PHYSICAL REVIEW D, VOLUME 65, 083520

Quintessence restrictions on negative power and condensate potentials

A. de la Macorra* and C. Stephan-Otto†
Instituto de Física, UNAM, Apartado Postal 20-364, 01000 México, Distrito Federal, Mexico
(Received 23 October 2001; published 9 April 2002)

We study the cosmological evolution of scalar fields that arise from a phase transition at some energy scale Λ_c. We focus on negative power potentials given by $V = c\Lambda_c^{4+n}\phi^{-n}$ and restrict the cosmologically viable values of Λ_c and n. We make a complete analysis of V and impose SN1a conditions on the different cosmological parameters. The cosmological observations ruled out models where the scalar field has reached its attractor solution. For models where this is not the case, the analytic approximated solutions are not good enough to determine whether a specific model is phenomenologically viable or not and the full differential equations must be solved numerically. The results are not fine-tuned since a change of 45% in the initial conditions does not spoil the final results. We also determine the values of N_c and N_f that give a condensation scale Λ_c consistent with gauge coupling unification, leaving only four models that satisfy unification and SN1a constraints.

DOI: 10.1103/PhysRevD.65.083520 PACS number(s): 98.80.Cq

I. INTRODUCTION

Recent cosmological results given by the superonova project SN1a [1] and the Maxima and Boomerang [2] observations have led us to conclude that the universe is flat and is expanding with an accelerating velocity. This conclusions show that the universe is now dominated by an energy density with a negative pressure with $\Omega_\phi = 0.7 \pm 0.1$ and $w_\phi < -2/3$ [3]. This energy is generically called the cosmological constant. An interesting parametrization of this energy density is in terms of a scalar field with only a gravitational interaction called quintessence [4]. The evolution of the scalar field has been widely studied and some general approaches can be found in [5,6]. The evolution of the scalar field ϕ depends on the functional form of its potential V and a late time accelerating universe constrains the form of the potential [6].

In this paper we will concentrate on negative power potentials because they lead to an acceptable phenomenology and because they are naturally obtained from gauge group dynamics. Negative power potentials have been extensively studied [4–11] first in [7] and then as tracker fields in [4]. Steinhardt *et al.* [4] showed that a scalar field with a negative power potential $V = c\phi^{-n}$ with $n > 5$ has already reached its tracker solution but is not cosmologically acceptable because it has $w_\phi > -0.52$. However, if the scalar field has not reached its tracker solution by today, we will show that the models may lead to an acceptable phenomenology and the final results depend on the initial conditions and on the value of n. Contrary to the tracker models, no analytic solution is good enough to determine the value of $w_{\phi o}$ (from now on the subscript o will refer to present day values) and it is sensitive to the whole dynamics. We will solve the differential equations numerically and we will constrain the values n of cosmologically viable models, including the big bang nucleosynthesis (NS) constraints [12]. We will also give approximate analytical solutions.

Tracker solutions are widely favored because they do not have a fine-tuning problem in the initial conditions. But even more, they are independent of the initial conditions since the range of the initial conditions can vary by up to 100 orders of magnitude. The models with $n < 5$ do depend on the initial conditions but it is important to remark that they *do not* have a fine-tuning problem. The initial conditions can vary by up to 45%; the solutions are still fine and the values of the initial conditions are completely ‘‘natural,’’ i.e., they are of the same order of magnitude as the other relevant cosmological parameters. So, to conclude, one thing is to have a model with no dependence on the initial conditions and another is to have a fine-tuning problem. ‘‘Natural’’ models in physics should not have fine-tuning problems but they do in general depend on the initial values as it is the case for our models. However, for any initial conditions we will end up with $-1 \leq w_{\phi o} \leq -2/(2+n) = w_{tr}$, where n gives the inverse power and w_{tr} is the tracker value.

Negative power potentials [9–11] can be obtained using the Affleck-Dine-Seiberg (ADS) superpotential [13]. The condensation scale Λ_c of the gauge group SU(N_c) can be determined from the high energy scale (Λ) using renormalization group equations in terms of N_c, N_f, and it is then natural to ask if it is possible to have a common gauge coupling unification with the standard model (SM) gauge groups [11]. We will show that this is indeed possible and we will give the values of N_c, N_f where gauge coupling unification is achieved.

The cosmological picture in the case of gauge coupling unification is very pleasing. We assume gauge coupling unification at a scale Λ for all gauge groups (as predicted by string theory) and then let all fields evolve. At the beginning all fields, SM and those from the SU(N_c) gauge group, are massless and redshift as radiation until we reach the condensation scale Λ_c. Below this scale the fields of the SU(N_c) group will dynamically condense and we use the ADS potential to study their cosmologically evolution. Interestingly enough, the relative energy density of the SU(N_c) group Ω_ϕ drops quickly, independently of the initial conditions, and it

*Email address: macorra@fisica.unam.mx
†Email address: stephan@servidor.unam.mx

is close to zero for a long period of time, which may include nucleosynthesis, until very recently (around one e-fold of inflation). The energy density of the universe is at present time dominated by the scalar field with $\Omega_\phi \simeq 0.7$ and a negative pressure $w_\phi < -2/3$ leading to an accelerating universe [3].

The paper is organized as follows. In Sec. II we give the general framework to derive the scalar potential for ϕ from non-Abelian gauge dynamics using the ADS potential. In Sec. III we analyze the cosmological evolution of ϕ and we concentrate on the nonattractor regime. We derive analytic formulas for $w_{\phi o}$ and ϕ_o as functions of the initial conditions and of n, and we discuss in detail the possible choices of initial conditions and show that the models do not have a fine-tuning problem. In Sec. IV we constrain the values of n in order to have $w_{\phi o} < -2/3$, while in Sec. V we comment on the possibility of having models with a gauge coupling constant unified with the couplings of the standard model and we explicitly give these models. In Sec. VI we give further examples. Finally, we summarize and conclude in Sec. VII.

II. POTENTIALS OF THE FORM $V=c\Lambda_c^{4+n}\phi^{-n}$

In this work we study the quintessence field (scalar) ϕ with negative power potentials that arise from a phase transition at some stage of the evolution of the universe. The energy scale of the phase transition is given by Λ_c and the initial value of the scalar field ϕ is naturally given by $\phi_i = \Lambda_c$ since it is the relevant scale for the transition. The potential we will consider is of the type

$$V = c^2 \Lambda_c^{4+n} \phi^{-n} \tag{1}$$

with c a constant (we will comment on the value of c in Sec. IV) and has a runaway behavior.

This class of models will have a vanishing potential $V \equiv 0$ for energy scales above Λ_c since the phase transition has not taken place yet and there is no ϕ field. At the phase transition energy scale Λ_c a potential $V(\Lambda_c) \simeq \Lambda_c^4$ and a field $\phi(\Lambda_c) = \Lambda_c$ are generated. Below Λ_c, the ϕ field becomes dynamic and it evolves to its minimum. The cosmological evolution depends on the functional form of V and for Eq. (1) with $n>0$ we expect ϕ to roll down its potential. This class of potential has been chosen because they can be obtained from a phase transition of non-Abelian gauge dynamics (see Sec. II A) [13] and because they lead to a quintessence interpretation of the ϕ field. However, if some other physical process also leads to an inverse power scalar potential the cosmological evolution studied in Sec. III and the conclusions remain valid.

The energy scale Λ_c is expected to be considerably smaller than the reduced Planck mass m_{Pl}, so the initial value $\phi_i / m_{Pl} = \Lambda_c / m_{Pl}$ is much smaller than 1 and this has interesting consequences for the cosmological evolution of ϕ (from now on we will set the reduced Planck mass to 1, $m_{Pl}^2 = G/8\pi \equiv 1$).

The normalization of the field is important and we will consider, for simplicity, the ϕ field to be canonically normalized, $L_k = (K_\phi^\phi)^{-1} |\partial_\mu \phi|^2$ with $K_\phi^\phi = 1$ and $K_\phi \equiv \partial K / \partial \phi$. However, the complete Kahler potential K is in general not known. The canonically normalized field ϕ' can be defined by $\phi' = g(\phi, \bar{\phi})\phi$ with the function g given by solving the differential equation $K_\phi^\phi = (g + \phi g_\phi + \bar{\phi} g_{\bar{\phi}})^2$. For $\phi \ll 1$ we do not expect any large contributions to the kinetic term but for $\phi \sim 1$ the Kahler potential could give a significant contribution and could spoil the runaway and quintessence behavior of ϕ. In order to see this, we can expand the Kahler potential as a power series $K = |\phi|^2 + \Sigma_i a_i |\phi|^{2i}/2i$ with a_i some constants of order 1 and to be determined by the specific model. If we approximate, for simplicity, the canonically normalized field ϕ' by $\phi' = (K_\phi^\phi)^{1/2}\phi$, the potential in Eq. (1) is then given by

$$\begin{aligned} V &= (K_\phi^\phi)^{-1} |W_\phi|^2 = c^2 \Lambda_c^{4+n} \phi^{-n} (K_\phi^\phi)^{-1} \\ &= c^2 \Lambda_c^{4+n} \phi'^{-n} (K_\phi^\phi)^{n/2-1}. \end{aligned} \tag{2}$$

For $n<2$ the exponent term of K_ϕ^ϕ in Eq. (2) is negative so it will not spoil the runaway behavior of ϕ, but for $n>2$ the extra terms could stabilize the potential. In the absence of a better understanding of K we will work with canonically normalized fields, but we should keep in mind that for $n<2$ the results are robust while for $n>2$ the contribution from the Kahler potential could spoil our results and must be determined.

A. ADS potential

The potential in Eq. (1) can be obtained from the nonperturbative dynamics of a non-Abelian asymptotically free gauge group $SU(N_c)$ with N_f chiral + antichiral fields Q in $N=1$ supersymmetric theory. At energy scales much larger than the condensation scale the gauge coupling constant is small and the Q fields are free elementary fields. As the universe expands and cools down, the energy of the elementary fields Q becomes smaller while the gauge coupling constant grows. When the gauge coupling constant has the critical value to condense the Q fields, then all the elementary fields will no longer be free and they will form "mesons" and "baryons," as in QCD. This effect takes place at the condensation scale Λ_c and below this scale the correct description of the dynamics of the non-Abelian gauge group is in terms of the condensates ϕ. In order to study the dynamics of these fields ϕ we use the ADS superpotential, which is exact (i.e., it does not receive radiative or nonperturbative contributions) and is given by $W(\phi) = (N_c - N_f)(\Lambda_c^{3N_c - N_f}/\det\langle Q\tilde{Q}\rangle)^{1/(N_c - N_f)}$ [13]. In terms of the gauge singlet combination of chiral and antichiral bilinear terms $\phi = \langle Q\tilde{Q}\rangle$ the globally supersymmetric scalar potential is given by Eq. (2) with $c = 2N_f$, $\det\langle Q\tilde{Q}\rangle = \Pi_{j=1}^{N_f} \phi_j^2$, and $n = 2 + 4N_f/(N_c - N_f)$ [9–11].

If we wish to study models with $0<n<2$, which are cosmologically favored as we will see in Sec. IV, we need to consider the possibility that not all N_f condensates ϕ_i become dynamic but only a fraction ν (with $N_f \geq \nu \geq 1$), and we also need $N_f > N_c$. It is important to point out that even though it has been argued that for $N_f > N_c$ there is no nonperturbative superpotential W generated [13] this is not always the case [14]. The possibility of having $\nu \neq N_f$ can be reached with a gauge group with unmatching numbers of chiral and antichiral fields or if some of the chiral fields are also charged under another gauge group. In this latter case we have $c=2\nu, n=2+4\nu/(N_c-N_f)$, and $N_f-\nu$ condensates fixed at their vaccum expectation value (VEV) $\langle Q\tilde{Q}\rangle = \Lambda_c^2$ [11].

III. EVOLUTION OF ϕ

We will now determine the cosmological evolution of a scalar field ϕ with an inverse power potential regardless of what physical process caused it. We will concentrate on potentials that have not reached the tracker solution yet, since they can give the correct value of $w_{\phi o}$, and we will give approximately analytic solutions for $w_{\phi o}$ and ϕ_o.

The cosmological evolution of ϕ with an arbitrary potential $V(\phi)$ can be determined from a system of differential equations describing a spatially flat Friedmann-Robertson-Walker universe in the presence of a barotropic fluid energy density ρ_γ that can be either radiation or matter; they are

$$\dot{H} = -\frac{1}{2}(\rho_\gamma + p_\gamma + \dot{\phi}^2),$$

$$\dot{\rho} = -3H(\rho+p),$$

$$\ddot{\phi} = -3H\dot{\phi} - \frac{dV(\phi)}{d\phi}, \tag{3}$$

where H is the Hubble parameter, $\dot{\phi}=d\phi/dt$, and ρ (p) is the total energy density (pressure). We use the change of variables $x \equiv \dot{\phi}/\sqrt{6}H$ and $y \equiv \sqrt{V}/\sqrt{3}H$ and Eqs. (3) take the following form [15,6]:

$$x_N = -3x + \sqrt{\frac{3}{2}}\lambda y^2 + \frac{3}{2}x[2x^2 + \gamma_\gamma(1-x^2-y^2)],$$

$$y_N = -\sqrt{\frac{3}{2}}\lambda xy + \frac{3}{2}y[2x^2 + \gamma_\gamma(1-x^2-y^2)],$$

$$H_N = -\frac{3}{2}H[2x^2 + \gamma_\gamma(1-x^2-y^2)], \tag{4}$$

where N is the logarithm of the scale factor a, $N \equiv \ln(a)$, $f_N \equiv df/dN$ for $f=x,y,H$, $\gamma_\gamma = 1+w_\gamma$, and $\lambda(N) \equiv -V'/V$ with $V'=dV/d\phi$. In terms of x,y the energy density parameter is $\Omega_\phi = x^2+y^2$ while the equation of state parameter is given by $w_\phi \equiv \rho_\phi/p_\phi = (x^2-y^2)/(x^2+y^2)$.

The Friedmann or constraint equation for a flat universe $\Omega_\gamma + \Omega_\phi = 1$ must supplement Eqs. (4), which are valid for any scalar potential as long as the interaction between the scalar field and matter or radiation is gravitational only. This set of differential equations is nonlinear and for most cases has no analytical solutions. A general analysis for arbitrary potentials was performed in [6]; the conclusion there is that all model dependence falls on two quantities: $\lambda(N)$ and the constant parameter γ_γ. In the particular case given by $V \propto 1/\phi^n$ we find $\lambda \to 0$ in the asymptotic limit. If we think the scalar field appears well after Planck times we have $\lambda_i = n\, m_{Pl}/\Lambda_c \gg 1$ (the subscript i corresponds to the initial value of a quantity). An interesting general property of these models is the presence of a many e-fold scaling period in which λ is practically a constant and $\Omega_\phi \ll 1$. Figure 1 shows the rapid arrival and long permanence of this parameter at its constant value, together with the final decay to zero. In this last regime we have $\lambda \to 0$, which implies $x_N/x<0$ and $y_N/y>0$ [6], leaving us with $\Omega_\phi \equiv x^2+y^2 \to 1$ and $w_{\phi o} \equiv (x^2-y^2)/(x^2+y^2) \to -1$, which are in accordance with a universe dominated by a quintessence field whose equation of state parameter agrees with positively accelerated expansion. The development of Ω_ϕ can be in agreement with the restriction of the nucleosynthesis stage $\Omega_\phi(\mathrm{NS})<0.1$ [12] as well as with the observational result $\Omega_{\phi o}=0.7$ (the subscript o refers to present day quantities). This can be observed in Fig. 2, together with the evolution of w_ϕ which satisfies the condition $w_{\phi o}<-2/3$ [3].

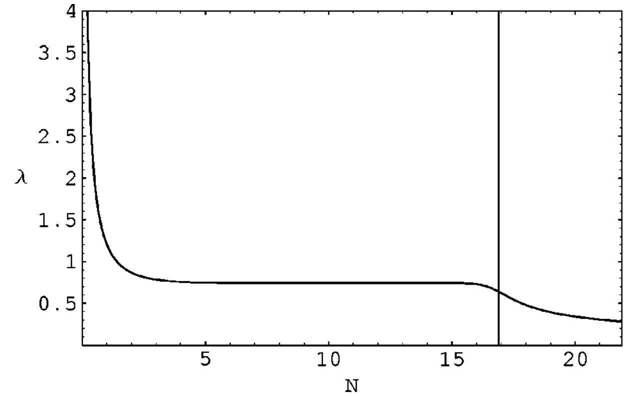

FIG. 1. Evolution of λ for $n=1$. The vertical line marks the time at $\Omega_{\phi o}=0.7$ with $N_{total}=17.04$.

The analysis of inverse power potentials has been extensively studied [4–11]. However, the analysis has not been specific enough to determine their viability in describing the late evolution of our universe. In [4] the scalar field was required to track before the present day and this imposes a constraint on n to be larger than 5, thus ruling this model out since it has $w_{\phi o}>-0.52$ in contradiction to the SN1a data.

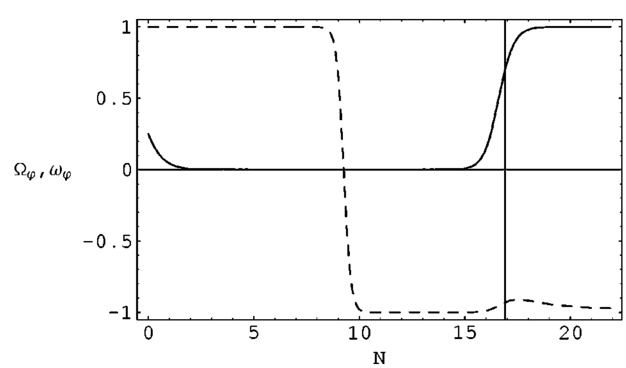

FIG. 2. Evolution of Ω_ϕ (solid curve) and w_ϕ (dashed curve) for $n=1$. The vertical line marks the time at $\Omega_{\phi o}=0.7$.

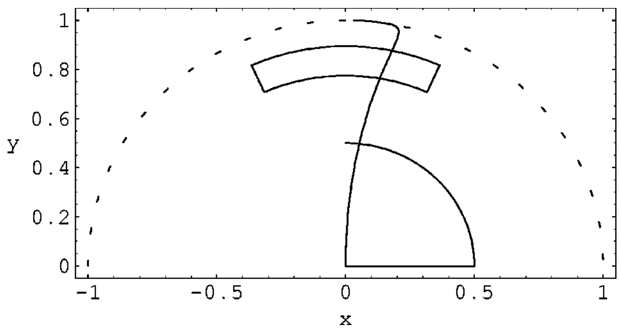

FIG. 3. Phase plane x-y for $n=1$. Starting point is (0,0.5). The region defined by $w_{\phi o} < -2/3$ and $\Omega_{\phi o} = 0.7 \pm 0.1$ is shown.

The models we will concentrate on are, therefore, models with $n<5$ where ϕ has not reached its tracker value.

For future reference we give now the scaling value of[1] ϕ [4]:

$$\phi_{sc} = \begin{cases} \phi_i + \sqrt{6\Omega_{\phi i}} & \text{for } \Omega_{\phi i} < 1/2, \\ \phi_i + \sqrt{6}\left[\frac{1}{\sqrt{2}} + \frac{1}{2}\log\left(\frac{\Omega_{\phi i}}{1-\Omega_{\phi i}}\right)\right] & \text{for } \Omega_{\phi i} > 1/2. \end{cases} \tag{5}$$

The scaling value depends only on the initial conditions $\Omega_{\phi i}$; it is independent of Λ_c, H_i, since $\phi_i \ll 1$. The tracker value of w is given by [4]

$$w_{tr} = -1 + \frac{n}{2+n}(w_\gamma + 1), \tag{6}$$

and it is an attractor solution valid for large n, when ϕ is already tracking. In the tracker limit [4], i.e., $n=5$, from Eq. (6) one has $w_{tr} = -0.28$ but the value obtained numerically is only $w_o = -0.52$ for $\Omega_{\phi i} > 0.25$. For smaller n the discrepancy is even worse since the scalar field has not reached its tracker value, obtained from Eqs. (6) and (15),

$$\phi_{tr} = \sqrt{\frac{n(2+n)\Omega_{\phi o}}{3}}, \tag{7}$$

which is larger than ϕ_{sc} if $\Omega_{\phi i} < [n(2+n)/18]\Omega_{\phi o}$.

A semianalytic approach is useful to study some properties of the differential equation system given by Eqs. (4). To do this we initially consider only the terms that are proportional to λ, since $\lambda \gg 1$; then we follow the evolution of x,y, and H so that every period has a characteristic set of simplified differential equations. The parameter Ω_ϕ is adequate to divide the process into four periods, the first one being a short lapse in which $\Omega_\phi = \text{const}$, easy to recognize in Fig. 3; the second is defined from the fall of this parameter to negligible values; the third is the so-called scaling period; and in the fourth Ω_ϕ is again considerable, eventually reaching the value 0.7.

The phase plane x-y provides an illustrative approach, useful for the analysis. We see, from Fig. 3, that the system follows a circular path at first with $\equiv x^2 + y^2 = \Omega_{\phi i}$ constant, and ends up with $x^2 \simeq \Omega_{\phi i} \gg y^2$. Then x and y decrease to negligible values; this situation prevails throughout the scaling period. Finally, a growth in both parameters causes Ω_ϕ to reach what we set as the final value 0.7 preferred by observational results. From the restriction over the equation of state parameter $w_\phi < -2/3$ and the observational range for $\Omega_{\phi o} = 0.7 \pm 0.1$, we can define a region limited by the expressions $y^2 = [(1-w_\phi)/(1+w_\phi)]x^2$ with $w_\phi = -2/3$ and $y^2 = \Omega_{\phi o} - x^2$ with $0.6 \leq \Omega_{\phi o} \leq 0.8$.

The minimal value y_{min} of y after its initial steep descent is given from Eq. (4) with $y_N = 0$, $x^2 \simeq \Omega_{\phi i}$, and $\lambda x\sqrt{3/2} = -H_N/H = 3\gamma_\gamma/2$ by

$$y_{min} = y_i\left(\frac{\Lambda_c}{\phi_{min}}\right)^{n/2},$$

$$\phi_{min} = \frac{n}{4}\sqrt{6\Omega_{\phi i}}, \tag{8}$$

and we have approximated $H_{min} \simeq H_i$ in Eq. (8). Shortly after y reaches its minimum value the scaling period begins. In this period we neglect the term proportional to λ in Eqs. (4) to find

$$\frac{y_N}{y} = -\frac{H_N}{N}, \tag{9}$$

which leads to $yH = H_{min}y_{min} \simeq H_i y_{min}$. Notice that the dependence of λ on y and H is given by $\lambda = A\ (yH)^{2/n}$ with A a constant; therefore from Eq. (9) we have $\lambda = \text{const}$ (i.e., $\phi = \text{const}$) during all of the scaling period; this holds for any n. Furthermore, we may neglect squared terms in x and y in the third equation of system (4), since they are small, to get the expressions

$$H = H_i e^{-3\gamma_\gamma N/2},$$

$$y = y_{min} e^{3\gamma_\gamma N/2}. \tag{10}$$

The quantity y has an increasing exponential form for almost the whole process, so the duration of this regime can be seen as the total time (see Fig. 4). Now, in order to calculate the number of e-folds from the initial value to the present day, we consider Eq. (10), to end up with

$$N_{total} = \frac{2}{3}\ln\left(\frac{y_o}{y_{min}}\right) \tag{11}$$

with y_{min} given by Eq. (8) and $y_o \simeq 0.8$ (to have $\Omega_{\phi o} = 0.7, w_{\phi o} < -2/3$). The evolution of ln($x$), ln($y$), and ln($H$) as functions of N is seen in Fig. 4.

If we consider Eqs. (10) and assume that the end of the scaling period is very close to today we get an approximated

[1]Our value of ϕ_{sc} differs in the case of $\Omega_{\phi i} > 1/2$ by a factor of $1/\sqrt{2}$ from that in [4] and the authors of [4] use $M_{Pl} = 1$ instead of $m_p = M_{Pl}/\sqrt{8\pi} = 1$ as we do.

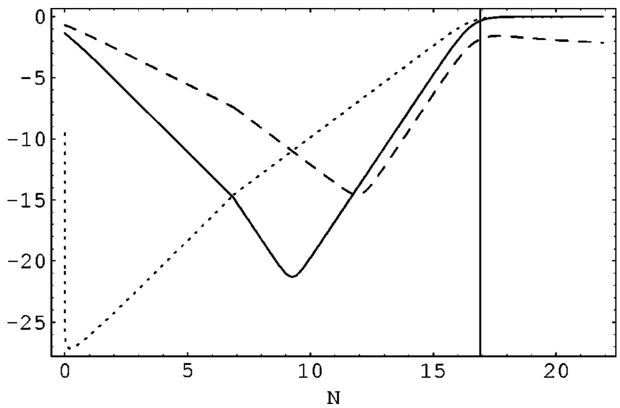

FIG. 4. Evolution for $n=1$ of $\ln(x)$, $\ln(y)$, and $\ln(\Omega_\phi)$ (dashed, dotted, and solid lines, respectively). The vertical line marks the time at $\Omega_{\phi o}=0.7$.

equation $y_o H_o \simeq H_{min} y_{min} = H_i y_i(\phi_i/\phi_{min})$. This, together with Eq. (8) and the definition of $y_i^2 = c^2\Lambda_c^4/3H_i^2$, establishes an expression for Λ_c, the energy scale at which the scalar field appears, in terms of y_o, H_o:

$$\Lambda_c = \left(\frac{3y_o^2 H_o^2}{c^2}\right)^{1/(4+n)} \phi_{min}^{n/(4+n)} \tag{12}$$

and $\phi_{min}=(n/4)\sqrt{6\Omega_{\phi i}}$. The latter expression is a semianalytic calculation of the initial energy scale of a specific model. Finally, the value y_o is set to be of the order of 0.8 to satisfy simultaneously $\Omega_{\phi o}=0.7$ and the condition $w_{\phi o}< -2/3$.

Of course, we could have guessed expression (12) using the definition of $y_o^2=(c^2\Lambda_c^{4+n}/3H_o^2)\phi_o^{-n}$ to give [9]

$$\Lambda_c = \left(\frac{3H_o^2 y_o^2 \phi_o^n}{c^2}\right)^{1/(4+n)} \simeq H_o^{2/(4+n)}, \tag{13}$$

and the last equality holds approximately since we expect to have y_o, ϕ_o of the order of 1.

Now, we wish to determine the values of $w_{\phi o}$, y_o, and ϕ_o. We use the differential equation for $\gamma_\phi = w_\phi + 1$ and Ω_ϕ [16]

$$(\gamma_\phi)_N = 3\gamma_\phi(2-\gamma_\phi)\left(\lambda\sqrt{\frac{\Omega_{\phi o}}{3\gamma_\phi}}-1\right),$$

$$(\Omega_\phi)_N = 3(\gamma_\gamma-\gamma_\phi)\Omega_\phi(1-\Omega_\phi). \tag{14}$$

We see that γ_ϕ is extremized at $w_\phi=\gamma_\phi-1=-1,1$ and at $w_\phi=-1+\lambda^2\Omega_\phi/3=-1+n^2\Omega_\phi/3\phi^2$. We have checked that the value of w_ϕ at the maximum evaluated at ϕ_o is a very good approximation (within 5% of the numerical value)

$$w_{\phi o} = -1 + \frac{n^2\Omega_{\phi o}}{3\phi_o^2}. \tag{15}$$

Equation (15) should be compared with [9] which differs by the factor of $\Omega_{\phi o}\simeq 0.7$. Of course, if we do not have the exact value of ϕ_o Eq. (15) is not very useful. In order to determine ϕ_o we evolve Eqs. (14) and Eq. (4) from present day values to the scaling regime where $w_{\phi sc}\simeq -1$ and $x_{sc}^2 \ll y_{sc}^2 \simeq \Omega_{\phi sc}$ with the condition $\Omega_{\phi o}=0.7$. This evolution is model independent if $0\leqslant w_\phi+1\ll 1$ [16] and from the definition of y^2 we have

$$\frac{c^2}{3}\Lambda_c^{4+n} = y_o^2 H_o^2 \phi_o^n = y_{sc}^2 H_{sc}^2 \phi_{sc}^n, \tag{16}$$

and we obtain

$$y_o^2\phi_o^n = \frac{y_{sc}^2 H_{sc}^2 \phi_{sc}^n}{H_o^2} = \phi_{sc}^n \Omega_{\phi o}, \tag{17}$$

where we have used $H_N/H \simeq -3(1-\Omega_\phi)/2, H_{sc}^2/H_o^2 = (1-\Omega_{\phi o})e^{3\Delta N}$, and $y_{sc}^2 \simeq \Omega_\phi = \Omega_{\phi o}e^{-3\Delta N}/(1-\Omega_{\phi o}+\Omega_{\phi o}e^{-3\Delta N})$, $\Delta N \gg 1$ [16]. Using Eqs. (17), (15), and (5) we can solve easily for ϕ_o and/or y_o in terms of $\Omega_{\phi o}$, n, and $\Omega_{\phi i}$ (via ϕ_{sc}),

$$\phi_o^2 - \phi_{sc}^n\phi_o^{2-n} - \frac{n^2}{6}\Omega_{\phi o} = 0 \tag{18}$$

or equivalently

$$y_o^2\left(\frac{n^2\Omega_{\phi o}^2}{6(\Omega_{\phi o}-y_o^2)}\right)^{n/2} = \phi_{sc}^n\Omega_{\phi o}. \tag{19}$$

In order to analytically solve Eqs. (18),(19) we need to fix the value of n, and we can determine $w_{\phi o}$ by putting the solution of Eq. (18) into Eq. (15). Equation (18) can be rewritten as $\phi_o=\phi_{sc}(1-n^2\Omega_{\phi o}/6\phi_o^2)^{-1/n}$ and we see that $\phi_o>\phi_{sc}$ and $\phi_o>n\sqrt{\Omega_{\phi o}/6}$ and that ϕ_o is of the order of 1 ($\Omega_{\phi o}\sim 0.7$) regardless of the initial conditions. However, the exact value does indeed depend on the initial conditions, but for any initial conditions we will have $-1\leqslant w_{\phi o}\leqslant w_{tr}$.

If $\gamma_{\phi o}=n^2\Omega_{\phi o}/6\phi_o^2\ll 1$ one has $\phi_o\simeq\phi_{sc}$ and for the simple cases of $n=1,2$, and 4 we can solve explicitly for ϕ_o. We find $\phi_o|_{n=1}=\phi_{sc}/2+\sqrt{9\phi_{sc}^2+6\Omega_{\phi o}}/6$, $y_o^2|_{n=1}=\phi_{sc}(-3\phi_{sc}+\sqrt{9\phi_{sc}^2+6\Omega_{\phi o}})$, $\phi_o|_{n=2}=\sqrt{\phi_{sc}^2+2\Omega_{\phi o}/3}$, $y_o^2|_{n=2}=3\phi_{sc}^2\Omega_{\phi o}/(3\phi_{sc}^2+2\Omega_{\phi o})$, and $\phi_o|_{n=4}=\sqrt{4\Omega_{\phi o}/3+\sqrt{9\phi_{sc}^2+16\Omega_{\phi o}}/3}$, $y_o^2|_{n=4}=\Omega_{\phi o}-8\Omega_{\phi o}^2/(4\Omega_{\phi o}+\sqrt{9\phi_{sc}^2+16\Omega_{\phi o}})$, respectively. Notice that the values of ϕ_o, $w_{\phi o}$ at $\Omega_{\phi o}=0.7$ *do not* depend on H_i or H_o and depend only on $\Omega_{\phi i}$ (through ϕ_{sc}) and n.

We show in Fig. 5 how $w_{\phi o}$ varies for different initial conditions $\Omega_{\phi i}$ with $n=18/7\simeq 2.57$ fixed. We see that for larger $\Omega_{\phi i}$ we end up with a smaller $w_{\phi o}$ and this is a generic result as can be seen from Eqs. (18),(5), and (15) since for larger $\Omega_{\phi i}$ one has a larger ϕ_{sc}, ϕ_o and therefore a smaller $w_{\phi o}$. This can be seen also from Fig. 6 where for small $\Omega_{\phi i}$ a plateau arises in w_ϕ (the field ϕ has already reached its tracker value by the present day). From Eqs. (15),(18) we notice that for smaller n one gets a smaller $w_{\phi o}$ (see Fig. 7).

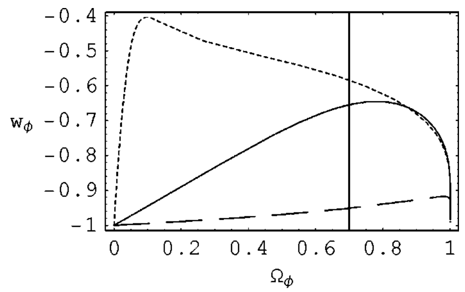

FIG. 5. Variations of $\Omega_{\phi i}$ lead to different physical situations given by $w_{\phi o}$. We have taken $\Omega_{\phi i}=0.9,0.25,10^{-10}$ (dashed, solid, and dotted lines, respectively). The vertical line marks the time at $\Omega_{\phi o}=0.7$.

A. Initial conditions

It is well known that an inverse power law potential leads to attractor solutions [4] that are therefore independent of the initial conditions (one has 100 orders of magnitude range for $\Omega_{\phi i}$). However, as pointed out by Steinhardt *et al.* [4], this is only the case for $n>5$; for smaller values of n the field ϕ has not necessarily reached its tracker value at the present time. However, even though in these cases the present day quantities depend on the initial conditions there is no fine-tuning problem since we can vary the initial conditions over a wide range of values (i.e., 45%) and the end results are still physically acceptable.

The differential equations given in Eqs. (4) depend on values of x,y, $\lambda=n/\phi$ but they do not depend on the absolute value of H, i.e., we have the same evolution for x,y, H as for x,y, $H'=kH$ where k is an arbitrary constant. This scaling freedom allows us to set the normalization of H as we wish, and in particular to have $\Omega_{\phi o}=0.7$ at $H=H_o$ for any values of the initial conditions x_i, y_i. This implies that we can have a quintessence model for arbitrary initial conditions. Once H_o is fixed we get $\Lambda_c \simeq H_o^{2/(4+n)}$ [Eq. (20)] and $H_i=c\Lambda_c^2/y_i$ (from the definition of y_i). One of the main differences from the tracker solution is that $w_{\phi o}$ and ϕ_o do

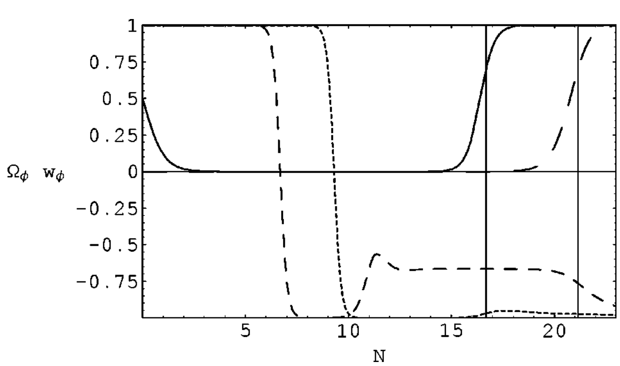

FIG. 6. Evolution of Ω_ϕ and w_ϕ for $n=1$. Parameter Ω_ϕ evolves in a similar way for $\Omega_{\phi i}=0.25$ (solid line) and $\Omega_{\phi i}=10^{-10}$ (long-dashed line), varying only in the total time, $N_{total}=16.7$ or 21.1 in the previous order. A discrepancy between the two cases at $\Omega_{\phi o}=0.7$ is seen for parameter w_ϕ because it has for $\Omega_{\phi i}=0.25$ a local maximum with $w_{\phi o}=-0.93$ (dotted line) while for $\Omega_{\phi i}=10^{-10}$ (short-dashed line) a plateau appears with $w_{\phi o}=-0.76$. The vertical lines mark the time at $\Omega_{\phi o}=0.7$.

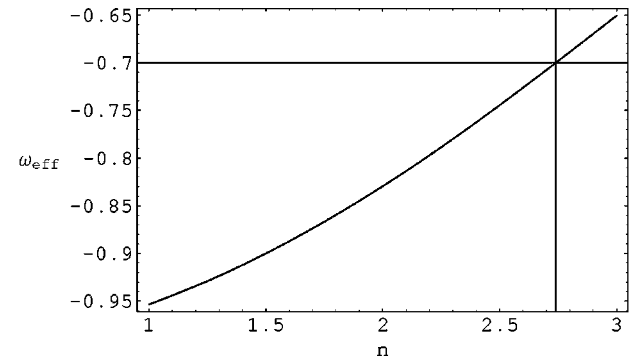

FIG. 7. Restriction on n from $w_{eff}<-0.7$ with $c=1$ and $\Omega_{\phi i}=0.35$.

not have the same values in all cases but depend on $\Omega_{\phi i}$ as can be seen from Eqs. (15) and (18). However, the dependence on $\Omega_{\phi i}$ is mild since, for example, for $n=1$ one has that ϕ_o only depends as $\sqrt{\Omega_{\phi i}}$. For this reason once H_o and $\Omega_{\phi i}$ are fixed we still have a wide range of values of $\Omega_{\phi i}$ giving the correct phenomenology, i.e., $\Delta\Omega_{\phi i}=45\%$. For $\Omega_{\phi i}\ll 1$, i.e., $y_i=c\Lambda_c/H_i\ll 1$, with a fixed Λ_c one has a larger H_i and the time of expansion up to the present day is also larger. In this case it is possible that the ϕ field has already reached its tracker value, even for $n<5$, and we would have $w_{\phi o}=w_{tr}=-2/(2+n)$. Indeed, it can be seen from Eq. (18) that $\Omega_{\phi i}\ll 1$ results in $\phi_o \sim n\sqrt{\Omega_{\phi o}/6}$ and independent of the initial conditions.[2] For any initial conditions we will end up with $-1\leqslant w_{\phi o}\leqslant w_{tr}$.

IV. QUINTESSENCE RESTRICTION ON *n*

Before analyzing the quintessence restriction imposed on n we would like to comment on the value of c in the potential Eq. (1). It can take different values with different physical interpretations.

For simplicity of argument let us assume that we have no kinetic energy at the beginning and that $\rho_{\phi i}=V_i=c^2\Lambda_c^4$. The initial energy density is $\Omega_{\phi i}=\rho_{\phi i}/\rho_c$. Taking $\rho_{\phi i}=c^2\Lambda_c^4$, with $c=2\nu$, and $\rho_c=E^4$ we have that the initial energy density is given by $\Omega_{\phi i}=(2\nu)^2(\Lambda_c/E)^4$. In this case we see that the initial energy density depends on the ratio between the condensation scale Λ_c and the critical energy density $E=\rho_c^{1/4}$.

Another possibility is to take $\rho_{\phi i}=g_\phi T^4$ and $\rho_c=g_{tot}T^4$ where g_{tot} and g_ϕ are the total and quintessence number of degrees of freedom, respectively, at a temperature T. In this case we have $\Omega_{\phi i}=g_\phi/g_{tot}$ and it depends only on the ratio of quintessence and total degrees of freedom and not on the energy scales. The condensation scale is then $\Lambda_c=(g_\phi/c^2)^{1/4}T$ and $\rho_{\phi i}=g_\phi T^4=c^2\Lambda_c^4=V_i$.

Let us now study the restrictions to the values of n. In both cases mentioned above we get similar restrictions so we will consider only the first one.

[2]We would like to stress that Eqs. (18) and (17) are valid only for $0\leqslant w_\phi+1\leqslant 1$ and in the tracker regime this condition is no longer satisfied.

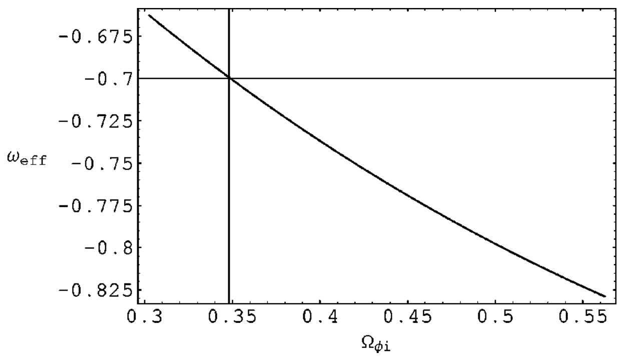

FIG. 8. Restriction $w_{eff}<-0.7$ avoided by increasing $\Omega_{\phi i}$ with $n=3$ $(c=1)$.

For a fixed n we take the following conditions: h_o must be in the range 0.7 ± 0.1 and $\Omega_{\phi o}$ must belong to the interval 0.6–0.8. We can restrict n according to different physical arguments. These restrictions are encountered while numerically solving the differential system settled by Eqs. (4). The first restriction comes from the observational value of the relevant parameter $w_{\phi o}$. The limit $w_{\phi o}<-2/3$ can be translated to $w_{eff}<-0.7$ [3]. Notice that in this potential $w_{eff}<w_{\phi o}$, contrary to the general arguments [4], and the reason is that w_ϕ is still growing by today. From this analysis we find that, in order to satisfy the w_{eff} condition, n must be smaller than 2.74 as shown in Fig. 7 for $\Omega_{\phi i}=0.25$. The value of $w_{\phi o}$ depends not only on n but also on $\Omega_{\phi i}$ and it decreases with increasing $\Omega_{\phi i}$. If we fix n and increase $\Omega_{\phi i}$ we find that examples originally discarded by the $w_{eff}<-0.7$ restriction now enter the physically permitted group. The example with $n=3$ is depicted in Fig. 8. An equipartition value of $\Omega_{\phi i}$ is 0.25 but if we allow $\Omega_{\phi i}$ to be as large as 0.75 then the restriction on n is only $n<5.2$.

A further restriction comes from big bang nucleosynthesis results that require $\Omega_\phi(\mathrm{NS})<0.1$ at the energy scale range of NS: 0.1–10 MeV [12]. To account for this either we have to consider $\Omega_{\phi i}<0.1$ or, for example, if $\Omega_{\phi i}=0.25$ we must take out $1.2<n<2.1$ because for this range the initial value H_i lies within the range of values of $H_{\rm NS}$. For all the values of n allowed by w_{eff} and NS restrictions, a variation of 45% on $\Omega_{\phi i}$ can be performed without disturbing the permanence in the observed ranges of H_o and $\Omega_{\phi o}$.

V. UNIFICATION OF GAUGE COUPLING CONSTANTS

The condensation scale Λ_c used in Eq. (1) is from an elementary particle point of view an arbitrary scale that sets the energy scale of the phase transition. However, if the inverse power potential Eq. (2) is obtained from a non-Abelian asymptotically free gauge group then we can relate the condensation scale Λ_c to other energy scales using the renormalization group equation. The one-loop evolution of the gauge coupling constant for a SU(N_c) gauge group with N_f chiral fields gives a condensation scale (strong coupling $g^{-2}\ll1$)

$$\Lambda_c=\Lambda_x e^{-1/2b_o g_x^2} \tag{20}$$

where $b_o=(3N_c-N_f)/16\pi^2$ is the one-loop beta function and Λ_x and g_x are the arbitrary energy scale and coupling constant, respectively, which include the high energy scale where the original chiral fields Q are weakly coupled.

It is well known that the gauge coupling constants of the standard model become unified at an energy scale $\Lambda_{GUT}\simeq10^{16}$ GeV with a coupling constant $g_{GUT}\simeq\sqrt{4\pi/25.7}$ [18].

We want to impose gauge coupling unification on our model, i.e., the coupling of the gauge group responsible for quintessence should be unified at Λ_{GUT} with the standard model gauge groups [11]. In this case we require Λ_x, g_x in Eq. (20) to take the values $\Lambda_x=\Lambda_{GUT}$, $g_x=g_{GUT}$ and we have $\Lambda_c=\Lambda_{GUT}\exp[-1/2b_o g_{GUT}^2]$. This is not a necessary condition but opens the possibility of thinking of the model as coming from string theory after compactifying the extra dimensions, or of a grand unification scheme where all gauge coupling constants are unified.

TABLE I. Models with $\Lambda_c=\Lambda_u$. The first five have $\Delta N_f<0.05$ while the other six have $\Delta N_f<0.10$ discrepancy from an integer.

Number	N_c	N_f	ν	n	Λ_c (GeV)
1	3	5.98	1	0.66	6×10^{-8}
2	6	14.97	3	0.66	6.9×10^{-8}
3	7	18.05	4	0.55	1.6×10^{-8}
4	8	5.97	5.97	13.83	1.3×10^{12}
5	8	6.96	3	13.55	7.6×10^{11}
6	3	1.90	1	5.66	1.3×10^{6}
7	5	3.91	2	9.38	4.7×10^{9}
8	6	5.09	2	10.85	3.7×10^{10}
9	7	5.08	5.08	12.64	3.9×10^{11}
10	8	20.90	4	6.75	2.4×10^{-7}
11	8	21.10	5	0.47	5.7×10^{-9}

TABLE II. Numerical solutions for different values of n with $c=1$ and $\Omega_{\phi i}=0.25$.

n	H_i (GeV)	$\rho_{\phi i}^{1/4}$ (GeV)	Λ_c (GeV)	$w_{\phi o}$	w_{eff}	N_{total}
1/2	1.33×10^{-35}	7.48×10^{-9}	3.74×10^{-9}	-0.97	-0.98	10.72
2/3	1.16×10^{-33}	6.99×10^{-8}	3.49×10^{-8}	-0.97	-0.98	12.96
1	3.46×10^{-30}	3.82×10^{-6}	1.91×10^{-6}	-0.93	-0.95	16.97
2	4.68×10^{-22}	4.44×10^{-2}	2.22×10^{-2}	-0.77	-0.83	26.33
18/7	1.68×10^{-18}	2.66	1.33	-0.65	-0.73	30.42
3	3.41×10^{-16}	37.92	18.77	-0.57	-0.65	33.07

TABLE III. Numerical solutions for different values of $\Omega_{\phi i}$ with $c=1$ and $n=3$.

$\Omega_{\phi i}$	H_i (GeV)	$\rho_{\phi i}^{1/4}$	Λ_c (GeV)	$w_{\phi o}$	w_{eff}	N_{total}
1×10^{-5}	5.48×10^{-14}	480.90	19.12	-0.55	-0.50	35.69
0.25	3.34×10^{-16}	37.54	18.77	-0.57	-0.65	33.07
0.5	2.85×10^{-16}	34.68	20.62	-0.76	-0.83	32.88
0.75	3.05×10^{-16}	35.88	23.61	-0.88	-0.92	32.75
0.99	5.13×10^{-16}	46.53	32.82	-0.97	-0.98	32.20

Of course, not all values of N_c, N_f will give an acceptable phenomenology. This is because the cosmological evolution of ϕ and the gauge coupling unification set independent constrains on the condensation scale Λ_c and on N_c, N_f. From a cosmological point of view Λ_c depends on the inverse power n [see Eq. (13)], which is a function of N_c, N_f, and from gauge unification Λ_c depends also on N_c, N_f through b_o [see Eq. (20)]. These two constraints drastically reduce the allowed values of N_c, N_f and we also require $N_f \geqslant 1$ and $N_c \geqslant 2$ to be integers.

In Table I we give the different values of N_c, N_f, ν for which we have gauge coupling unification. We can see that there are only a small number of possible models (11). The first five models have an N_f that differs from an integer by less than 0.05, while the other six models differ at most by 0.10. All other combinations of N_c, N_f, ν have a larger discrepancy and do not lead to $\Lambda_c = \Lambda_u$. If we further constrain the models to agree with the cosmological observations (i.e., $w_{\phi o} < -0.7$ requiring $n<5$) we are left with only four models (numbers 1,2,3,11 of Table I). All of these four models have $n<2$ and the quantum corrections to the Kahler potential are, therefore, not dangerous. Notice as well that only two models (4,9) have $\nu = N_f$ and in both cases $n>12$.

As an example of a model with gauge coupling unification we have a gauge group $N_c=3$ with $N_f=6$ and $\nu=1$ which entails a value $n=2/3$ according to $n=2+4\nu/(N_c - N_f)=2/3$ [11]. For this model we find, from the numerical solution, a total time $N_{total}=12.96$ which does not superpose on the NS range 19.6–27.2. The $w_{eff}<-0.7$ restriction is satisfied with $w_{eff}=-0.98$ and the condition from experimental central values $h_o=0.7$ and $\Omega_{\phi o}=0.7$ is also satisfied, taking $\Omega_{\phi i}=0.25$. The condensation scale is $\Lambda_c = 4.2\times10^{-8}$ GeV. A full analysis of this model is presented in [17].

VI. FURTHER EXAMPLES

We have already given some example in the previous sections. In Table II and III we show the numerical results for different values of n with initial condition $\Omega_{\phi i}=0.25$ and for different initial conditions with $n=3$ fixed, respectively.

Other interesting examples are when $\nu = N_f$. The condition $n<2.74$ (for $\Omega_{\phi i} \leqslant 0.25$) requires $N_f/N_c < 0.15$ and therefore $N_c > 7$. For $N_c=8$, $N_f=1$ one has $n=18/7 \simeq 2.57$, and using $\Omega_{\phi i}=0.25$ one obtains $w_{\phi o}=-0.65$, $w_{eff}=-0.73$, $\phi_o=2.01$, $N_o=30.3$, $M_i=2.3$ GeV, and $\Lambda_c=1.6$ GeV, while for $N_c=7$, $N_f=1$ one has $n=8/3$, $w_{\phi o}=-0.64$, $w_{eff}=-0.71$, $\phi_o=2.04$, $N_o=30.9$, $M_i=4.2$ GeV, and $\Lambda_c=3$ GeV. However, these models do not have $\Lambda_c=\Lambda_u$, i.e., they are not unified with the SM gauge groups.

VII. CONCLUSIONS

We studied negative power potentials and constrained the initial conditions and the power of the potential to satisfy the SN1a results. For n larger than 5, the scalar field ϕ has already reached its tracker value and $w_{\phi o}$ is too large. So we need to concentrate on potentials with $n<5$ to comply with SN1a results. We gave a semianalytic solution to $w_{\phi o}$ and Λ_c in terms of H_o, $\Omega_{\phi i}$, and n and we solved numerically for some relevant cases. We obtained the result that $w_{\phi o}$ depends on $\Omega_{\phi i}, n$; it decreases with increasing $\Omega_{\phi i}$ while it becomes smaller for larger n. If we assume equipartition initial conditions with $\Omega_{\phi i} \leqslant 0.25$ then n is constrained to be smaller than 2.74; however, if we allow for $\Omega_{\phi i}=0.75$ the constraint is relaxed to $n<5.2$. We have shown that one can vary the initial conditions by up to 45% without spoiling the observational cosmological values at the present time. For any initial conditions we will end up with $-1 \leqslant w_{\phi o} \leqslant w_{tr}$.

We have seen that the negative power potentials can be derived from the Affleck-Dine-Seiberg potential and in order to avoid problems with the Kahler potential one requires $n<2$, which implies that $N_f > N_c$ and that not all condensates become dynamical (i.e., $\nu \neq N_f$). For $\nu = N_f$ one needs $N_f/N_c < 0.15$ to have $w_{\phi o} < -2/3$. Furthermore, we have shown that it is possible to have a quintessence model with gauge coupling unification for all gauge groups, standard model and the gauge group responsible for quintessence, but the number of models is quite limited (four).

ACKNOWLEDGMENTS

This work was supported in part by CONACYT project 32415-E and DGAPA, UNAM project IN-110200.

[1] A.G. Riess *et al.*, Astron. J. **116**, 1009 (1998); S. Perlmutter *et al.*, Astrophys. J. **517**, 565 (1999); P.M. Garnavich *et al.*, *ibid.* **509**, 74 (1998).
[2] P. de Bernardis *et al.*, Nature (London) **404**, 955 (2000); S. Hannany *et al.*, Astrophys. J. Lett. **545**, L1 (2000).
[3] S. Perlmutter, M. Turner, and M.J. White, Phys. Rev. Lett. **83**, 670 (1999); T. Saini, S. Raychaudhury, V. Sahni, and A.A. Starobinsky, *ibid.* **85**, 1162 (2000).
[4] I. Zlatev, L. Wang, and P.J. Steinhardt, Phys. Rev. Lett. **82**, 896 (1999); P.J. Steinhardt, L. Wang, and I. Zlatev, Phys. Rev. D **59**, 123504 (1999).
[5] A.R. Liddle and R.J. Scherrer, Phys. Rev. D **59**, 023509 (1999).
[6] A. de la Macorra and G. Piccinelli, Phys. Rev. D **61**, 123503 (2000).
[7] P.J.E. Peebles and B. Ratra, Astrophys. J. Lett. **325**, L17 (1988); B. Ratra and P.J.E. Peebles, Phys. Rev. D **37**, 3406 (1988).
[8] J.P. Uzan, Phys. Rev. D **59**, 123510 (1999).
[9] P. Binetruy, Phys. Rev. D **60**, 063502 (1999); Int. J. Theor. Phys. **39**, 1859 (2000).
[10] A. Masiero, M. Pietroni, and F. Rosati, Phys. Rev. D **61**, 023509 (2000).
[11] A. de la Macorra and C. Stephan-Otto, Phys. Rev. Lett. **87**, 271301 (2001).
[12] K. Freese, F.C. Adams, J.A. Frieman, and E. Mottola, Nucl. Phys. **B287**, 797 (1987); M. Birkel and S. Sarkar, Astropart. Phys. **6**, 197 (1997).
[13] I. Affleck, M. Dine, and N. Seiberg, Nucl. Phys. **B256**, 557 (1985).
[14] C.P. Burgess, A. de la Macorra, I. Maksymyk, and F. Quevedo, Phys. Lett. B **410**, 181 (1997).
[15] E.J. Copeland, A. Liddle, and D. Wands, Phys. Rev. D **57**, 4686 (1998).
[16] A. de la Macorra, Int. J. Mod. Phys. D **9**, 661 (2000).
[17] A. de la Macorra, hep-ph/0111292.
[18] U. Amaldi, W. de Boer, and H. Furstenau, Phys. Lett. B **260**, 447 (1991); P. Langacker and M. Luo, Phys. Rev. D **44**, 817 (1991).