Electron relaxation induced by electron–longitudinal-acoustic-phonon scattering in single and coupled quantum dots in external magnetic and electric fields

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Our calculations show that the relaxation time due to electron–LA-phonon scattering for an electron in the first excited state in coupled quantum dots depends strongly on the barrier thickness between the two dots and external magnetic or electric fields. We also show that, for electrons in higher excited states, the relaxation time in both single and coupled quantum dots is dominated by multiple LA-phonon scattering processes.

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Electron-phonon scattering is severely suppressed in semiconductor quantum dots (QDs) due to discrete electron energy levels in such systems.[1] In lateral or vertical QDs where separation between the energy levels is small as compared to the longitudinal-optical (LO) phonon energy, the electron–longitudinal-acoustic- (LA-)phonon interaction is dominant.[2] It is the most important scattering mechanism limiting applications of these quantum dots for devices, such as quantum logic gates and single-electron QD devices. Electron relaxation and charge decoherence in two coupled QDs due to electron–LA-phonon interactions have been studied recently.[3,4] Due to the linear dispersion relation of the LA-phonon modes, efficient electron–LA-phonon scattering involves the long-wavelength phonons, i.e., the low-energy ones (typically, a few meV). Calculations concerning the direct scattering rate between two levels in a quantum dot are straightforward.[5] The scattering rate demonstrates oscillations as a function of confinement length and external magnetic field.

In this work, we study the electron relaxation processes in single and coupled QDs in the presence of external magnetic and electric fields. The electron is considered photoexcited or electrically injected into an excited state. Attention is paid not only to the lowest excited state but also to higher states. For an electron in the first excited state in two coupled QDs, we show that its relaxation time depends strongly on the barrier thickness between the dots and external fields. For higher excited states, relaxation to the ground state is related to the intermediate levels. In this case, the electron inelastic relaxation rates are dominated by a multiscattering process where the long-wavelength phonon plays a crucial role. An external magnetic field often enhances the multiple-phonon scattering and, consequently, the electron relaxation. In a recent work,[6] however, it was claimed that the relaxation can be strongly suppressed in coupled quantum dots by an external magnetic field. The present investigation does not support this conclusion. Notice that, in few-electron QDs, it has been shown that the electron relaxation rate is reduced by electron-electron correlation.[7] But the relaxation time still behaves qualitatively in the same way.

We model the single (two coupled) $GaAs/Al_{0.3}Ga_{0.7}As$ QDs with a parabolic confinement in the xy plane and a single quantum well (two coupled symmetric quantum wells) in the z direction: $V(\mathbf{r})=\frac{1}{2}m^*\Omega^2(x^2+y^2)+V_{QW}(z)$. When an external magnetic field $\mathbf{B}$ is applied in the z direction, the electron energy levels are given by $E_{nml}=(2n+|m|+1)\hbar\omega+\frac{1}{2}m\hbar\omega_c+E_l$, where $\omega=\sqrt{\Omega^2+\omega_c^2/4}$, $\omega_c=eB/m^*$, $n=0,1,2,\ldots$, $m=0,\pm1,\pm2,\ldots$, and $l=0,1,\ldots$. The wave functions are $\Psi_{nml}(x,y,z)=\phi_{nm}(x,y)\ \psi_l(z)$ with the Fork-Darwin wave function $\phi_{nm}(x,y)$ in the xy plane and the wave function $\psi_l(z)$ describing the electron state in the z direction. The width and barrier height of the quantum well for the confinement in the z direction are W_z and V_0, respectively. For a $GaAs/Al_{0.3}Ga_{0.7}As$ structure, $V_0=240$ meV. Usually, the confinement in the z direction is much stronger than that in the xy plane so that we only consider the $l=0$ state for a single quantum dot (SQD). In two coupled quantum dots (CQDs), the energy splitting Δ between the two lowest levels in the z direction depends on the barrier thickness W_B and height V_0 between the two quantum wells. For two symmetric quantum wells, there are the bonding ($l=0$) and the antibonding ($l=1$) states in the z direction.

The electron–LA-phonon scattering rate can be obtained by the Fermi golden rule[5] as long as the energy difference between the two levels is much smaller than the LO-phonon energy. This is the case we considered here with $\hbar\Omega$ being a few meV. The scattering rate at zero temperature from the (n,m,l) to the (n',m',l') state due to deformation potential LA phonons is given by

$$\tau^{-1}_{(n,m,l)\to(n'm'l')} = \frac{D^2q_0^3}{2\pi\rho\hbar v_s^2}\int_0^{\pi/2} d\theta\sin\theta \times|\langle\phi_{n'm'}|e^{-iq_0r\sin\theta}|\phi_{nl}\rangle|^2|\langle\psi_{l'}|e^{-iq_0z\cos\theta}|\psi_l\rangle|^2, \quad (1)$$

where $q_0=q^{n'm'l'}_{nml}=(E_{n'm'l'}-E_{nml})/\hbar v_s$, D is the deformation potential, ρ_s is the density of the semiconductor material, and v_s is the sound speed ($\omega_q=v_sq$). In the calculation, we take $D=8.6$ eV, $\rho_s=5300$ kg/m^3, and $v_s=3700$ m/s.

When an electron is injected into an excited state in the quantum dots, it relaxes to the ground state by emitting LA phonons. The corresponding relaxation time τ_R is an important parameter for understanding the related electronic properties as well as the photoluminescence spectrum.[8] For an electron initially in the first excited state, τ_R is given by the direct scattering rate from this state to the ground state. However, for electrons in higher excited states, different relax-

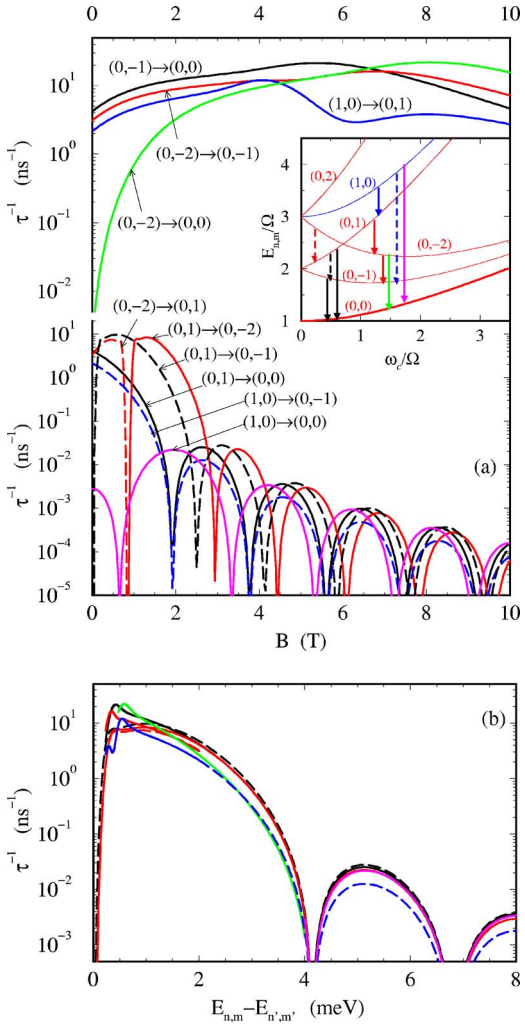

FIG. 1. (Color online) Scattering rate from the (n,m) to the (n',m') state in a SQD as a function of (a) magnetic field and (b) the energy difference between the two levels. The inset gives the energy levels and the corresponding transitions are indicated by vertical arrows.

ation channels exist. The relaxation time can be obtained by using Matthiessen's rule.

We consider the states associated with the lowest energy in the z direction ($l=0$) in a SQD. In this case, the electron states can be labeled by two indices (n,m). The ground state is $(0,0)$ and the first excited state is $(0,-1)$. Figure 1(a) shows the scattering rates between two different levels in a SQD as a function of magnetic field. In the calculation, we took $\hbar\Omega=2$ meV and $W_z=5$ nm. The electron energy levels are plotted in the inset and the corresponding transitions are indicated by the vertical arrows. The scattering rate depends strongly on the initial and final states as well as on the magnetic field. But, when we plot the scattering rates as a function of the energy difference between the corresponding levels in Fig. 1(b), it is seen that the scattering rates are basically determined by this energy difference or $q_0=(E_{n'm'}-E_{nm})/\hbar v_s$. For the same q_0, the scattering rates are very close to each other though they are from different initial and final states and can be at different magnetic fields. Strong electron–LA-phonon scattering occurs at small q_0 (long-wavelength phonons) where τ is on the order of 10^{-1} ns. The scattering rate τ^{-1}decreases rapidly with increasing q_0. Notice that τ^{-1} is zero at $q_0=0$ because the phonon density of states vanishes. We also see that in a wide range of magnetic field, $\tau^{-1}_{(0,-1)\to(0,0)}$ is the largest. At larger magnetic fields, $\tau^{-1}_{(0,-2)\to(0,0)}$ becomes the largest because the two states have the same parity. We need to mention that the scattering rate depends on the confinement potential, i.e., the confinement frequency Ω in the xy plane and the thickness W_z in the z direction. However, for a fixed W_z, the energy dependence of the scattering rate as shown in Fig. 1(b) is almost the same. The minima and maxima of the scattering rates occur at the same energy values independent of Ω. For different W_z, this energy dependence changes quantitatively. In general, the scattering rate decreases with increasing W_z. At the same time, the energy space between two minima becomes smaller, too.

Strong scattering at small energy means that the transition probability for an electron from an excited level to its adjacent levels is much larger than to other levels. As a consequence, the transitions between any two levels can be dominated by a multiscattering process if there are intermediate states between them in energy space. Because the energy difference between two adjacent levels is always smaller than $\hbar\Omega$ and decreases with increasing magnetic field, the magnetic field obviously enhances the multiple scattering.

For an electron in the first excited state $(0,-1)$, its relaxation time τ_R is determined by the direct scattering rate $\tau^{-1}_{(0,-1)\to(0,0)}$. For an electron in higher excited states, relaxation to the ground state is a multichannel process.[9] For instance, an electron in the $(0,-2)$ state can relax to the ground state through the direct transition $(0,-2)\to(0,0)$ or through the indirect transitions $(0,-2)\to(0,-1)\to(0,0)$ and $(0,-2)\to(0,1,)\to\cdots\to(0,0)$ at small magnetic fields. The relaxation times of an electron from $(0,-1)$, $(0,-2)$, and $(0,-3)$ to the ground state $(0,0)$ are obtained according to Matthiessen's rule. For scattering occurring through parallel channels, the total scattering rate is given by the sum of the scattering rates of each independent channel. If they are serial, however, the total scattering rate is determined by the sum of the scattering times. The obtained results are plotted in Fig. 2(a). The direct scattering rates from $(0,-2)$ and $(0,-3)$ to the $(0,0)$ state are also shown in thin curves which are much smaller than the multiscattering rates. Figure 2(b) shows the relaxation time from $(0,+1)$ to the ground state. Different from the $(0,-m)$ states ($m=1,2,3,\ldots$), the energy difference between this level and the ground state increases with increasing magnetic field. Consequently, the direct scattering from $(0,+1)$ to the ground state decreases rapidly. However, more indirect transition channels turn on with increasing magnetic field. The level $(0,+1)$ crosses the levels $(0,-m)$ ($m=1,2,3,\ldots$). Close to the crossing, transitions between the two levels are efficient. As soon as an electron is scattered from the $(0,+1)$ to the $(0,-j)$ state, it relaxes rapidly to the ground state. Our results indicate that the relaxation channel $(0,1)\to(0,-j)\to(0,-j+1)\to\cdots\to(0,0)$ is usually very efficient as shown in Fig. 2(b). For the present quantum

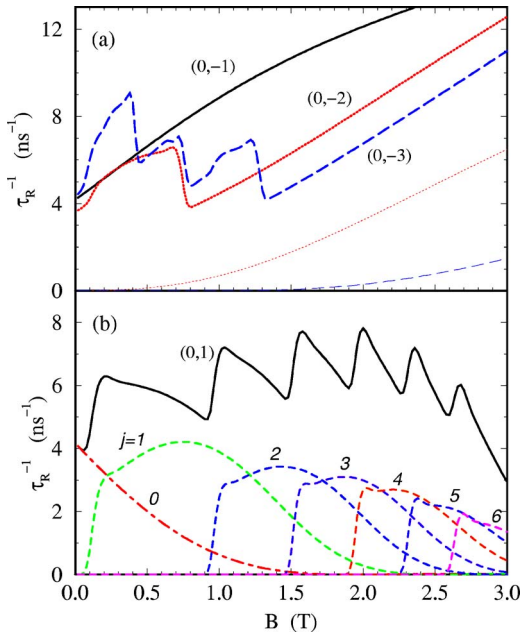

FIG. 2. (Color online) Electron energy relaxation time from (a) $(0,-1)$, $(0,-2)$, $(0,-3)$ and (b) $(0,1)$ to the ground state $(0,0)$ as a function of magnetic field in a SQD. The thin curves in (a) show the direct scattering rates. The dashed curves in (b) present the multiscattering rates through different channels.

dot, the multichannel relaxation processes due to LA-phonon scattering lead to a relaxation time from an excited state to the ground state on the order of subnanoseconds.

The tunneling energy Δ in CQDs depends on the barrier thickness W_B between the two dots. When $\Delta > \hbar\omega - \frac{1}{2}\hbar\omega_c$, the first excited state is $(n,m,l)=(0,-1,0)$. Otherwise, it is the state $(0,0,1)$. Tunneling between the dots affects strongly the electron–LA-phonon interaction because the electron wave function spreads in the two dots leading to different phonon wavelengths matching the electron-phonon interference. Figure 3(a) shows, in the absence of external fields, the two lowest levels in the bonding $(l=0)$ and in the antibonding $(l=1)$ states as a function of W_B in the CQDs of $\hbar\Omega=2$ meV and $W_z=10$ nm. The first excited state is $(0,0,1)$ for $W_B>4.3$ nm ($\Delta<\hbar\Omega$). The LA-phonon scattering rates for different transitions [as indicated by vertical arrows in Fig. 3(a)] are plotted in Fig. 3(b). It shows that, though the energy difference between the $(0,\pm1,0)$ and $(0,0,0)$ states is a constant, the scattering rate between these two states oscillates as a function of W_B. The reason is that the wave functions spread in the two dots and the effective width of the whole system increases with increasing W_B. The scattering rates between other levels oscillate even more strongly as a function of W_B since their corresponding energy differences depend also on the barrier thickness. We notice that $\tau^{-1}_{(0,0,1)\to(0,0,0)}$ turns out to be larger than $\tau^{-1}_{(0,\pm1,0)\to(0,0,0)}$ when $(0,0,1)$ becomes the first excited state for $W_B>4.3$ nm.

The magnetic field dependence of the scattering rate $\tau^{-1}_{(0,-1,0)\to(0,0,0)}$ in CQDs of different barrier thickness is shown in Fig. 4(a). The corresponding scattering rate in the

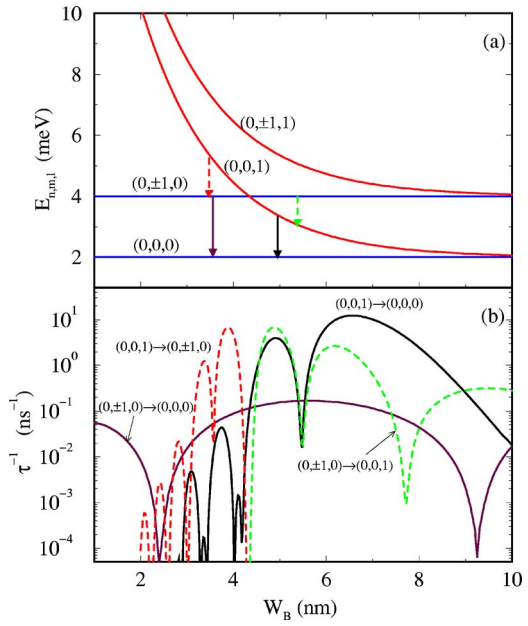

FIG. 3. (Color online) (a) Energy levels and (b) scattering rates in the CQDs as a function of the barrier thickness in the absence of external fields. $\hbar\Omega=2$ meV and $W_z=10$ nm.

SQDs of $W_z=10$ and 20 nm are given by the dashed curves. They are the limits for infinite and zero barrier thickness of the CQDs. It shows that a thin barrier strongly modifies the scattering rate. At the small (large) magnetic field side in Fig. 4(a), $\tau^{-1}_{(0,-1,0)\to(0,0,0)}$ increases (decreases) with increasing barrier thickness up to $W_B=1$ nm. For a thick barrier $W_B>8$ nm (not shown in the figure), considering the superposition of the bonding and antibonding levels, the scattering rate

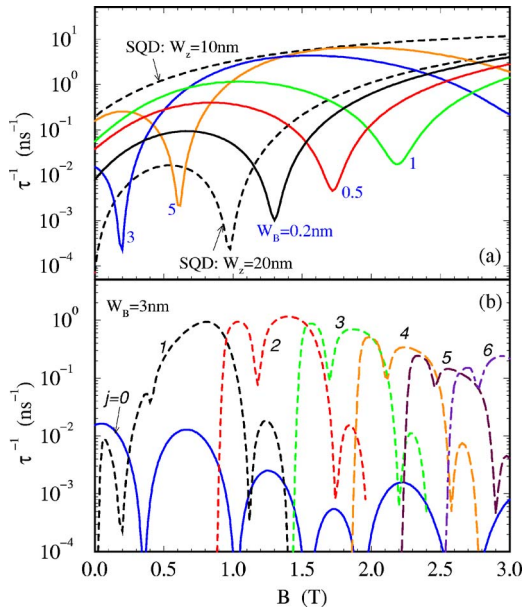

FIG. 4. (Color online) (a) Scattering rate from the $(0,-1,0)$ to the $(0,0,0)$ state as a function of magnetic field in CQDs of different W_B. (b) Multiscattering rate from the $(0,+1,0)$ to the (0,0,0) state through different scattering channels in CQDs of $W_B=3$ nm. $\hbar\Omega=2$ meV, and $W_z=10$ nm.

in the SQD of $W_z=10$ nm can be recovered as the limit.

Multichannel relaxation occurs in a very similar way in the CQDs. Figure 4(b) shows the magnetic field dependence of the multiscattering rate from the $(0,+1,0)$ to the $(0,0,0)$ state through different channels in the CQDs of $W_B=3$ nm. Here, the tunneling energy is large ($\Delta=4.486$ meV) and, consequently, the states $(n,m,l=1)$ are not important. The scattering through the channel $j=0$ is the direct transition. Channel j ($j=1,2,3,4,5$) indicates the transition $(0,1,0) \rightarrow (0,-j,0) \rightarrow (0,-j+1,0) \rightarrow \cdots \rightarrow (0,0,0)$. With increasing magnetic field, more relaxation channels from $(0,+1,0)$ to the ground state are introduced. For $B>2.64$ T, channel 6 becomes dominant, which corresponds to the transition $(0,1,0) \rightarrow (0,-6,0) \rightarrow (0,-4,0) \rightarrow (0,-2,0) \rightarrow (0,0,0)$. These indirect transitions to the ground state are much faster than the direct one ($j=0$). The total relaxation time is determined by all the possible relaxation channels using Matthiessen's rule. When Δ is small, more relaxation channels related to the antibonding state can be involved resulting in a faster electron relaxation time.

When an electric field is applied to the CQDs in the z direction, the electron tends to localize in one of the dots. The scattering rate in the corresponding SQD can be recovered in a strong electric field by the scattering rate $\tau^{-1}_{(nml)\rightarrow(n'm'l)}$ (i.e., for $l=l'$). This is shown in Fig. 5(a) for the scattering from the $(0,-1,0)$ to the (0,0,0) state. The electric field affects most strongly the scattering rate from the $(n,m,l=1)$ to the $(n',m',l'=0)$ state since the states with $l=0$ tend to localize in one dot and those with $l=1$ in the other. The scattering rate $\tau^{-1}_{(nml)\rightarrow(n'm'l')}$ for $l\neq l'$ decays rapidly with increasing electric field because the overlap between the initial and the final states decreases exponentially. Figure 5(b) shows $\tau^{-1}_{(0,0,1)\rightarrow(0,0,0)}$ as a function of magnetic field in a CQD at different electric fields.

In summary, our calculations show, in both SQDs and CQDs, that the electron–LA-phonon scattering depends strongly on the dimension of the quantum dots and the external magnetic and electric fields. For quantum dots with a definite confinement potential, the scattering rate between two levels is basically determined by the energy difference between them. Because the LA-phonon scattering between two levels with small energy difference is strong, the transition probability for an electron from an excited level to its adjacent levels is much larger than to other levels. As a consequence, the electron relaxation from an excited state (except the lowest) to the ground state can be dominated by a multiscattering process. A magnetic field reduces the energy space between adjacent levels and, consequently, enhances the LA-phonon scattering and electron relaxation.

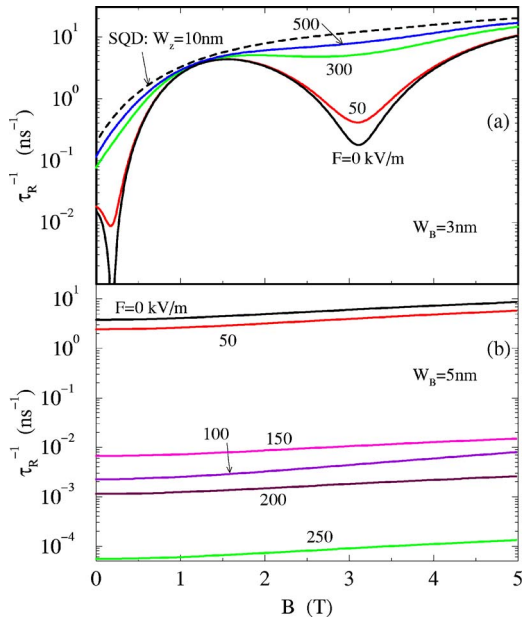

FIG. 5. (Color online) Relaxation time from the first excited state (a) $(0,-1,0)$ for $W_B=3$ nm and (b) $(0,0,1)$ for $W_B=5$ nm to the ground state as a function of magnetic field in CQDs at different electric fields. The dashed curve in (a) gives the result for the SQD of $W_z=10$ nm.

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