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Diffraction of electromagnetic plane wave by an infinitely long conducting strip on dielectric slab

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ABSTRACT

Diffraction of electromagnetic plane wave by an infinitely long conducting strip which is placed on a dielectric slab of finite thickness is formulated rigorously. Both the principal polarizations have been considered. The method of analysis is Kobayashi potential. Imposition of boundary conditions result in dual integral equations. These dual integral equations are reduced to matrix equations with infinite number of unknowns. The elements of the matrix equations are given in terms of infinite integrals. These integrals are hard to solve analytically, so computed numerically. Diffracted far field patterns for different angle of incidence have been computed. Current distributions on the strip are also presented. We have compared our field patterns with those of obtained through physical optics. The agreement is good.

1. Introduction

Scattering from PEC/non-PEC strips is a well-known topic of study in electromagnetics. It has received the attention of various investigators working in the field [1–10]. Senior [6] examined scattering from resistive strips. Senior and Liepa [7] used diffraction techniques in order to study strips with a nonconstant or tapered resistivity. Peters and Newmann [8] examined TM scattering by a resistive sheet located in a dielectric half space using method of moments and spectral domain Green's functions. Strips with superconducting materials had been treated in [9]. Imran et al. studied the diffraction from an impedance strip using Kobayashi potential method [10].

In this work, diffraction of electromagnetic waves from a perfectly conducting strip which is placed on a dielectric slab has been studied. This problem may be considered as a simple model of a micro-strip antenna. Few investigations, which are close to our study, are [11–14]. Michalski and Butler [13] studied the problem of the current density induced on the strip embedded in a dielectric slab. Using appropriate expansion and testing functions, they developed singular integro-differential equations and utilized moment method. Cheng and Chew [14] studied electromagnetic scattering of finite strip array on a dielectric slab. Present analysis is based on the Kobayashi potential (KP) method. This method may be utilized as an alternative approach to study these kind of problems. Kobayashi potential (KP) method is an analytical technique for solving the mixed boundary value problem. It was developed by Iwao Kobayashi in the beginning of 1930s. In his original work, Kobayashi, firstly, introduced the idea that the solutions of potential problems associated with conducting disc and strip can be effectively constructed by using the discontinuous properties of Weber–Schafheitlin's integrals [15]. He also discussed the properties of Jacobi's polynomials which were used as the basis of functional space in this method. The application of KP method to wave phenomena has been studied extensively by Nomura [16–20], Hongo [21–27] and their coworkers.

In KP method, we assume the solution in the form of unknown weighting functions. Enforcement of boundary conditions yield dual integral equations for the weighting functions. These equations are solved using the discontinuous properties of the Weber–Schafheitlin's Integrals. At this stage we can incorporate the edge conditions in the solution. The resulting equations is then converted into matrix equations by applying the projection method with a functional space that consists of a set of Jacobi's polynomials [28] as basis functions. The elements of the matrix equations are usually infinite integrals which have branch points as well as poles. These integrals are hard to solve analytically. Therefore, we use numerical techniques to evaluate these integrals. Illustrative computations are given for far field patterns and the current induced on the strip. We have compared our results with those of obtained through physical optics (po) and the comparison is good.

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2. Formulation of the problem

2.1. E-Polarization

The geometry of the problem and the coordinate system are shown in Fig. 1. The conducting strip is located on a dielectric slab with finite thickness 'd'. Width of the strip is $2a$. The constitutive parameters of medium occupying the slab $(-d < y < 0)$ are (ε, μ_0), while rest of the space has medium with constitutive parameters (ε_0, μ_0). Incident field E_z^i may be written as

$$E_z^i = \exp[jk_0(x\cos\phi_0 + y\sin\phi_0)] \tag{1a}$$

The space has been divided into three regions. Region 1 is above the strip. Region 2 is the dielectric slab while region 3 is below the dielectric slab. In each region, scattered field may be assumed in terms of unknowns. The expressions for scattered fields E_z^u in region 1, E_z^d in the slab, and E_z^l in region 3, can be assumed in the form

$$E_z^u = \int_0^\infty [g_1(\xi)\cos(\xi x_a) + g_2(\xi)\sin(\xi x_a)] \times \exp(-uy_a)d\xi, \quad y > 0 \tag{1b}$$

$$E_z^d = \int_0^\infty \cos(\xi x_a)[f_1(\xi)\exp(\upsilon y_a) + h_1(\xi)\exp(-\upsilon y_a)]d\xi + \int_0^\infty \sin(\xi x_a)[f_2(\xi)\exp(\upsilon y_a) + h_2(\xi)\exp(-\upsilon y_a)]d\xi, \quad -d < y < 0 \tag{1c}$$

$$E_z^l = \int_0^\infty [\ell_1(\xi)\cos(\xi x_a) + \ell_2(\xi)\sin(\xi x_a)]\exp[u(y_a + d_a)]d\xi, \quad y < -d \tag{1d}$$

where $k_0 = \sqrt{\mu_0\epsilon_0}$ and $k = \sqrt{\mu_0\epsilon}$ are the propagation constants of free space and dielectric slab, respectively, and $u = \sqrt{\xi^2 - \kappa_0^2}$, $\upsilon = \sqrt{\xi^2 - \kappa^2}$, $\kappa_0 = k_0 a$, $\kappa = ka$, $d_a = \frac{d}{a}$, $x_a = \frac{x}{a} y_a = \frac{y}{a}$ and the functions $f_{1,2}(\xi)$, $g_{1,2}(\xi)$, $h_{1,2}(\xi)$ $l_2(\xi)$ are the weighting functions to be determined from the boundary conditions.

The required boundary conditions of the problem are given by

$$\begin{aligned}
&\text{(i)} \quad E_z \text{ is continuous at } y = 0 \text{ for all values of } x \\
&\text{(ii)} \quad E_z \text{ and } H_x \text{ are continuous for } y = -d \\
&\text{(iii)} \quad H_x \text{ is continuous at } y = 0 \text{ and for } |x_a| \geqslant 1 \\
&\text{(iv)} \quad E_z^i + E_z^r + E_z^u = 0 \text{ at } y = 0 \text{ and for } |x_a| \leqslant 1
\end{aligned} \tag{2}$$

Using boundary conditions (i) and (ii) we have obtained the following relations:

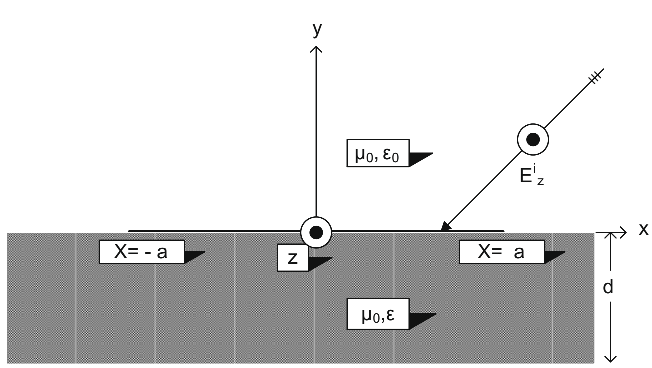

Fig. 1. Geometry of the problem.

$$f_1(\xi) + h_1(\xi) = g_1(\xi) \tag{3a}$$
$$f_2(\xi) + h_2(\xi) = g_2(\xi) \tag{3b}$$
$$f_1(\xi)\exp(-p) + h_1(\xi)\exp(p) = \ell_1(\xi) \tag{3c}$$
$$f_2(\xi)\exp(-p) + h_2(\xi)\exp(p) = \ell_2(\xi) \tag{3d}$$
$$f_1(\xi)\exp(-p) - h_1(\xi)\exp(p) = u\ell_1(\xi) \tag{3e}$$
$$f_2(\xi)\exp(-p) - h_2(\xi)\exp(p) = u\ell_2(\xi) \tag{3f}$$

where $p = \upsilon d_a$. Using boundary conditions (iii) and (iv) we have

$$\int_0^\infty [\{ug_1(\xi) + \upsilon[f_1(\xi) - h_1(\xi)]\}\cos(\xi x_a) + \{ug_2(\xi) + \upsilon[f_2(\xi) - h_2(\xi)]\}\sin(\xi x_a)]d\xi = 0 \quad \text{for } |x_a| \geqslant 1 \tag{3g}$$

$$\int_0^\infty [g_1(\xi)\cos\xi x_a + g_2(\xi)\sin\xi x_a]d\xi = -(1 + R_E)\exp[jk_0 x\sin\phi_0] \quad \text{for } |x_a| \leqslant 1 \tag{3h}$$

The reflection coefficient R_E of the slab for E-polarization can be calculated as

$$R_E = \frac{(1 - P_E^2)(e^* - e)}{\Delta_E}, \quad P_E = \frac{Y_0\sin\phi_0}{Y\sin\phi_t}, \quad e = \exp(jkd\cos\phi_t),$$
$$e^* = \exp(-jkd\cos\phi_t), \quad \Delta_E = (1 + P_E)^2 e - (1 - P_E)^2 e^*$$

where $Y_0 = \sqrt{\frac{\epsilon_0}{\mu_0}}$ and $Y = \sqrt{\frac{\epsilon}{\mu_0}}$ are intrinsic admittances of free space and dielectric slab, respectively. Permeability is assumed to be constant all over the space. The angle of refraction ϕ_t is related with the angle of incident ϕ_0 by

$$k\cos\phi_t = k_0\cos\phi_0$$

Manipulating the expressions ((3a)–(3f)), we get

$$f_1(\xi) = \frac{(u + \upsilon)\exp(p)}{(\upsilon + u)\exp(p) + (\upsilon - u)\exp(-p)} g_1(\xi) \tag{4a}$$
$$h_1(\xi) = \frac{(\upsilon - u)\exp(-p)}{(\upsilon + u)\exp(p) + (\upsilon - u)\exp(-p)} g_1(\xi) \tag{4b}$$
$$\ell_1(\xi) = \frac{2\upsilon}{(\upsilon + u)\exp(p) + (\upsilon - u)\exp(-p)} f_1(\xi) \tag{4c}$$
$$f_2(\xi) = \frac{(u + \upsilon)\exp(p)}{(\upsilon + u)\exp(p) + (\upsilon - u)\exp(-p)} g_2(\xi) \tag{4d}$$
$$h_2(\xi) = \frac{(\upsilon - u)\exp(-p)}{(\upsilon + u)\exp(p) + (\upsilon - u)\exp(-p)} g_2(\xi) \tag{4e}$$
$$\ell_2(\xi) = \frac{2\upsilon}{(\upsilon + u)\exp(p) + (\upsilon - u)\exp(-p)} g_2(\xi) \tag{4f}$$

Eqs. (3g) and (3h) are the dual integral equations, so making use of discontinuous properties of Weber–Schafheitlin's integrals and incorporating the edge and radiation conditions we can write

$$ug_1(\xi) + \upsilon[f_1(\xi) - h_1(\xi)] = \sum_{m=0}^\infty A_m J_{2m}(\xi) \tag{5a}$$

$$ug_2(\xi) + \upsilon[f_2(\xi) - h_2(\xi)] = \sum_{m=0}^\infty B_m J_{2m+1}(\xi) \tag{5b}$$

where A_m and B_m are constants to be determined and $J_m(\xi)$ be the Bessel's function. Using Eqs. (4), weighting functions $g_1(\xi)$ and $g_2(\xi)$ can be expressed as

$$g_1(\xi) = \frac{u + \upsilon + (\upsilon - u)\exp(-2p)}{(\upsilon + u)^2 - (\upsilon - u)^2\exp(-2p)} \sum_{m=0}^\infty A_m J_{2m}(\xi) \tag{6a}$$

$$g_2(\xi) = \frac{u + \upsilon + (\upsilon - u)\exp(-2p)}{(\upsilon + u)^2 - (\upsilon - u)^2\exp(-2p)} \sum_{m=0}^\infty B_m J_{2m+1}(\xi) \tag{6b}$$

Putting these values of $g_{1,2}(\xi)$ in (3h), comparing even and odd functions and then projecting the resulting equations into the functional space with elements $u_n^{\pm\frac{1}{2}}(x_a^2)$ [25], we get

$$\sum_{m=0}^{\infty} A_m G_G(2m, 2n) = -(1+R_E) J_{2n}(\kappa_0 \cos\phi_0) \tag{7a}$$

$$\sum_{m=0}^{\infty} B_m G_G(2m+1, 2n+1) = -j(1+R_E) J_{2n+1}(\kappa_0 \cos\phi_0) \tag{7b}$$

where $G_G(v, \mu)$ is defined by

$$G_G(v,\mu) = \int_0^{\infty} \frac{u + v + (v-u)\exp(-2p)}{(v+u)^2 - (v-u)^2 \exp(-2p)} J_v(\xi) J_\mu(\xi) d\xi \tag{7c}$$

In obtaining (7a) and (7b), we have used of the following relations:

$$\cos x = \sqrt{\frac{\pi x}{2}} J_{-1/2}(x)$$

$$\sin x = \sqrt{\frac{\pi x}{2}} J_{1/2}(x)$$

$$x^{-m/2} J_m(\xi\sqrt{x}) = \sum_{n=0}^{\infty} \frac{\sqrt{2}(2n+m+\frac{1}{2})\Gamma(n+m+\frac{1}{2})}{\Gamma(n+1)\Gamma(m+1)} \frac{J_{2n+m+\frac{1}{2}}(\xi)}{\sqrt{\xi}} u_n^m(x)$$

$$u_n^m(x) = \frac{\sqrt{2}\Gamma(n+1)\Gamma(m+1)}{\Gamma(n+m+\frac{1}{2})} x^{-m/2} \int_0^{\infty} \frac{J_m(\sqrt{x}\xi) J_{2n+m+\frac{1}{2}}(\xi)}{\sqrt{\xi}} d\xi$$

where $u_n^m(x)$ be the Jacobi's polynomials.

2.1.1. Field patterns

Since the geometry supports surface waves. If the observation point is far from the surface of the strip then this contribution can be neglected and diffracted fields dominate. Substituting $g_1(\xi)$ and $g_2(\xi)$ from (6a) and (6b) in expression (1b) and then using saddle point method, we can calculate the far diffracted field for region $y > 0$ as

$$E_z^u(\phi_0, \phi) = \frac{\sqrt{\epsilon_r - \cos^2\phi} + \sin\phi + \left(\sqrt{\epsilon_r - \cos^2\phi} - \sin\phi\right)\exp\left(-j2k_0 d\sqrt{\epsilon_r - \cos^2\phi}\right)}{\left(\sqrt{\epsilon_r - \cos^2\phi} + \sin\phi\right)^2 - \left(\sqrt{\epsilon_r - \cos^2\phi} - \sin\phi\right)^2 \exp\left(-j2k_0 d\sqrt{\epsilon_r - \cos^2\phi}\right)} \times \sum_{m=0}^{\infty}\left[A_m J_{2m}(\kappa_0\cos\phi) + jB_m J_{2m+1}(\kappa_0\cos\phi)\right] \tag{8}$$

where ϕ is the angle of observation and $\epsilon_r = \frac{\epsilon}{\epsilon_0}$. Constants A_m and B_m can be computed from Eq. (7).

2.1.2. Current induced on the strip

Current density, induced on the strip, may be computed from the expression

$$\begin{aligned} J_z &= -H_x|_{y=0+} + H_x|_{y=0-} \\ &= \frac{Y_0}{j\kappa} \sum_{m=0}^{\infty} \int_0^{\infty} [A_m J_{2m}(\xi)\cos(\xi x_a) + B_m J_{2m+1}(\xi)\sin(\xi x_a)] d\xi \\ &= \frac{Y_0}{j\kappa}(1 - x_a^2)^{-\frac{1}{2}} \sum_{m=0}^{\infty} \{A_m \cos[2m \sin^{-1} x_a] \\ &\quad + B_m \sin[(2m+1)\sin^{-1} x_a]\} \end{aligned} \tag{9}$$

2.2. H-Polarization

The formulation for H-polarization can be conducted in a manner similar to E-polarization. The expressions corresponding to ((1a)–(1d)) for H-polarization may be written as

$$H_z^i = \exp[jk_0(x\cos\phi_0 + y\sin\phi_0)] \tag{10a}$$

$$H_z^u = \int_0^{\infty} [g_1(\xi)\cos(\xi x_a) + g_2(\xi)\sin(\xi x_a)] \exp(-u y_a) d\xi, \quad y > 0 \tag{10b}$$

$$\begin{aligned} H_z^d &= \int_0^{\infty} \cos(\xi x_a)[f_1(\xi)\exp(v y_a) + h_1(\xi)\exp(-v y_a)] d\xi \\ &\quad + \int_0^{\infty} \sin(\xi x_a)[f_2(\xi)\exp(v y_a) + h_2(\xi)\exp(-v y_a)] d\xi, \\ &\quad -d < y < 0 \end{aligned} \tag{10c}$$

$$\begin{aligned} H_z^l &= \int_0^{\infty} [\ell_1(\xi)\cos(\xi x_a) + \ell_2(\xi)\sin(\xi x_a)] \\ &\quad \exp[u(y_a + d_a)] d\xi, \quad y < -d \end{aligned} \tag{10d}$$

The required boundary conditions are given by

$$\begin{aligned} &\text{(i)} \quad E_x \text{ is continuous at } y = 0 \text{ for all values of } x \\ &\text{(ii)} \quad H_z \text{ and } E_x \text{ are continuous for } y = -d \\ &\text{(iii)} \quad H_z \text{ is continuous at } y = 0 \text{ and for } |x_a| \geqslant 1 \\ &\text{(iv)} \quad E_x^i + E_x^r + E_x^u = 0 \text{ at } y = 0 \text{ and for } |x_a| \leqslant 1 \end{aligned} \tag{11}$$

Using these boundary conditions and proceeding in the similar manner as last section, we have dual integral equations:

$$\begin{aligned} &\int_0^{\infty} [\{g_1(\xi) - f_1(\xi) - h_1(\xi)\}\cos(\xi x_a) + \{g_2(\xi) - f_2(\xi) \\ &\quad - h_2(\xi)\}\sin(\xi x_a)] d\xi = 0 \quad \text{for } |x_a| \geqslant 1 \end{aligned} \tag{12a}$$

$$\begin{aligned} &\int_0^{\infty} u[g_1(\xi)\cos\xi x_a + g_2(\xi)\sin\xi x_a] d\xi \\ &\quad = -j\cos\phi_0 \kappa_0 (1 - R_H)\exp[jk_0 x \sin\phi_0] \quad \text{for } |x_a| \leqslant 1 \end{aligned} \tag{12b}$$

and unknown coefficients

$$f_1(\xi) = -\frac{\epsilon_r u}{v} \frac{(v + \epsilon_r u)}{(\epsilon_r u + v) + (\epsilon_r u - v)\exp(-2p)} g_1(\xi) \tag{13a}$$

$$h_1(\xi) = \frac{\epsilon_r u}{v} \frac{(\epsilon_r u - v)\exp(-2p)}{(\epsilon_r u + v) + (\epsilon_r u - v)\exp(-2p)} g_1(\xi) \tag{13b}$$

$$\ell_1(\xi) = -\frac{2\epsilon_r u \exp(-p)}{(\epsilon_r u + v) + (\epsilon_r u - v)\exp(-2p)} g_1(\xi) \tag{13c}$$

$$f_2(\xi) = -\frac{\epsilon_r u}{v} \frac{(v + \epsilon_r u)}{(\epsilon_r u + v) + (\epsilon_r u - v)\exp(-2p)} g_2(\xi) \tag{13d}$$

$$h_2(\xi) = \frac{\epsilon_r u}{v} \frac{(\epsilon_r u - v)\exp(-2p)}{(\epsilon_r u + v) + (\epsilon_r u - v)\exp(-2p)} g_2(\xi) \tag{13e}$$

$$\ell_2(\xi) = -\frac{2\epsilon_r u \exp(-p)}{(\epsilon_r u + v) + (\epsilon_r u - v)\exp(-2p)} g_2(\xi) \tag{13f}$$

where R_H be the reflection coefficient of the slab for H-polarization and it can be calculated as

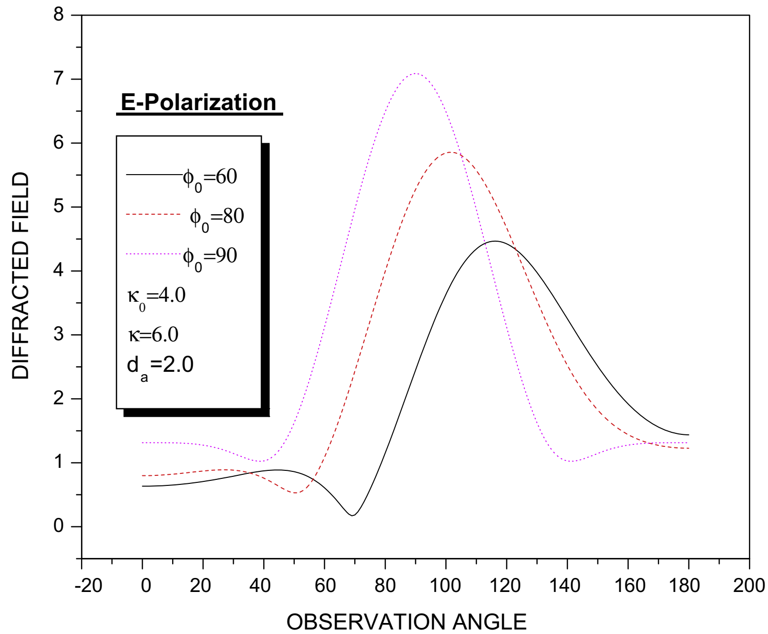

Fig. 2. Variation of diffracted fields with angle of incidence.

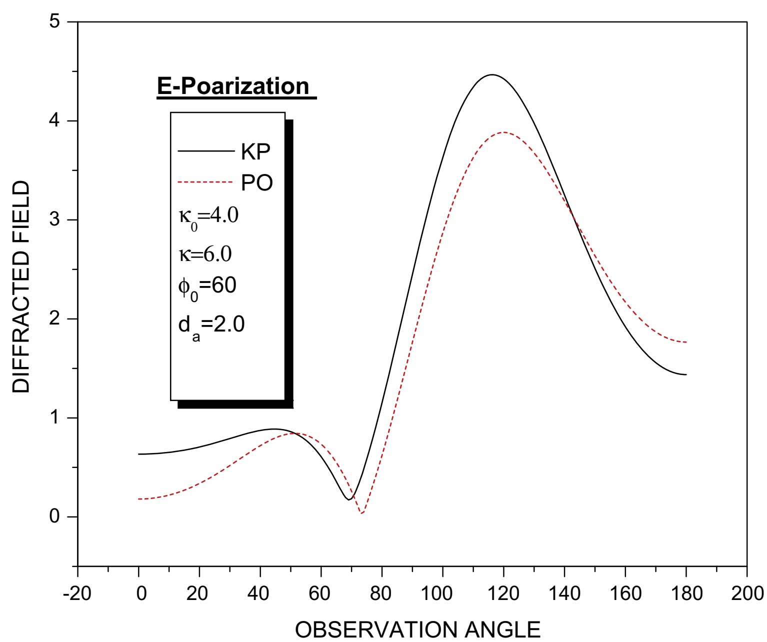

Fig. 3. Comparison of field patterns obtained through KP and PO methods.

$$R_H = \frac{(1-P_H^2)(e^* - e)}{\Delta_H}, \quad P_H = \frac{Z_0 \sin\phi_0}{Z \sin\phi_t}, \quad e = \exp(jkd\cos\phi_t),$$

$$e^* = \exp(-jkd\cos\phi_t), \quad \Delta_H = (1+P_H)^2 e - (1-P_H)^2 e^*$$

where $Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ and $Z = \sqrt{\frac{\mu_0}{\epsilon}}$ are intrinsic impedance of free space and dielectric slab, respectively. It is assumed that permeability is constant all over the space. The angle of refraction ϕ_t is related with the angle of incident ϕ_0 by

$$k\cos\phi_t = k_0\cos\phi_0$$

Eqs. (12a) and (12b) are the dual integral equations. Using discontinuous properties of Weber–Schafheitlin's integral, we can write

$$g_1(\xi) - f_1(\xi) - h_1(\xi) = \sum_{m=0}^{\infty} A_m \frac{J_{2m+1}(\xi)}{\xi} \tag{14a}$$

$$g_2(\xi) - f_2(\xi) - h_2(\xi) = \sum_{m=0}^{\infty} B_m \frac{J_{2m+2}(\xi)}{\xi} \tag{14b}$$

Substituting $f_{1,2}(\xi)$ and $h_{1,2}(\xi)$ given by (13a), (13b) and (13d), (13e) into (14), we can determine the weighting functions $g_1(\xi)$ and $g_2(\xi)$ as

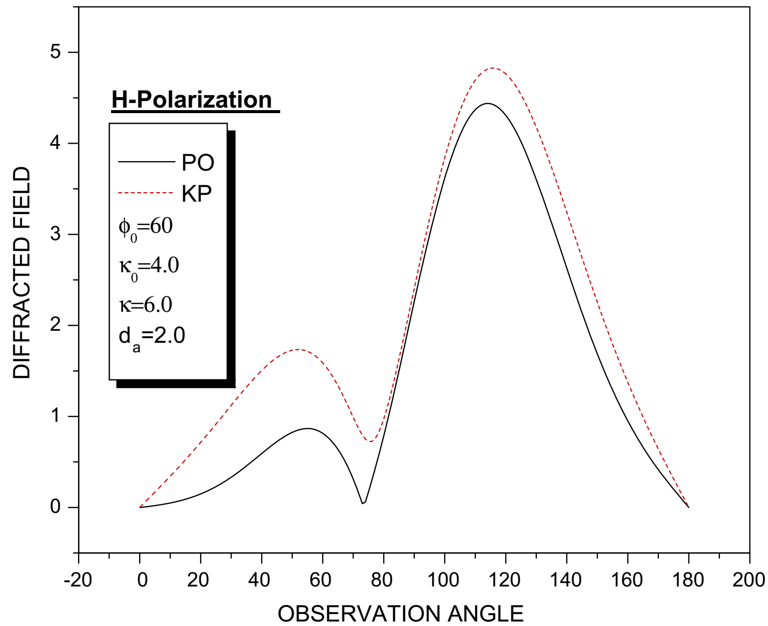

Fig. 4. Comparison of diffracted field patterns (H-Polarization).

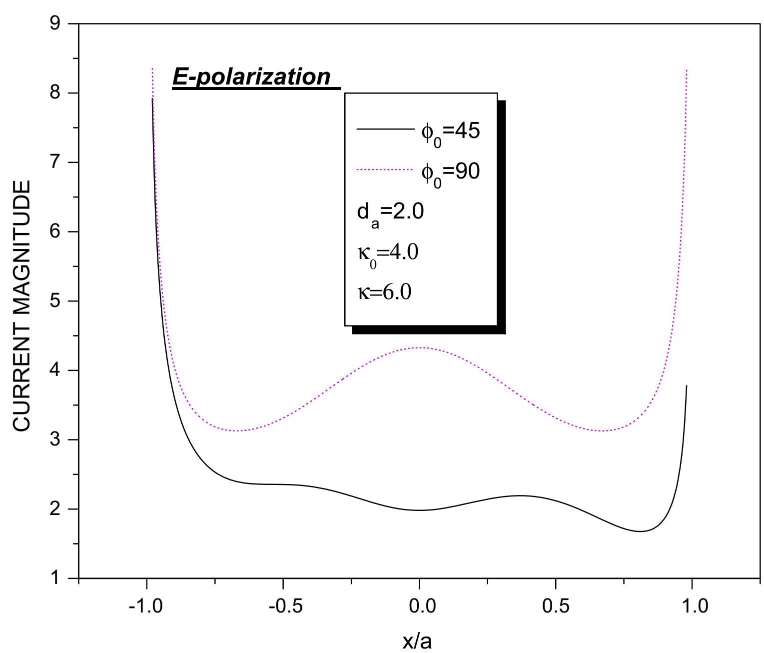

Fig. 5. Current distribution on the strip (E-polarization).

$$g_1(\xi) = v\frac{(\epsilon_r u + v) + (\epsilon_r u - v)\exp(-2p)}{(\epsilon_r u + v)^2 - (\epsilon_r u - v)^2\exp(-2p)}\sum_{m=0}^{\infty} A_m \frac{J_{2m+1}(\xi)}{\xi} \tag{15a}$$

$$g_2(\xi) = v\frac{(\epsilon_r u + v) + (\epsilon_r u - v)\exp(-2p)}{(\epsilon_r u + v)^2 - (\epsilon_r u - v)^2\exp(-2p)}\sum_{m=0}^{\infty} B_m \frac{J_{2m+2}(\xi)}{\xi} \tag{15b}$$

Substituting Eqs. (15) into (10b), and projecting the resulting equations into the functional space with elements $v_n^{\pm\frac{1}{2}}(x_a^2)$ [25]. Then we have

$$\sum\nolimits_{m=0}^{\infty} A_m K_K(2m+1, 2n+1) = -j\frac{\cos\phi_0}{\sin\phi_0}(1 - R_H)J_{2n+1}(\kappa\cos\phi_0) \tag{16a}$$

$$\sum\nolimits_{m=0}^{\infty} B_m K_K(2m+2, 2n+2) = \frac{\cos\phi_0}{\sin\phi_0}(1 - R_H)J_{2n+2}(\kappa\cos\phi_0) \tag{16b}$$

where $K_K(v,\mu)$ is defined by

$$K_K(v,\mu) = \int_0^{\infty} uv\frac{(\epsilon_r u + v) + (\epsilon_r u - v)\exp(-2p)}{(\epsilon_r u + v)^2 - (\epsilon_r u - v)^2\exp(-2p)}\frac{J_v(\xi)J_\mu(\xi)}{\xi^2}d\xi \tag{16c}$$

2.2.1. Field patterns

Diffracted far fields for region $y > 0$ can be calculated using saddle point method, and are given below

$$H_z^u(\phi_0,\phi)=\frac{\left(\epsilon_r\sin\phi+\sqrt{\epsilon_r-\cos^2\phi}\right)+\left[\epsilon_r\sin\phi+\sqrt{\epsilon_r-\sin^2\phi}\right)\exp\left(-j2k_0d\sqrt{\epsilon_r-\cos^2\phi}\right)}{\left(\sqrt{\epsilon_r-\cos^2\phi}+\epsilon_r\sin\phi\right)^2-\left(\epsilon_r\sin\phi-\sqrt{\epsilon_r-\cos^2\phi}\right)^2\exp\left(-j2k_0d\sqrt{\epsilon_r-\cos^2\phi}\right)}$$
$$\times\sum_{m=0}^{\infty}[A_mJ_{2m+1}(\kappa_0\cos\phi)+jB_mJ_{2m+2}(\kappa_0\cos\phi)]\frac{\sin\phi}{\cos\phi} \tag{17}$$

where ϕ be the angle of observation and $\epsilon_r=\frac{\epsilon}{\epsilon_0}$.

2.2.2. Current induced on the strip

Current density may be computed from the expression

$$\begin{aligned}J_x&=H_z|_{y=0+}-H_z|_{y=0-}\\&=\sum_{m=0}^{\infty}\int_0^{\infty}\left[A_m\frac{J_{2m+1}(\xi)}{\xi}\cos\xi x_a+B_m\frac{J_{2m+1}(\xi)}{\xi}\sin\xi x_a\right]d\xi\\&=\sum_{m=0}^{\infty}\left\{\frac{A_m}{2m+1}\cos[2m+1\sin^{-1}x_a]\right.\\&\left.\quad+\frac{B_m}{2m+2}\sin[(2m+2)\sin^{-1}x_a]\right\}\end{aligned}\tag{18}$$

3. Results and discussion

Far field patterns as a function of angle of observation, have been computed using Eq. (8) for E-polarization and Eq. (17) for H-polarization. These expressions contain expansion coefficients A_m and B_m. These expansion coefficients have been computed using Eq. (7) for E-polarization and Eq. (16) for H-polarization. $K_K(n,m)$ and $G_G(n,m)$ are the matrices, the elements of which are in terms of infinite integrals and have been computed as discussed in Appendix. These integrals have been computed for finite values of m and n values. The matrix size is taken as $m\times n=(2\kappa_0+1)\times(2\kappa_0+1)$. Fig. 2 gives the field patterns for different values of angle of incidence for E-polarization. It is evident from the patterns that as we increase the angle of incidence, the corresponding main lobe shifts towards the lower value of angle of observation ϕ. To check the validity of these patterns we have compared our results with those of obtained through physical optics (po). Figs. 3 and 4 give the comparisons for E- and H-polarized field, respectively, for $\phi_0=60$, $\kappa_0=4.0$, $\kappa=6.0$, $d_a=2.0$. The comparison seems good. We have also obtained the current distributions on the strip. Figs. 5 and 6 give the same. It is evident from the graphs, that the current density is maximum at the strip edges for E-polarization and reverse is true for H-polarization. Figs. 7 and 8 give the effects of strip widths on the field patterns. The graphs show, as we increase the strip width, the side lobes become prominent.

Appendix. Evaluation of $K_K(\nu,\mu)$ and $G_G(\nu,\mu)$

Integrals $G_G(\nu,\mu)$ and $K_K(\nu,\mu)$ can be represented in the form

$$\begin{aligned}G_G(\nu,\mu)&=\int_0^{\infty}\frac{1}{2}\frac{J_\nu(\xi)J_\mu(\xi)}{\sqrt{\xi^2-\kappa^2}}d\xi+\int_0^{\infty}F_G(\xi,\nu,\mu)d\xi\\&=\frac{G(\nu,\mu;\kappa)}{2}+\int_0^{\infty}F_G(\xi,\nu,\mu)d\xi\end{aligned}$$

$$\begin{aligned}K_K(\nu,\mu)&=\frac{1}{1+\epsilon_r}\int_0^{\infty}\frac{\sqrt{\xi^2-\kappa^2}}{\xi^2}J_\nu(\xi)J_\mu(\xi)d\xi+\int_0^{\infty}F_K(\xi,\nu,\mu)d\xi\\&=\frac{K(\nu,\mu;\kappa)}{1+\epsilon_r}+\int_0^{\infty}F_K(\xi,\nu,\mu)d\xi\end{aligned}$$

where

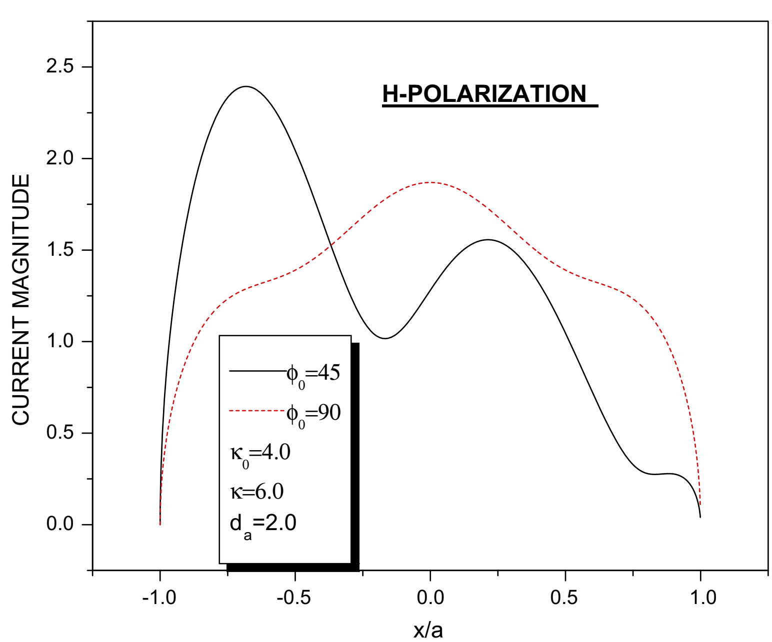

Fig. 6. Current distribution on the strip (H-Polarization).

$$K(v,\mu;\kappa) = \int_0^\infty \frac{\sqrt{\xi^2-\kappa^2}}{\xi^2} J_v(\xi) J_\mu(\xi) d\xi$$
$$G(v,\mu;\kappa) = \int_0^\infty \frac{J_v(\xi) J_\mu(\xi)}{\sqrt{\xi^2-\kappa^2}} d\xi$$
$$F_G(\xi; v,\mu) = \frac{v^2-u^2+(v-u)(3v-u)\exp(-2p)}{2v[(v+u)^2-(v-u)^2\exp(-2p)]} J_v(\xi) J_\mu(\xi) d\xi$$
$$F_K(\xi; v,\mu) = v\frac{(\epsilon_r u+v)[(1+\epsilon_r)\xi-(\epsilon_r u+v)]+(\epsilon_r u-v)[(1+\epsilon_r)\xi+(\epsilon_r u-v)]\exp(-2p)}{(1+\epsilon_r)[(v+\epsilon_r u)^2-(\epsilon_r u-v)^2\exp(-2p)]}\frac{J_v(\xi)J_\mu(\xi)}{\xi^2} d\xi$$

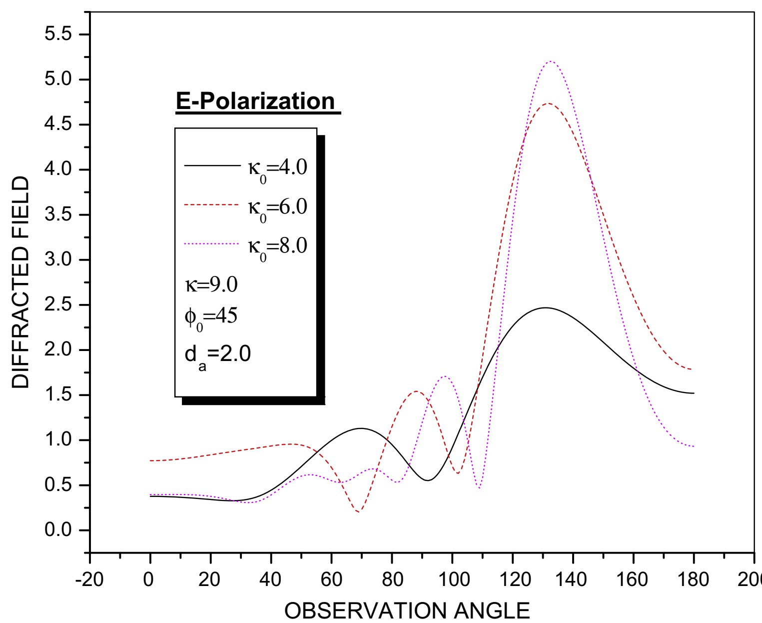

Fig. 7. Variation of diffracted field as a function of strip width.

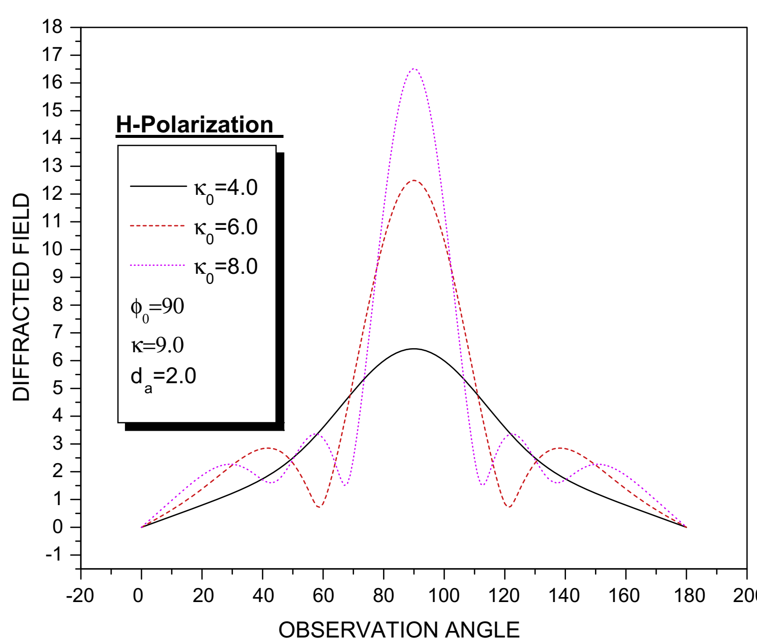

Fig. 8. Variation of field patterns with width of strip.

How to compute the integral $G(\nu,\mu;\kappa)$ and $K(\nu,\mu,\kappa)$ is discussed by Hongo [21]. The "correction integrals" $F_K(\xi;\nu,\mu)$ and $K_K(\nu,\mu)$ can be computed by the standard methods. The integrands of correction integrals have poles between $\kappa_0 \leqslant \xi \leqslant \kappa$. These poles can be avoided by deforming the path of integration.

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