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Breathers in one-dimensional binary metamaterial models

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ABSTRACT

The existence and stability of discrete breathers is studied theoretically for a model magnetic metamaterial composed of a periodic binary array of split-ring resonators with resonance frequency mismatch. It is demonstrated that breathers can be excited spontaneously by frequency chirping of the driving field, a method that is well suited for experiments.

1. Introduction

The contemporary notion of metamaterials includes a wide range of artificially structured materials that exhibit electromagnetic properties and functionality unattainable from natural materials [1–4]. A specific class of metamaterials that exhibit significant magnetic properties at Terahertz and optical frequencies is represented by the magnetic metamaterials (MMs) [5,6], which are customarily composed of split-ring resonators (SRRs). Dynamic control over metamaterials that makes possible the real-time tuning of their effective parameters is of great importance for potential applications. Thus, the possibility of dynamically tuning MMs using nonlinearity has been explored (see Refs. [3,7] and the references therein), and experiments on second harmonic generation (SHG) in such materials have been reported [8]. Recently, nonlinear MMs operating in microwave frequencies which are dynamically tunable by varying the input power have been realized [9].

The combination of nonlinearity and discreteness in MMs allows for the excitation of intrinsic localized modes or discrete breathers (DBs), that oscillate for long times in a localized region of space and may be produced generically in discrete lattices of weakly coupled nonlinear elements [10]. Their existence has been rigorously proved [11], and they can be constructed with standard numerical algorithms [12,13]. DBs may appear spontaneously in a lattice either statistically [14,15] or by a purely deterministic mechanism that relies on a fundamental instability for wave propagation in nonlinear media (known as modulational instability, MI) [16,17]. Dissipative DBs (DDBs) in particular can be produced by frequency chirping of an alternating driving field [17]. The existence and stability of DBs in nonlinear SRR-based MM models, localized either in the bulk [18,19] or at the surface [20,21], have been demonstrated numerically. Moreover, domain wall [22] and solitonic excitations [23] may also appear in such systems.

Recently, a novel MM composed of two types of SRRs was suggested and was demonstrated that in the nonlinear regime it is suited for the observation of phase-matched parametric interaction and enhanced SHG [24]. In the present work we investigate the existence of both energy-conserving and dissipative DBs, which are localized either in the bulk or at the surface of a model nonlinear MM, composed of a one-dimensional (1D) array of two types of SRRs [25]. There have been many studies on DBs that can be excited in binary arrays in different context. A recent example is that of the study of DBs in binary arrays of weakly coupled waveguides within the framework of the discrete nonlinear Schrödinger (DNLS) equation [26,27]. In the context of MMs, the corresponding dynamic equation (see Section 2 below) modelling a binary SRR chain is of second order, it includes damping and forcing terms and, moreover, neighboring elements are coupled through the second time-derivatives due to the nature of the interaction. That makes our model radically different than previously used ones. However, surface DBs in our system are very similar to those in a DNLS model for a semi-infinite binary waveguide array [27].

2. Model binary metamaterial

In its simplest version an SRR is just a metallic ring with a slit, where a capacitance C is built up. Under certain conditions, it can be treated as a series RLC circuit, featuring an inductive–capacitive resonance at $\omega_r \simeq 1/\sqrt{LC}$, with L being the self-inductance of the

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SRR and assuming that Ohmic losses are low ($R \simeq 0$). An SRR becomes nonlinear either by insertion of a nonlinear dielectric [28] or a varactor diode [9,29,30] in its slit. Both ways result in a voltage-dependent SRR capacitance that can be approximated by a cubic nonlinearity. The construction of an array of nonlinear SRRs, that are weakly coupled through magnetic interactions [18,19,22] results in a nonlinear tunable MM [9]. Consider a 1D array in the axial geometry (Fig. 1), composed of two types of SRRs, a and b, with resonance frequencies ω_a and ω_b, respectively, placed at even-numbered and odd-numbered sites of the array [25]. That nonlinear binary array is placed in an alternating magnetic field perpendicular to the planes of the SRRs. Then, the normalized dynamic equation for the charge q_n accumulated in the capacitor of the n-th SRR reads [25]

$$\frac{d^2}{d\tau^2}[\lambda q_{n-1}+q_n+\lambda q_{n+1}]+\gamma\frac{dq_n}{d\tau}+\omega_n^2 q_n-\chi\omega_n^6 q_n^3=\varepsilon_0 \sin(\Omega\tau), \qquad (1)$$

where λ is the coupling parameter ($\lambda > 0$ in the axial geometry), γ is the loss coefficient, χ is the nonlinearity parameter, $\omega_n^2 = 1/\delta$ ($\omega_n^2=\delta$) for n an odd (even) integer, with $\delta \equiv \omega_a/\omega_b$ being the resonance frequency mismatch (RFM) parameter. The term on the right-hand side of Eq. (1) is the electromotive force of frequency Ω induced in each SRR due to the applied field. The dispersion of low amplitude magnetoinductive waves [31] in the binary MM is obtained as [25]

$$\Omega_{\pm}^2=\frac{\Delta \pm \sqrt{\Delta^2-4(1-4\lambda^2\cos^2\kappa)}}{2(1-4\lambda^2\cos^2\kappa)}, \qquad (2)$$

where κ is the normalized wavenumber and $\Delta=\delta+(1/\delta)$. This dispersion relation has two branches separated by a gap. For $\gamma=0$ and $\varepsilon=0$, Eqs. (1) can be obtained from the Hamiltonian $\mathcal{H}=\sum_n \mathcal{H}_n$, where $\mathcal{H}_n$ is the discrete Hamiltonian density, given by

$$\mathcal{H}_n=\tfrac{1}{2}\{\dot{q}_n^2+\lambda\dot{q}_n(\dot{q}_{n-1}+\dot{q}_{n+1})\}+V_n, \qquad (3)$$

with the nonlinear (cubic) on-site potential

$$V_n \equiv V(q_n)=\tfrac{1}{2}(\omega_n q_n)^2[1-\tfrac{1}{2}\chi\omega_n^2(\omega_n q_n)^2]. \qquad (4)$$

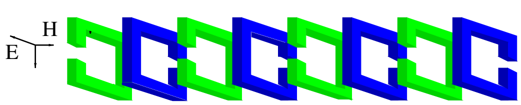

Fig. 1. Schematic of a binary split-ring resonator array.

3. Hamiltonian breathers

For the construction of DBs for the Hamiltonian (i.e., energy-conserving) system, the standard algorithm relying on Newton's method [11,12] has been employed. We start from the anticontinuous limit, where the coupling parameter vanishes ($\lambda \to 0$), and construct a trivial breather. For this purpose, we identify the amplitude q_a of a solution of a single SRR and calculate its frequency ω_B. For the existence of Hamiltonian DBs it is required that ω_B and its multiples do not fall into the linear frequency band(s). For constructing a trivial breather we set the 'coordinate' q_n of a selected SRR, say the one at $n=n_B$, equal to q_a, i.e., $q_{n_B}=q_a$, and all the other q_ns' equal to zero (and also $dq_n/d\tau=0$ for any n). Then, by continuation of the trivial DB solution to finite couplings λ we can construct DBs up to a maximum value where they cease to exist. The DBs formed in this way oscillate with frequency ω_B. We have constructed several types of Hamiltonian DBs using Newton's method that are localized either at the surface or in the bulk. Importantly, one can get approximately the same results using the rotating wave approximation (RWA) [25]. Typical single-site Hamiltonian DB profiles are shown in Fig. 2 for $\omega_B=0.77$ ($T_B=2\pi/\omega_B \simeq 8.16$), where we have set $q_a=1.7085$ and 1.8164 for the DBs localized at odd and even sites, respectively. The one shown in Fig. 2 a is a true surface DB since it is localized exactly at the left end of the array ($n=1$). The next two (Figs. 2 b and c) be also characterized as surface DBs, since they are localized very close to the surface ($n=2$ and 3, respectively), but actually they are cross-over states between surface and bulk DBs. Since the DBs shown here are highly localized, they obtain their bulk form within a distance of only a few sites from the surface, so that the DB shown in Fig. 2 d (localized at $n=4$) can be considered as a bulk DB.

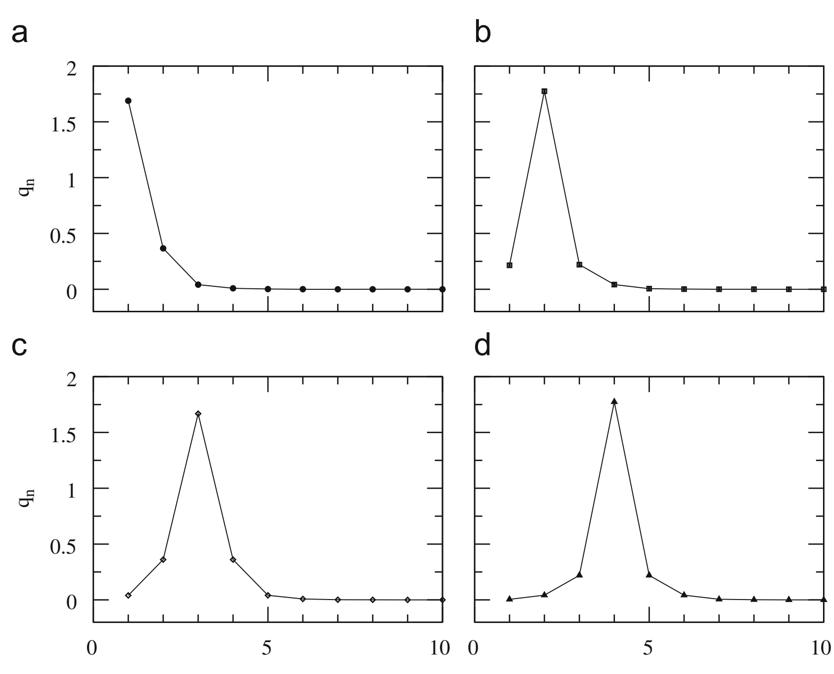

Fig. 2. Typical Hamiltonian single-site surface breather profiles at maximum amplitude in a magnetoinductive binary array for $\delta=0.9$, $\lambda=0.1$, $\omega=0.77$, $\chi=+\frac{1}{6}$, $N=40$.

4. Dissipative breathers

Construction of DDBs: In typical experiments involving MMs, the metamaterial is driven by an electromagnetic field of appropriate polarization which can be chosen so that only its magnetic component excites an electromotive force in the SRRs. For constructing DDBs we use the algorithm designed in Ref. [13], appropriately modified for the case of the binary array. We start from the anticontinuous limit and identify attractors of the damped-driven single SRR oscillators. In order to construct trivial DDBs for the binary array we need to find, for at least one of the oscillators, two different amplitude attractors. A single SRR oscillator with cubic nonlinearity has either one or three attractors. By varying a parameter, the number of attractors can jump from one to three or vice versa through a pitchfork bifurcation at some critical value of that parameter. Then, one stable (unstable) attractor can suddenly split into three attractors, from which two are stable (unstable) or vice versa. Those attractors can be obtained accurately with RWA applied to a single SRR oscillator [25]. For the parameters in Fig. 3 we obtained for even-numbered SRRs a single stable attractor at $q_1^e = 0.5822$, while for odd-numbered ones we obtained two stable attractors at $q_1^o = 1.330$ and $q_2^o = 0.09968$. A trivial DDB localized at $n=1$ is constructed as $q_1 = 1.330$, $q_{2n} = 0.5822$ and $q_{2n-1} = 0.09968$ ($n>1$). Then, by continuation of the trivial DB to finite λ we get DDBs up to $\lambda = \lambda_{max} \simeq 0.071$. The obtained DDB profiles for $\lambda = 0.02$ and 0.05 are shown in Figs. 3 a and b, respectively. Two more profiles, for DDBs localized at $n=3$ and 19 (both for $\lambda = 0.02$) are shown in Figs. 3 c and d, respectively. DDB profiles obtained for a different parameter set are shown in Fig. 4 for several values of λ. In this case we obtained two stable attractors both for even- and odd-numbered SRRs. Specifically, for even-numbered SRRs we obtained stable attractors at $q_1^e = 1.334$ and $q_2^e = 0.02286$, while for odd-numbered ones at $q_1^o = 4.067$ and $q_2^o = 0.1602$. A trivial DDB localized at $n = n_B = 19$ is now constructed as $q_{n_B} = -4.067$, $q_{2n} = -0.02286$ and $q_{2n-1} = -0.1602$ ($n \neq (n_B+1)/2$), whose continuation results in the generation of DDBs up to $\lambda = \lambda_{max} \simeq 0.19$.

Induction of DDBs through frequency chirping: For a frequency gapped linear spectrum, some of the linear modes become unstable at large amplitude. If the curvature of the dispersion curve in the region of that mode is negative and the lattice potential is hard then, the large amplitude mode becomes unstable with respect to the formation of a DDB in the gap above the linear spectrum [17]. Below we exploit MI in order to generate spontaneously DDBs in the driven binary array.

For the parameters in Fig. 5, the top of the upper linear band is located at $\Omega_0 \simeq 1.42$ where the curvature is negative. Moreover, the SRRs are subjected to hard on-site potentials for $\chi < 0$. The (large amplitude) driver is initiated with its frequency just below Ω_0 and is then chirped with time to produce enough vibrational amplitude to induce MI of the uniform mode, which then leads to spontaneous DDB generation. At the end of the frequency chirping phase, the driver frequency is well above Ω_0, and only supplies energy into the formed DDB(s). During that phase, a large number of DDBs may be generated, which can move and collide and eventually coalesce into a small number of high amplitude DDBs that are frequency locked to the driver and, because of that, they are trapped at particular SRRs. After that, the driving frequency is kept constant and the high amplitude DDBs (and even some low-amplitude ones) continue to receive energy falling into a stationary state. When the driver is switched off all DDBs die out in a short time interval.

In Fig. 5, the contours of the energy density $\mathcal{H}_n$ on the $\tau - n$ plane identify the evolution of the DDBs formed by the procedure described above. There, the chirping phase lasts for $2000T_0 \simeq 8850$ time units ($T_0 = 2\pi/\Omega_0$), where the frequency varies linearly from $\Omega_i = 0.997\Omega_0$ to $\Omega_f = 1.020\Omega_0$. The driver is subsequently kept at constant frequency Ω_f until it is switched off after another $2000T_0$ time units. The upper and lower panels in Fig. 5 correspond to the regions of the array close to the right and left ends, respectively, where several DDBs have survived after the

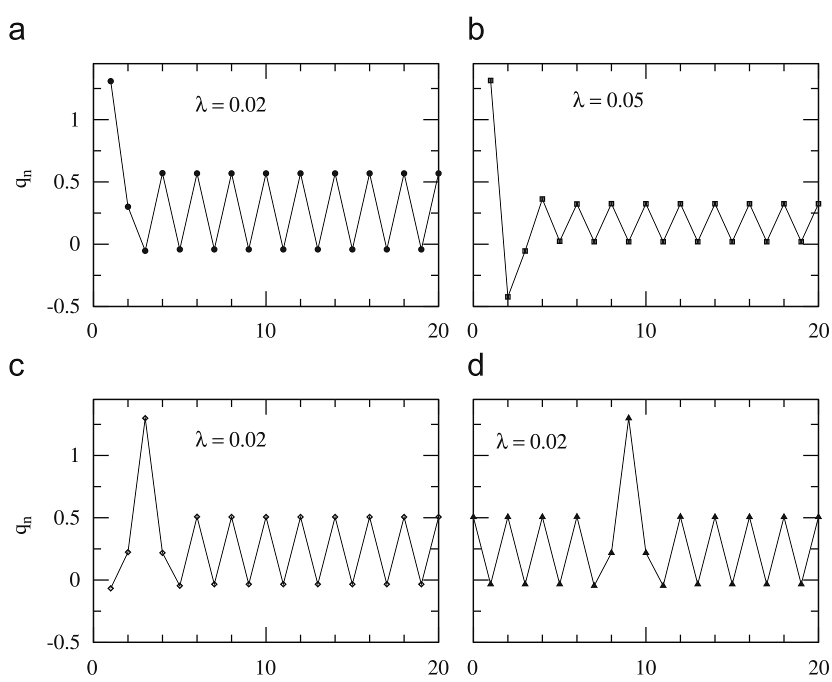

Fig. 3. Dissipative low-amplitude surface breather profiles at maximum amplitude for $\delta = 0.8$, $\Omega = 0.92$, $\chi = +\frac{1}{6}$, $\gamma = 0.01$, $\varepsilon_0 = 0.04$, $N = 40$, and λ as shown in the figure. The central breather site is located on an odd-numbered SRR oscillator.

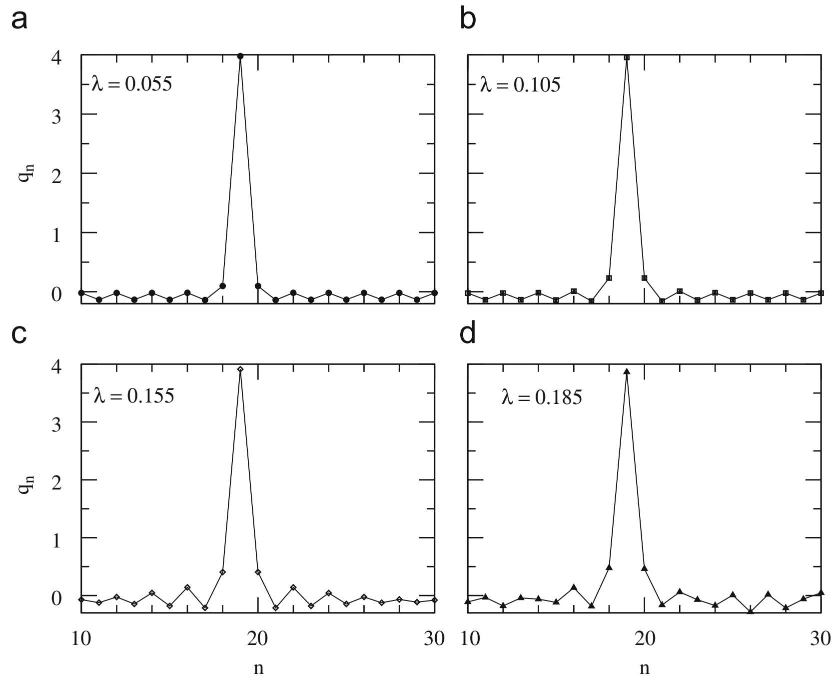

Fig. 4. Dissipative, high-amplitude bulk breather profiles at maximum amplitude for $\delta=2$, $\Omega=0.5$, $\chi=+\frac{1}{6}$, $\gamma=0.01$, $\varepsilon_0=0.04$, $N=40$ and different values of the coupling parameter λ as shown in the figure. The central breather site is located on an odd-numbered SRR oscillator ($n=19$) in the middle of the array.

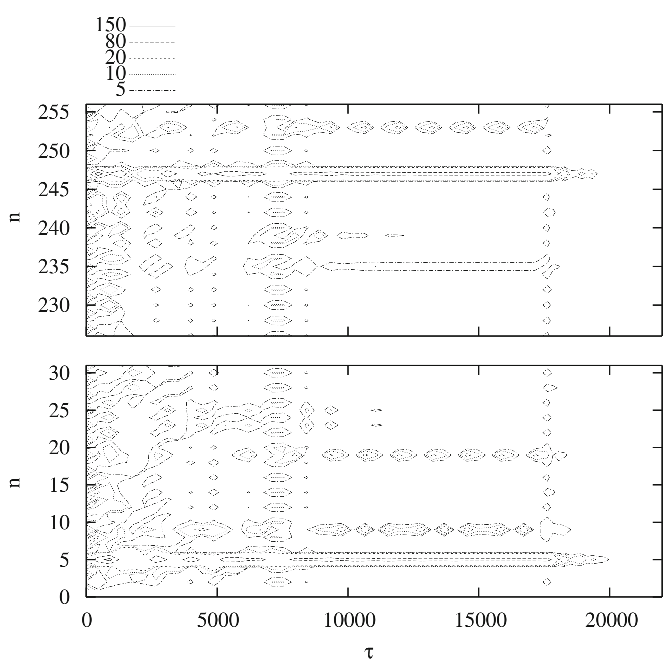

Fig. 5. Contours of the energy density $\mathcal{H}_n$ for the binary array obtained with frequency chirping, for $\delta=2$, $\Omega_0=1.42$, $\chi=-\frac{1}{6}$, $\gamma=0.001$, $\varepsilon_0=2.85$, and $\lambda=0.05$. Only the parts of the array ($N=256$) where breathers exist in the constant frequency phase of the driver are shown.

chirping phase. There we observe clearly two high amplitude DDBs at $n=5$ and 247, along with some other DDBs of considerably lower amplitude, that survive until the end of the constant frequency phase. There are also some other DDBs of even lower amplitude that are not locked to the driver and die out during that phase.

5. Concluding remarks

The existence of energy-conserving and dissipative DBs in a model binary MM with RFM is demonstrated, which can be localized either in the bulk or at the surface. For the construction of those excitations we have used standard numerical algorithms along with the frequency chirping method that has been applied for DDB generation both in actual experiments and simulations of micromechanical cantilever arrays [17]. Magnetic metamaterials are driven by alternating fields and thus it is expected that dissipative DBs are relevant to these type of experiments when nonlinearity is present. Since SRR-based MMs with approximately cubic capacitive nonlinearities have been already constructed, at least in the microwave frequency range [9], the realization of a binary array is in principle possible. We propose that an experiment with frequency chirped applied field can lead to DDB generation in a fashion very similar to that described above.

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