Strange Star Surface: A Crust with Nuggets

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We reexamine the surface composition of strange stars. Strange quark stars are hypothetical compact stars which could exist if strange quark matter was absolutely stable. It is widely accepted that they are characterized by an enormous density gradient (10^{26} g/cm^4) and large electric fields at the surface. By investigating the possibility of realizing a heterogeneous crust, comprised of nuggets of strange quark matter embedded in an uniform electron background, we find that the strange star surface has a much reduced density gradient and negligible electric field. We comment on how our findings will impact various proposed observable signatures for strange stars.

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The conjecture that matter containing strange quarks could be absolutely stable is several decades old [1–4]. In the intervening years numerous authors investigated how such matter would manifest in nature (for recent reviews see Refs. [5–7]). Possibilities include tiny (dimensions of a few Fermi) quark lumps called strange nuggets and large (dimensions $\sim$ km) compact stars made entirely of strange quark matter [8–10]. Strange stars, as discussed to date, have bulk homogeneous quark matter containing up, down, and strange quarks extending all the way to the surface, which is uniquely qualified by (i) a steep density drop $\Delta\rho \sim 10^{15}$ g/cm^3 over a distance of several Fermi; and (ii) large electric fields due to less rapid variation of the electron density [10]. In this Letter, we reexamine the surface region of these strange stars and find that, contrary to conventional wisdom, a heterogeneous (solid) crust made of strange nuggets and electrons is likely, leading to a much reduced density gradient and negligible electric fields in the surface region. Our proposal differs from the conventional picture, suggested by Alcock *et al.* [10], of a tiny nuclear crust suspended a few hundred Fermis above the quark star, supported by large electric fields near the surface. It shares some apparent features of the quark-alpha crust scenario, based on absolutely stable strange quark states called quark-alphas [11,12], but is otherwise fundamentally different. Although Ref. [9] mentions the possibility of a crust with nuggets, not even a qualitative study of its consequences exists. Our work is the first study of this type of crust in a general and model-independent context.

In the vicinity of the strange star surface hydrostatic equilibrium requires the pressure to become vanishingly small. At the surface the pressure is identically zero. The pressure of stable quark matter vanishes at a finite and large quark density $n \simeq 1$ quark/fm^3, corresponding to a quark chemical potential of $\mu \simeq 300$ MeV. Since the strange quark mass, m_s, is large compared to the up and down quark masses, homogeneous quark matter needs electrons to ensure charge neutrality. In normal (nonsuperconducting) quark matter the electron chemical potential needed to ensure neutrality is $\mu_e \simeq m_s^2/4\mu$. Consequently, when the total pressure ($P_{\rm quark} + P_{\rm electron}$) is close to zero, the pressure due to quarks is negative. We show that in this regime a heterogeneous mixed phase with nuggets and electrons may be favored if surface and Coulomb costs are small. Deeper in the crust the mixed phase is characterized by voids filled by an electron gas embedded in quark matter. This mixed phase resembles the mixed phase of nuclei and electrons in the crust of normal neutron stars and shares several features with the mixed phase of quark drops and nuclear matter in hybrid stars [13]. As usual, the size of the charged nuggets or voids in the mixed phase will be determined by minimizing the surface, Coulomb, and other finite size contributions to the energy. At low temperature, the mixed phase is a solid. Using typical quark model parameters, we find that strange stars will have a relatively large crust with radial extent $\Delta R \simeq 50$ m for a star with mass $M \simeq 1.4 M_\odot$ ($M_\odot \simeq 2 \times 10^{33}$ g is the mass of the sun) and radius $R = 10$ km. The electron density decreases to zero over this length scale.

To prove that a heterogeneous phase is favored when surface and Coulomb energies are small enough, we adopt a model-independent approach which is valid when $\mu_e \ll \mu$. In this case, the quark pressure may be expanded in powers of μ_e; to second order in μ_e, it is given by

$$P_{\rm q}(\mu, \mu_e) = P_0(\mu) - n_Q(\mu)\mu_e + \frac{1}{2}\chi_Q(\mu)\mu_e^2, \qquad (1)$$

where $n_Q(\mu) = -\partial P/\partial \mu_e$ is the positive charge density, $\chi_Q(\mu) = \partial^2 P/\partial \mu_e^2$ is the charge susceptibility, and P_0 is the pressure of the electron-free quark phase. They depend on μ, m_s, and strong interactions. To perform a model-independent analysis we treat P_0, n_Q, and χ_Q as (μ-dependent) parameters. To appreciate their typical magnitude, we note that in the bag model description $n_Q = m_s^2\mu/2\pi^2$, $\chi_Q = 2\mu^2/\pi^2$, and $P_0 = 3(\mu^4 - m_s^2\mu^2)/4\pi^2 - B$, where B is the bag constant. To investigate the regime where the electron contribution to the pressure is relevant, we should keep terms up to fourth order in μ_e. However, this greatly complicates the analytic treatment and does not provide much new insight. For the time being, assuming the μ_e^3 and μ_e^4 terms in the quark pressure to be numerically small compared to the electron pressure (which we include explicitly), we work to second order in μ_e and return to a more complete treatment of the problem later.

A heterogeneous state of positively charged quark matter coexisting with negatively charged electron gas is possible if $P_q = 0$ and $\partial P_q/\partial \mu_e \leq 0$. Since electrons reside both inside and outside quark matter, Gibbs phase equilibrium requires $P_q = 0$. Also, $\partial P_q/\partial \mu_e \leq 0$ ensures that quark matter is positively charged, thus satisfying global charge neutrality. At fixed μ, from Eq. (1) we see that P_q is zero and quark matter is positively charged when μ_e takes on the value

$$\tilde{\mu}_e = \frac{n_Q}{\chi_Q}(1 - \sqrt{1-\xi}) \quad \text{where } \xi = \frac{2P_0\chi_Q}{n_Q^2}. \tag{2}$$

Hence a mixed phase is possible when $0 < \xi < 1$. In this regime, the mixed phase has lower free energy (larger pressure) than homogeneous matter. Relaxing the condition of local charge neutrality allows us to reduce the strangeness fraction in quark matter and thereby lower its free energy. $\xi = 1$ characterizes the critical point where this becomes possible. The pressure at the critical point $P_c = (\mu_e^c)^4/12\pi^2$, where $\mu_e^c = n_Q/\chi_Q$ is the electron chemical potential there. In this phase, electrons contribute to the pressure while quarks contribute to the energy density—much like the mixed phase with electrons and nuclei in the crust of a conventional neutron star.

Although P_0, n_Q, and χ_Q all depend on μ, and, consequently, change across the mixed phase, we find that only the variation in P_0 is relevant because P_0 varies rapidly with μ inside the mixed phase. For example, in the bag model, μ changes by less than a percent across the interval $0 \leq \xi \leq 1$ so that to a good approximation, we can treat n_Q and χ_Q as constants throughout the mixed phase. Further, since μ is nearly constant across the mixed phase the variation of the energy density inside nuggets is negligible. In what follows, ϵ_0 denotes the energy density inside nuggets.

To characterize the mixed phase we need to determine how the electron chemical potential and the volume fraction of the quark phase change with ξ. We have already obtained Eq. (2) which determines how μ_e changes with ξ. The volume fraction of the quark phase, denoted by x, is determined by the condition of global charge neutrality $Q(\tilde{\mu}_e)x = n_e(\tilde{\mu}_e)$ where $Q(\tilde{\mu}_e) = -(\partial P_q/\partial \mu_e)_{\mu_e=\tilde{\mu}_e}$ is the quark charge density in the mixed phase. We find

$$x = \frac{\tilde{\mu}_e^3}{3\pi^2 n_Q}\left(1 - \frac{\chi_Q\tilde{\mu}_e}{n_Q}\right)^{-1}. \tag{3}$$

The mixed phase will be penalized by Coulomb, surface, and other finite size contributions to the energy. Its stability at fixed pressure is guaranteed if its Gibbs free energy (per quark) is lower than the homogeneous phase. The Gibbs energy per quark $g = (E + PV)/N$, where E is the energy, P is the pressure, and N is the number of quarks in volume V. In the homogeneous phase, $g_{\rm H} = \mu_{\rm H}$ where $\mu_{\rm H}$ is the quark chemical potential. Similarly, $g_M = \mu_M$ in the mixed phase if finite size contributions are neglected. We now calculate the Gibbs free energy gain $\Delta g = \mu_{\rm H} - \mu_M$ in the mixed phase.

Using the local charge neutrality condition $\mu_e = n_Q/\chi_Q$ and Eq. (1), we find the pressure of the homogeneous phase $P_{\rm H}(\mu) = P_0(\mu) - n_Q^2/2\chi_Q$. The Gibbs energy $\mu_{\rm H}$ at fixed total pressure P is then determined by the equation $P_0(\mu_{\rm H}) = P + n_Q^2/2\chi_Q$. Since we expect $\Delta g \ll \mu$, a Taylor series expansion of the form

$$P_0(\mu_{\rm H}) = P_0(\mu_M) + n\Delta g + \mathcal{O}(\Delta g^2\mu^2), \tag{4}$$

where $n = (\partial P_0/\partial \mu)_{\mu=\mu_M}$ is justified. When $\mu_e \ll \mu$, n is the quark number density inside nuggets. Using Eq. (2) and $\tilde{\mu}_e = (12\pi^2 P)^{1/4}$, the gain in Gibbs energy per quark is

$$\Delta g = \frac{n_Q^2}{2\chi_Q n}\left(1 - \frac{2\chi_Q\tilde{\mu}_e}{n_Q} + \frac{\chi_Q^2\tilde{\mu}_e^2}{n_Q^2}\right). \tag{5}$$

In the bag model when $P = 0$, $\Delta g = m_s^4/16\pi^2 n$. For $m_s = 150$ MeV and $n = 1/\text{ fm}^3$, $\Delta g \simeq 0.4$ MeV per quark. The surface and Coulomb energy cost in the mixed phase has been studied in the context of the nuclear mixed phase [14]. Using these results, which are valid when corrections due to Debye screening and curvature energy is negligible, we find that the Coulomb and surface energy cost per quark

$$\epsilon_{s+C} = \frac{6\pi}{n(16\pi^2)^{1/3}}[(e^2\sigma d n_Q)^2 f_d(x)]^{1/3}, \tag{6}$$

where σ is the surface tension, d is the dimensionality ($d = 3$ for spheres, $d = 2$ for rods, and $d = 1$ for slabs), and the function $f_d(x)$ depends on the dimensionality and the volume fraction x of the rarer phase. Explicit forms for $f_d(x)$ may be found in Ref. [14]. From Eqs. (5) and (6), the mixed phase is favored when

$$\sigma \leq \frac{n_Q^2}{6\sqrt{3\pi f_d(x)}e^2 \mathrm{d}\chi_Q^{3/2}}. \tag{7}$$

In the bag model, this condition may be written in terms of m_s and μ as follows

$$\sigma \lesssim 36\left(\frac{m_s}{150\text{ MeV}}\right)^3 \frac{m_s}{\mu}\ \text{MeV/fm}^2. \tag{8}$$

The surface tension between quark matter and vacuum is poorly known. Using the bag model, Berger and Jaffe [15] estimate the surface energy of strangelets $\sigma \simeq 8$ MeV/fm^2

for $m_s = 150$ MeV and $\mu \simeq 300$ MeV, while $\sigma \simeq 5\ \mathrm{MeV/fm^2}$ for $m_s = 200$ MeV (numerical values for the surface tension are extracted from surface energies quoted in Ref. [16]). The condition in Eq. (8) implies that a structured mixed phase is favored even for $m_s = 150$ MeV. The sensitivity to m_s in Eq. (8) and other sources of finite size contributions to the energy which we have neglected here does not allow us to make a definitive claim about the stability of the mixed phase. Clearly, this warrants further work which should include the curvature energy [16], Debye screening [17–19], shell effects [20], and better estimates of the surface tension. For now, we proceed by assuming that surface and Coulomb costs are small enough to favor the mixed phase.

We had assumed that μ_e^3 and μ_e^4 terms in the quark pressure were small compared to the electron contribution to facilitate a simplified model-independent analysis. We now relax this assumption and work within the bag model, retaining terms to all orders in m_s and μ_e. For $B = 65\ \mathrm{MeV/fm^3}$ and $m_s = 150$ MeV, the quark component of the pressure of homogeneous matter is zero when $\mu = \mu^c \simeq 300$ MeV and $\mu_e = \mu_e^c \simeq 18$ MeV. The critical pressure below which homogeneous quark matter cannot exist is given by $P_c = (\mu_e^c)^4/12\pi^2 \simeq 1.2 \times 10^{-4}\ \mathrm{MeV/fm^3}$. The electron chemical potential decreases from $\mu_e = \mu_e^c$ at the critical point to zero at zero pressure. In the mixed phase the pressure is due to electrons, and is given by $P_{\mathrm{mixed}} = \tilde{\mu}_e^4/12\pi^2$. While the energy density is due to nuggets, and is given by $\epsilon_{\mathrm{mixed}} = x\epsilon_0$. The variation of pressure and energy density across the mixed phase are shown in Fig. 1. For our choice of bag model parameters, we find $\epsilon_0 \simeq 283\ \mathrm{MeV/fm^3}$. The inset in Fig. 1 shows how the volume fraction of the quark phase changes with μ_e. Near the critical point, x and consequently the equation of state (EOS) through $\epsilon_{\mathrm{mixed}} = x\epsilon_0$, varies rapidly. Except for the region very close to P_c, most of the mixed phase is characterized by small x. Here, sparsely distributed spherical nuggets are preferred.

To estimate the radial extent ΔR of the mixed phase crust, consider a strange star with mass M and radius R. For simplicity, ignoring special and general relativistic corrections

$$GM \int_R^{R+\Delta R} \frac{dr}{r^2} = \int_{P_c}^{P=0} \frac{dP}{\epsilon_{\mathrm{mixed}}}, \tag{9}$$

$$\text{when } \Delta R \ll R, \qquad \Delta R = \frac{R^2}{GM} \int_0^{P_c} \frac{dP}{\epsilon_{\mathrm{mixed}}}, \tag{10}$$

where P_c is the critical pressure at which the transition to the mixed phase occurs. Since the energy density $\epsilon_{\mathrm{mixed}} = x\epsilon_0$ and the pressure $P = \mu_e^4/12\pi^2$, we may use Eq. (3) to obtain

$$\Delta R = \frac{R^2 n_Q}{GM\epsilon_0} \int_0^{\mu_e^c} d\mu_e \left(1 - \frac{\chi_Q \mu_e}{n_Q}\right) = \frac{R}{R_s} \frac{n_Q^2}{\chi_Q \epsilon_0} R, \tag{11}$$

where $R_s = 2GM \simeq 3(M/M_\odot)$ km is the Schwarzschild radius of the star. From Eq. (10) we can also estimate the mass in the crust $M_{\mathrm{crust}} \simeq 1.4(\mu_e^c/20\ \mathrm{MeV})^4 \times (R/10\ \mathrm{km})^4 10^{-5} M_\odot$.

For $m_s = 150$ MeV and $\mu^c \simeq 300$ MeV we find that $n_Q \simeq 0.045\ \mathrm{fm^{-3}}$, $\chi_Q \simeq 92$ MeV/fm, and $\epsilon_0 \simeq 283\ \mathrm{MeV/fm^3}$. Substituting these values in Eq. (11) we find that $\Delta R \simeq 100$ meters for a star with mass $M = 1.4M_\odot$. The Newtonian estimate for ΔR and M_{crust} which were obtained using Eq. (1) provides useful insight. To obtain a more accurate value for ΔR, we use the bag model EOS shown in Fig. 1 and numerically solve general relativistic equations for hydrostatic equilibrium. The resulting density profile of the crust is shown in Fig. 2. Here we find that $\Delta R \simeq 40$ m and $M_{\mathrm{crust}} \simeq 6 \times 10^{-6} M_\odot$ for a $1.4M_\odot$ strange star.

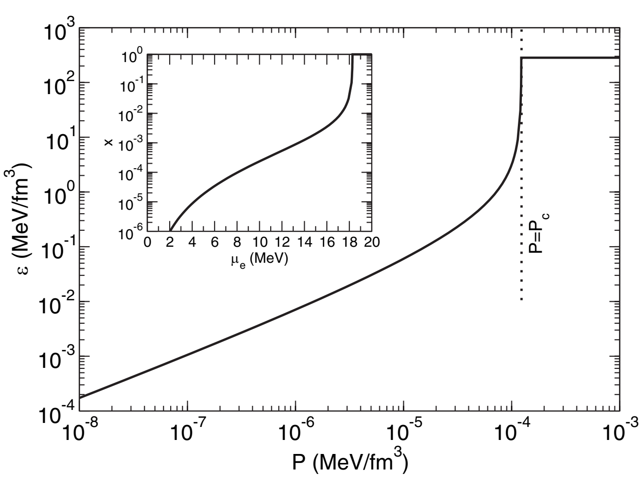

FIG. 1. The equation of state of the mixed phase. Inset shows the variation of the volume fraction of the quark phase as μ_e changes across the mixed phase.

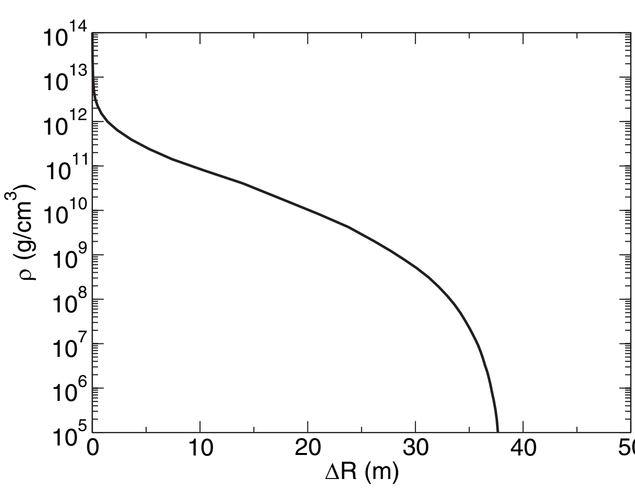

FIG. 2. Density profile of the crust for a strange star with mass $M = 14M_\odot$ and radius $R = 10$ km.

We expect the presence of a crust with nuggets to have several phenomenological consequences.

(i) *Photon radiation.*—In the conventional picture, the high electron chemical potential at the surface of a bare strange star leads to the formation of an electrosphere. The high plasmon frequency at its inner edge leads to distinguishing spectral features in the photon radiation from bare strange stars [21,22], especially in the gamma-ray region. In our scenario the presence of the crust obviates the need for the electrosphere, a large electric field, or any remarkable spectral features thereof [23].

(ii) *Moment of inertia.*—The moment of inertia of the solid crust $I_{\rm crust} \sim I_0 M_{\rm crust}/M_{\rm star}$ where I_0 is the moment of inertia of the star and $M_{\rm crust}/M_{\rm star}$ is the fractional mass in the crust. From our earlier estimate for $M_{\rm crust}$ we may conclude that $I_{\rm crust}/I_0 \simeq 10^{-6}$–$10^{-5}$. If glitches originate from crustal cracking, the observed giant glitches in the Vela pulsar requires $I_{\rm crust}/I_0 \simeq \Delta\Omega/\Omega \simeq 5 \times 10^{-6}$. This indicates that star quakes in the strange crust are not a likely explanation for this phenomenon.

(iii) *Thermal conductivity.*—The small mean free path for electrons scattering off nuggets implies that the thermal conductivity in the crust is much smaller than in the core. Using the results of Ref. [24], we find that the thermal conductivity of the nugget crust to be similar to that of a nuclear crust. This will influence thermal evolution since the crust will act as an insulator effectively keeping the surface temperature low. Scattering off nuggets is also likely to impact neutrino transport during the early evolution of the strange star subsequent to its birth in a supernova event [25].

To reiterate our main findings, a homogeneous and locally charged neutral phase of quarks and electrons could become unstable to phase separation at small pressure. Our main result, Eq. (7), provides a model-independent means to assess if a mixed phase is energetically favored. When the inequality in Eq. (7) is satisfied, a heterogeneous solid crust with strange nuggets embedded in a degenerate electron gas appears; deeper inside, one has voids filled with electrons embedded in quark matter. Such a crust closely resembles the conventional nuclear crust on normal neutron stars, and its thermal and transport properties are dramatically altered from those of a bare surface. An interesting exception is color-flavor-locked (CFL) quark matter [26] where quark pairing ensures equal numbers of up, down, and (massive) strange quarks [27,28]. The CFL phase is neutral in the bulk without electrons. Consequently, such stars do not require a crust but will be characterized by a bare surface and an extended electron layer at the surface [23].

Ultimately, the question of whether strange stars have strange crusts depends on the value of the surface tension between strange quark matter and the vacuum. To ascertain if strange nuggets or voids are indeed stable with respect to fusion at low pressure requires a proper account of Debye screening and curvature energy; these finite size contributions to the energy and their model dependence are being investigated, and will be reported elsewhere. If these are small enough, then almost all strange stars should have a crust and strange nuggets at zero pressure should have a finite stable size.

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