with in-plane PhC devices, the structured crystal is perpendicular (out-of-plane) to the propagation plane. Out-of-plane PBG structures have been studied since 1978 in optical fiber domain,[8] and have led to hollow-core PhC fibers[9] that have become the most advanced manifestation of 2D PBG structures, enabling the guidance of light in a hollow core with low attenuation on kilometer length scales.[10] Out-of-plane crystal acts on the transverse component of the field leading to a crystal pitch longer than the wavelength and therefore to a weaker sensitivity of the confined field to fabrication imperfections. This scale factor is of primary importance for building high Q resonators at microwave to millimeter wave frequencies, and in this letter we report the demonstration of a hollow-core resonator based on out-of-plane 2D PBG crystal cladding.

The resonator we investigate is composed of an out-of-plane 2D PBG crystal formed by an array of silica rods (in air) with a triangular lattice, which is sandwiched between two copper plates [Fig. 1]. The hollow-core is obtained by removing one or several rods in the center of the lattice. Silica rods are fabricated with the help of the optical fiber fabrication facilities at XLIM Research Institute. A silica bar is drawn to rods with diameter accuracy better than 1%, while the permittivity of the silica is measured with the microwave resonator method.[11] A permittivity of $3.78-j11.96\times10^{-4}$ is deduced by retrosimulation from the measurement of the frequency of the resonant peak associated to the transverse electric mode TE_{011} confined in the silica bar sample placed inside a specially designed resonator cavity. Diameter of silica rods ($d=2.5$ mm) and crystal pitch ($\Lambda=12.5$ mm) are chosen to allow the fundamental PBG frequency to be around 40 GHz. Effective indices (n_{eff}) of the modes supported by the PBG crystal are simulated with the help of commercial and home-made software based on finite element method. Within specific wavelength and effective index ranges, the rods in the cladding (depending on the optogeometrical parameters of the crystal) act as coupled resonating-waveguides.[12] As a result allowed and forbidden photonic bands are formed within the mode spectrum of the PhC cladding [Fig. 2(a)]. These band gaps can extend to effective values below the refractive index of air, so field trapping is then possible by introducing a hollow-core (a defect area) in the PhC (realized by removing one dielectric rod in the center of the 2D crystal) provided that the hollow-core supports guided mode within the forbidden band (i.e., band gap). The effective index versus frequency curve inside the band-gap [Fig. 2(a)] corresponds to the fundamental mode (hybrid mode HE_{11}) confined by the PBG effect in a hollow-core. A computed intensity picture of this mode is shown in the inset of Fig. 2(a). The strength of the field confinement in the hollow-core depends on the band-gap ef-

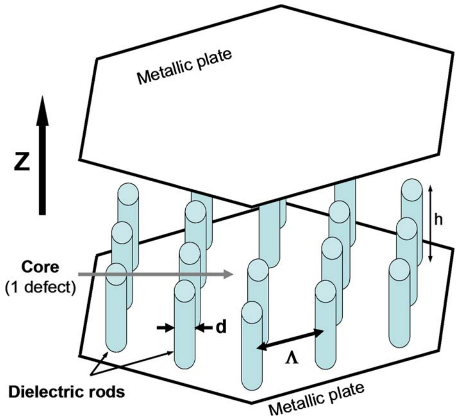

FIG. 1. (Color online) Schematic of the resonator structure.

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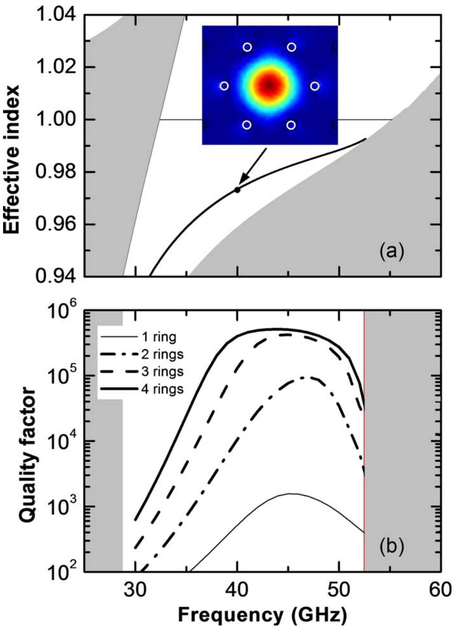

FIG. 2. (Color online) (a) Dispersion diagram of effective indices vs frequency of modes supported by the PBG crystal with one rod removed in the center (hollow-core). Gray areas show domains where the cladding array supports modes delimiting the band gap (white area). (Inset) Computed intensity picture of the fundamental mode confined in the hollow-core. Silica rods are delimitated by white circles. (b) Evolutions of the unloaded Q-factor of the hollow-core PBG crystal for different number of rings of rods disposed around the hollow-core.

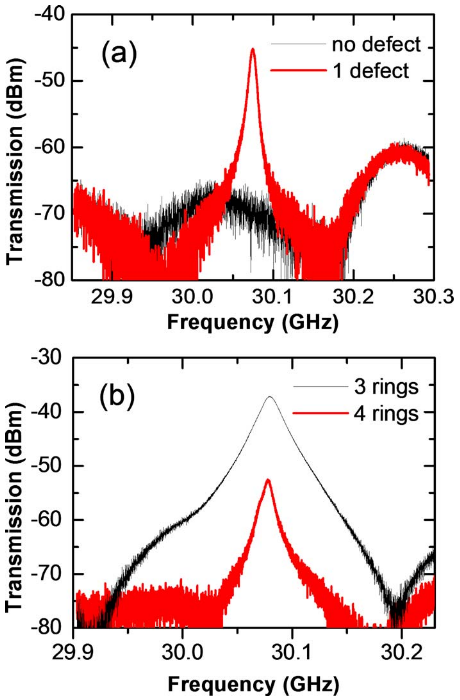

FIG. 3. (Color online) Transmission spectra measured with a vector network analyzer of a resonator composed of a PBG crystal, (a) without or with a hollow-core in the center of four rings of silica rods and (b) with a hollow-core surrounded by three or four rings of silica rods.

ficiency and thickness of the PBG crystal. Field confinement efficiency of the PBG crystal could be illustrated by the Q-factor (of an unloaded resonator) given by the relation $Q=\beta/2\alpha$, where β and α are the propagation constant $[\beta=k_0\,\mathrm{Re}(n_{\mathrm{eff}})]$ and the attenuation constant $[\alpha=k_0\,\mathrm{Im}(n_{\mathrm{eff}})]$, respectively; with k_0 the wave-number. The Q-factor has a maximum within the band-gap frequency range [Fig. 2(b)]. As expected, field confinement efficiency increases with the number of rod rings disposed around the hollow-core. Near optimum value of the Q-factor is obtained using three rings, as it is shown in Fig. 2(b), adding more rings is of little value as the Q-factor increases marginally due to the limit imposed by the dielectric loss due to the rods. Even though the Q-factor is limited by dielectric loss in this regime, due to the small amount of electromagnetic energy in the rods, the Q-factor is larger than the Q-factor of silica (1.9×10^4), corresponding to full confinement of the field in bulk silica. Only two rings of rods are enough to beat the loss tangent of the silica. Q-factor as high as 5.1×10^5 may be obtained for four rings of silica-rods that is more than one order of magnitude higher than a bulk silica resonator, emphasizing the potential of this out-of-plane 2D PhC for building high quality resonators at extremely high microwave frequencies.

To demonstrate the principle of resonance based on out-of-plane 2D PBG crystal, confinement along the propagation axis (z) is achieved by a Fabry–Perot cavity composed of two copper plates as shown in Fig. 1. This kind of cavity is not optimized for a high Q resonator. However, we use it to prove the principle and to study its behavior at microwave frequencies, and it was added in the three-dimensional model to calculate the Q-factor. The highest computed Q-factor of a resonator composed of four rings of silica rods drops from 5.1×10^5 to 4.7×10^3 when perfect metallic reflectors are replaced by copper reflectors with a conductivity of $4.1\times10^7\ (\Omega\ \mathrm{m})^{-1}$. The copper plate on the top is held by three micrometer adjusters to tune the spacing between both plates (i.e., cavity height) by graduations of 25 μm. The top plate is drilled to let the silica rods protrude through. Mode excitation in the hollow-core is done with a magnetic loop to couple to the axial component of the magnetic field. A second magnetic loop is used to measure scattering parameters with a vector network analyzer. The loops are placed between both copper plates and on the sides of the PBG crystal in order to not introduce coupling losses in the quality factor measurement. A PBG crystal without hollow-core and with a hollow-core formed by one missing silica rod (one defect) are studied in the resonator configuration with a cavity height (h) of 5.24 mm. Comparison of measurements done in transmission mode for both PBG crystals [Fig. 3(a)] demonstrates the existence of a resonant mode due to the hollow-core at frequency $f_r=30.074$ GHz with an unloaded Q factor of 4×10^3. Confinement of an electromagnetic field in a hollow-core is achieved when its effective index is below unity. For this resonator structure, the effective index is easily deduced by the resonance relation of a Fabry–Perot cavity: $n_{\mathrm{eff}}\ (2\pi f_r/c)\ h=\pi$. The effective index obtained $(n_{\mathrm{eff}}=0.95)$ confirms the assumption of field confinement in the hollow-core by the PBG crystal.

The PBG nature of the resonant mode is emphasized by the strong dependency of the unloaded Q factor to the number of silica rods around the hollow-core. As shown in Fig. 3(b), the addition of a fourth ring of rods yields an increase of the unloaded Q factor from 1.6×10^3 to 3.4×10^3 without a shift of the resonant frequency. This is associated with a diminution of the measured intensity illustrating the reduction of confinement losses and field intensity at the external boundary of the PBG crystal. The transmission spectrum is

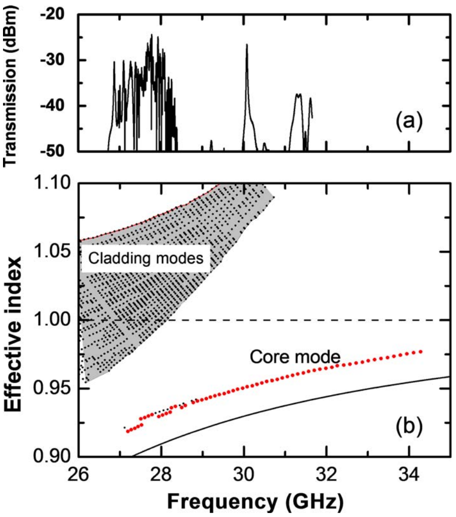

FIG. 4. (Color online) (a) Transmission spectra measured over a wider span with a vector network analyzer of a resonator composed of a PBG crystal of four rings of silica rods with one single silica rod removed in the center. (b) Dispersion diagram of effective indices of modes supported by the resonator deduced from measures as a function of the frequency (dots). Simulated effective index curve of the fundamental mode (HE_{11}) confined in the hollow-core (solid curve).

shown in Fig. 4(a) with a wider span, which includes the resonant peak and a band of peaks at lower frequencies. A nice feature is that the operational confined mode naturally exists in a spurious mode free region of 3 GHz, which is important for low noise oscillator applications. Properties of these peaks are investigated by recording transmission spectra at different cavity height. Effective indices of the modes related with all the peaks are deduced from the resonance relation of a Fabry–Perot cavity. The measured dispersion diagram [Fig. 4(b)] is in good agreement with the simulated diagram of the modes supported by the PBG crystal shown in Fig. 2. The evolution of the band of peaks is similar to the allowed band of modes that delimits the PBG band in the low frequency edge. These peaks are induced by cladding modes supported by the crystal outside the PBG band. The values of the mode effective index confined in the hollow-core (core mode) is close to the simulated values. The discrepancy between simulated and measured curves is about 2%. It falls to 1.3% when the central hole of the copper plate is roughly filled with a metallic rod. The discrepancy might be reduced further by taking into account, in the simulation model, all the holes drilled trough the copper plate instead of considering a perfect copper plate.

The highest Q-factor of 4×10^3 measured for four rings of rods is in good agreement with the computed one for copper plates (4.7×10^3). The Q-factor for a dielectric loaded cavity depends on the electric energy filling factor, the metal surface resistance, and the geometric factor of the mode. For our case the Q-factor is mainly limited by the metallic surface resistance losses rather than the loss tangent. To improve the Q-factor dependence on metal surface resistance the end plates may be changed to include Bragg reflectors, as achieved in Ref. 13, which developed high-Q resonators of 5.6×10^5 at 39 GHz with axial mode numbers of order 25. By combining our work with similar Bragg reflectors, this work has the potential to achieve a similar Q-factor with an axial mode number of order unity. Furthermore, the outcomes of this study could be extended at optical frequencies where interests of cavities based on 1D Bragg mirrors above and below a 2D PhC have been demonstrated only for in-plane 2D PhC.[14,15]

Based on simulations and on different experiments, we have demonstrated the realization of a hollow-core resonator based on out-of-plane 2D PBG crystal cladding. Following this demonstration of principle, the high potential of this technique will be exploited. The Q-factor will be improved by using a low-loss dielectric such as alumina and Bragg reflectors in the axial dimension.

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