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Enhancement of the Aharonov-Bohm effect of neutral excitons in semiconductor nanorings with an electric field

A. V. Maslov* and D. S. Citrin
School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332-0250
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This work demonstrates that the Aharonov-Bohm effect for excitons, practically indistinguishable from the numerical noise without an applied electric field, becomes clearly evident in the optical absorption once the electric field is applied in the plane containing the nanoring. The enhancement arises as a result of the field-induced delocalization of the relative electron-hole motion around the entire ring. The excitonic effects are essential to describe even qualitatively the absorption spectra.

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The (hc/e) Aharonov-Bohm (AB) effect[1] is an example of a geometrical phase[2] depending on the spatial coherence of the wave function, and as such it is a purely quantum mechanical effect without a classical analog. The AB effect, moreover, underlies the phenomenon of universal conductance fluctuations,[3] and thus must be regarded as a central feature of mesoscopic transport.

In fact, the AB effect has lain almost exclusively in the domain of transport. Recently, however, nanoscale semiconductor quantum rings [nanorings (NR)] have been fabricated that exhibit pronounced optical features associated with the optical generation of excitons.[4–6] This has spurred a search for optical signatures of the AB effect associated with excitonic resonances—an effect of interest, in part, due to the charge neutrality of the exciton. Predicted in Refs. 7 and 8, the effect was later shown to be disappointingly small in NR's of technologically accessible dimensions,[9,10] and the experimental search for the effect has not born fruit.[11] One of the possible problems that preclude the observation of AB effect is that the spatial extention of the internal electron-hole $(e\text{-}h)$ motion within the low-lying excitons (exciton Bohr radius) is much smaller than accessible NR diameters; therefore, neither the electron nor the hole individually samples the entire NR to any appreciable degree, leading to a negligibly small excitonic AB effect.

In the following, we discuss our theoretical work that indicates a large enhancement of the excitonic AB effect may be possible by applying an electric field in the NR plane. Let us begin with a brief review of the AB effect. If an electron confined to move in a ring threaded by magnetic flux Φ is in an eigenstate $\Psi(\phi_e)$ with energy E, then $\Psi(\phi_e)e^{-i\phi_e}$ is the eigenstate with the same energy for flux $\Phi+\Phi_0$, where $\Phi_0 = hc/e$ is the flux quantum. For example, the ground state changes from $\Psi(\phi_e)=1/\sqrt{2\pi}$ at $\Phi=0$ to $\Psi(\phi_e) = (1/\sqrt{2\pi})e^{-in\phi_e}$ at $\Phi=n\Phi_0$. This results in a periodic variation of the transmission of an electron through the ring.

In semiconductor NR's,[4] an optical field creates $e\text{-}h$ pairs. Since the $e\text{-}h$ pair does not possess a net charge, it is not immediately apparent whether the sensitivity to the magnetic flux, pertaining to each particle separately, still exists for the $e\text{-}h$ pair. However, it is easy to show that in a narrow NR in the absence of the Coulomb interaction between the electron and hole, the optically allowed ground state of the $e\text{-}h$ pair changes from $\Psi(\phi_e,\phi_h)=1/2\pi$ for $\Phi=0$ to $\Psi(\phi_e,\phi_h) = (1/2\pi)\exp[-in(\phi_e-\phi_h)]$ for $\Phi=n\Phi_0$. In such a state, the optically created electron and hole circulate within the NR in the opposite directions. Similar alteration occurs for the states with higher energies. Thus, the properties of a noninteracting electron and hole are still very sensitive to the magnetic flux even though their net charge is zero.

This sensitivity should in principle be preserved even if the Coulomb interaction is included. It was predicted that the excitonic properties such as the binding energy and the height of absorption peak vary periodically with threading magnetic flux.[7,8] Later, numerical calculations with the NR parameters close to the experimental values[4] showed that the excitonic Aharonov-Bohm effect is very small indeed and is probably not measurable in experiments.[9,10] To enhance the effect one can resort to smaller NRs in which the electron and hole may still individually probe the whole ring circumference. However, this also requires strong magnetic fields to produce significant fluxes. The purpose of this work is to show that strong enhancement of the excitonic Aharonov-Bohm effect is possible for large NR's in the presence of a lateral electric field. We note that a completely different way of enhancement was suggested recently—due to in-plane radial polarization of excitons.[12]

A complete analysis of the excitonic absorption in semiconductor NR's must account for the band-structure degeneracies, confining potential, the $e\text{-}h$ interaction, warping (anisotropy), and perhaps strain, disorder, etc. It is prohibitively difficult to carry out such an analysis for semiconductors in their full complexity. We can, however, consider a model that contains all the important physics and yet is simple enough to be solved numerically.

Let us assume that the carrier confinement inside the NR is strong enough so that the exciton envelope function can be approximated by

$$\Psi(\mathbf{r}_e,\mathbf{r}_h)=\psi(\phi_e,\phi_h)g_e(\mathbf{r}_e)g_h(\mathbf{r}_h), \tag{1}$$

where $g_e(\mathbf{r}_e)$ and $g_h(\mathbf{r}_h)$ are the lowest subband wave functions for the electon and hole, respectively; ϕ_e and ϕ_h are the polar angles. The envelope $\psi(\phi_e,\phi_h)$ obeys the Schrödinger equation (SE)

$$i\hbar\frac{\partial\psi}{\partial t}=\mathcal{H}\psi-ed\mathcal{E}(t)\,\delta(\phi_e-\phi_h)-i\hbar\gamma\psi \tag{2}$$

with the Hamiltonian

$$\mathcal{H}=\frac{\hbar^2}{2m_eR^2}\left(-i\frac{\partial}{\partial\phi_e}+\frac{\Phi}{\Phi_0}\right)^2+eRF\cos\phi_e+\frac{\hbar^2}{2m_hR^2}\times\left(-i\frac{\partial}{\partial\phi_h}-\frac{\Phi}{\Phi_0}\right)^2-eRF\cos\phi_h+V_{\rm coul}(\phi_e-\phi_h) \tag{3}$$

where F is the lateral electric field and Φ is the threading magnetic flux. The second term on the right-hand side of Eq. (2) describes the e-h excitation by an optical field with $\mathcal{E}(t)$ the time-dependent envelope; d is the interband dipole moment; γ is the phenomenological dephasing rate. We only account for heavy holes.

The effective Coulomb potential in Eq. (3) is

$$V_{\rm coul}(\phi_e-\phi_h)=-\frac{e^2}{\epsilon_b}\int d\mathbf{r}_e d\mathbf{r}_h\frac{|g_e(\mathbf{r}_e)|^2|g_h(\mathbf{r}_h)|^2}{|\mathbf{r}_e-\mathbf{r}_h|}. \tag{4}$$

The specific form of $V_{\rm coul}(\phi)$ depends on the shape of NR cross section. However, for narrow NR's one can use the cusp-type potential which agrees very well with the actual potential in quantum wires[14]

$$V_{\rm coul}(\phi_e-\phi_h)=-\frac{e^2}{\epsilon_b}\left(2R\sin\left|\frac{\phi_e-\phi_h}{2}\right|+a_0\right)^{-1}, \tag{5}$$

where parameter a_0 is proportional to the lateral size of the NR cross section.

To find the optical absorption, we solve Eq. (2) for the e-h pair dynamics after excitation by a short optical pulse. The optical susceptibility in the frequency domain is found after Fourier transforming the complex optical polarization $\mathcal{P}(t)=d^*\int_0^{2\pi}d\phi\psi(\phi,\phi)$.

To simplify SE (2), we eliminate the explicit dependence of Hamiltonian (3) on Φ by using substitution

$$\psi(\phi_e,\phi_h)=U(\phi_e,\phi_h)e^{-if\phi_e+if\phi_h},\quad f=\Phi/\Phi_0.$$

Function $U(\phi_e,\phi_h)$ obeys SE (2) with Hamiltonian (3) evaluated at $\Phi=0$. The effect of Φ is entirely included in the boundary conditons which, instead of simple equality of $\psi(\phi_e,\phi_h)$ at the angles 0 and 2π, are $U(0,\phi_h)=U(2\pi,\phi_h)e^{-i2\pi f}$, $U(\phi_e,0)=U(\phi_e,2\pi)e^{+i2\pi f}$ for $0\leqslant\phi_{e,h}<2\pi$. We use the following parameters: $R=500$ Å, $a_0=50$ Å, $m_e=0.07m_0$, $m_h=0.4m_0$ (with m_0 the free-electron mass), $\epsilon_b=12$, and $\hbar\gamma=0.2$ meV.[15]

To solve Eq. (2), we use the usual finite-difference discretization scheme. The spatial step is $\Delta\phi_e=\Delta\phi_h=2\pi/320$. To integrate Eq. (2) in time we use an explicit method with a typical time step 0.2 fs. The total integration time $t_{\rm max}$ is chosen in such a way that the optical polarization becomes practically zero at the end of integration, $t_{\rm max}=6.0/\gamma$.

The optical absorption in the NR in the absence of electric and magnetic fields is dominated by a strong exciton peak at about -10.22 meV [Fig. 1(a)]. (We measure energy with respect to the free-particle interband transitions and normalize the absorption spectra to the exciton peak for $F=0$.) This is related to the suppression of the e-h continuum in quantum wires (reduced quasi-one-dimensional Sommerfeld factor).[13,14] In the present case, the quasi-one-dimensional e-h continuum is broken into a set of discrete states by the requirement of periodicity imposed by the circular topology. The Sommerfeld factor increases with energy and for higher-energy states the difference between the optical absorption with and without excitonic effects decreases [Fig. 1(b)].

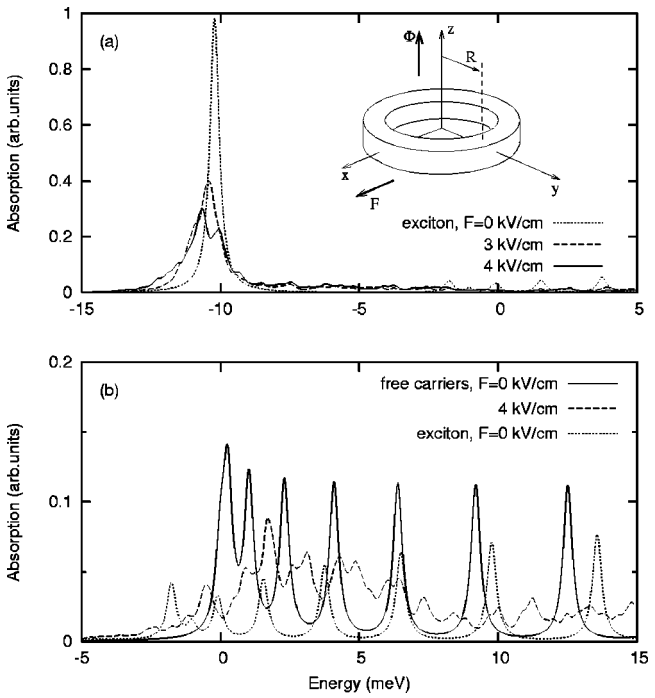

FIG. 1. Optical absorption of NR (with and without excitonic effects) for different electric-fields F in the absence of magnetic-flux Φ. Inset in (a) shows schematic of a NR in the presence of F and Φ.

Although the free-carrier interband absorption is never realized in practical situation, it is still of theoretical interest as it allows one to clearly identify the role of excitonic effects. Without excitonic effects [Fig. 1(b)], the absorption spectrum consist of Lorentzian peaks at energies $E=E_0n^2$ with $E_0=\hbar^2/(2m_rR^2)=0.256$ meV, $m_r^{-1}=m_e^{-1}+m_h^{-1}$, $n=0,\pm1,\pm2$, etc. Each state for $n\neq0$ is doubly degenerate.

When a dc field F is applied [Fig. 1(a)], the exciton peak reduces in height, broadens, and moves slightly to the red. Unlike quantum wires, in which application of F mostly results in the broadening of the peak due to field-induced tunneling,[16] in NR's the continuum is discrete and this allows one to observe a complicated structure in the exciton peak. The broadened peak is in fact composed of large number of states, which become optically-allowed in the presence of F. In the absence of excitonic effects [Fig. 1(b)],[17] the absorption spectrum becomes extended below the band gap and also becomes a significantly smoother function due to the appearance of many optically allowed transitions between the single-particle electron and hole states. The low intensity of the below-band-gap absorption is explained by the small

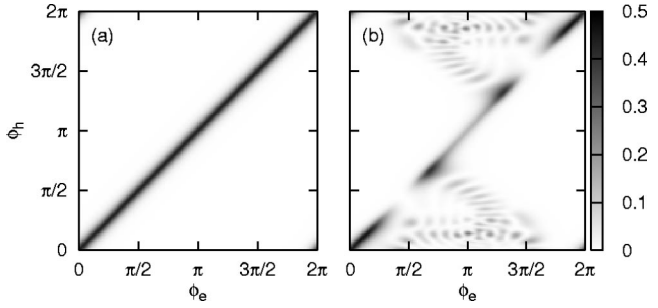

FIG. 2. Density plots of the normalized excitonic distribution function $|\psi(\phi_e,\phi_h)|^2$ for cw optical excitation at ω in the absence of Φ. Optical field is tuned at the exciton peak $\hbar\omega = -10.22$ meV for $F=0$ (a) and $\hbar\omega=-10.66$ for $F=4$ kV/cm (b).

overlap between the electron and hole states localized in the opposite ends of the NR. Well above the band gap, the density of states decreases and with it the absorption.[18]

The change in the excitonic properties with F is best illustrated by the steady-state wave function $\psi(\phi_e,\phi_h)$. To find it, we solve SE (2) subjected to a steplike optical pulse with fixed carrier frequency. After transient processes are completed, at $t \gg \gamma$, the e-h wave-function $\psi(\phi_e,\phi_h)$ contains all necessary information about the steady state. Figure 2(a) shows that for $F=0$, $|\psi(\phi_e,\phi_h)|^2$ peaks at $\phi_e=\phi_h$ and decreases very rapidly as the e-h separation increases. Note that for $F=0$, $|\psi(\phi_e,\phi_h)|^2$ depends only on the relative e-h coordinate $|\phi_e-\phi_h|$. This simple dependence disappears when the electric field is applied; F causes the strong reduction of $|\psi(\phi_e,\phi_h)|^2$ at the angles $\pi/2$ and $3\pi/2$, where the direction of F is parallel to the NR circumference and thus the exciton can be most easily ionized. One can estimate the field strength needed to cause ionization. For $F=0$, the half-width of the excitonic wave function in Fig. 2(a) is roughly 0.2 rad. For the given radius of 500 Å, this corresponds to the distance of 100 Å. The field needed for a particle to gain 10 meV in a 100 Å distance is 10 kV/cm which is the same order of magnitude as 4 kV/cm in Fig. 2(b). In addition to strongly reduced e-h density at angles $\pi/2$ and $3\pi/2$, the wave function becomes significantly delocalized around the entire NR. The complex structure of $|\psi(\phi_e,\phi_h)|^2$ indicates a strongly nonperturbative e-h behavior at $F=4$ kV/cm. We would like to emphasize that even though $|\psi(\phi_e,\phi_h)|^2$ in Fig. 2(b) is stationary, it is not an eigenfunction of e-h Hamiltonian (3). Due to finite dephasing rate γ, there are several eigenstates of Eq. (3) that may lie withing the range of $\hbar\gamma$ and contribute to the steady-state function created by the cw optical field.

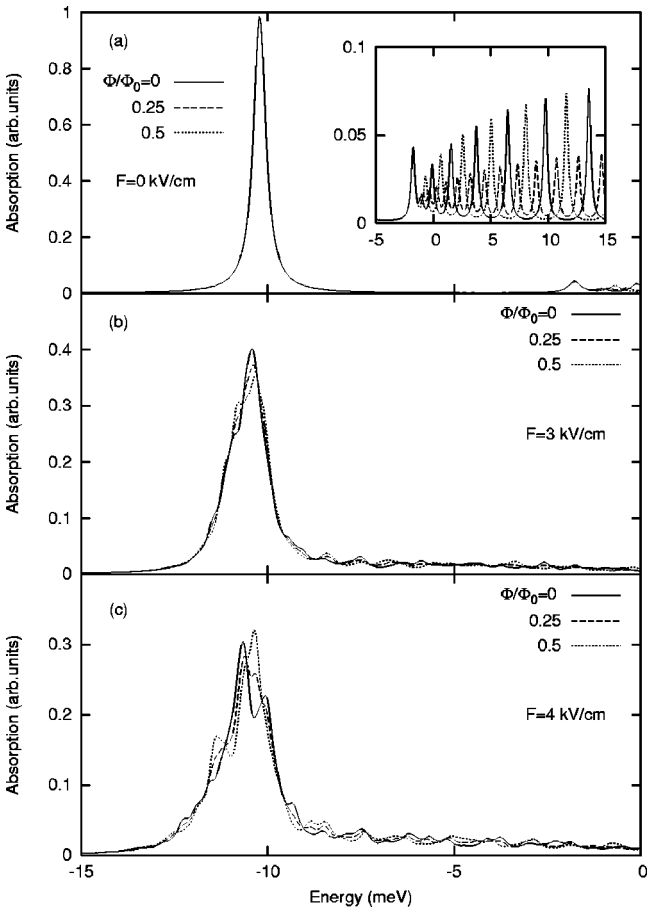

FIG. 3. Optical absorption of NR with excitonic effects for different F and Φ. In (a), all curves are superimposed for energies below the band gap.

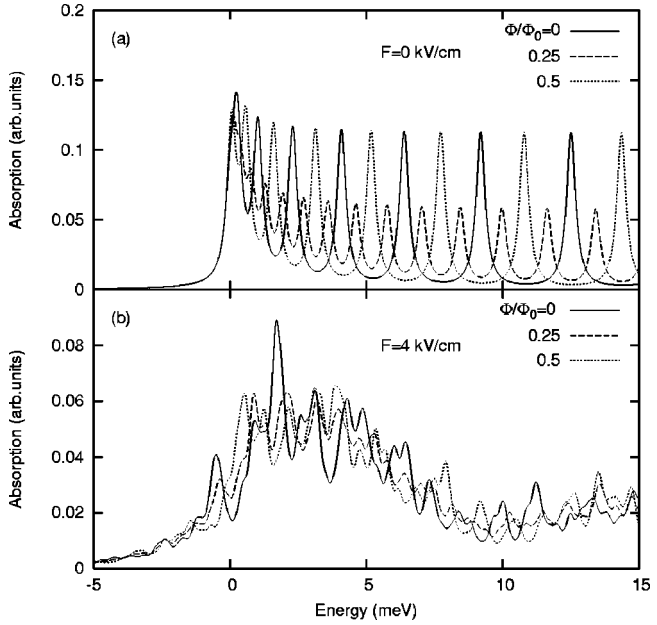

FIG. 4. Optical absorption of NR without excitonic effects for different F and Φ.

Having studied the effect of the electric field on the optical absorption, we now can include the magnetic field (Fig. 3). For $R=500$ Å the magnetic induction B needed to create flux $\Phi=\Phi_0$ is about 0.5 T. Without the electric-field F, the strong exciton peak does not change noticebably with Φ. The only changes observed are for the states well above the band gap which behave very similar to the case without excitonic effects [compare inset in Fig. 3(a) and Fig. 4(a)]. Thus, neither the energy nor the height of the exciton peak

changes with Φ for the given parameters. This agrees with the results presented in Refs. 9 and 10.

However, the picture changes drastically with F present [see Fig. 3(b,c)]. The presence of many states that lie within the peak gives rise to substantial changes of the overall peak shape with Φ. The origin of high sensitivity to Φ can be understood as follows. Each state that lies withing the peak is rather extended as illustrated in Fig. 2(b). Thus, the magnetic flux causes shift in the energy and the absorption strength for each of the states. This results in significant changes in the shape of the peak on the order of 10%. The changes become more pronounced as F increases.

It is interesting to compare the effect of the magnetic field on excitonic absorption (Fig. 3) and free-carrier absorption (Fig. 4). Without F, the free-carrier absorption spectrum consists of Lorentizian peaks at energies $E=E_0(n+\Phi/\Phi_0)^2$, $n=0,\pm 1,\pm 2$, etc. So the magnetic field simply shifts equally the energy of each peak. With F present, the single-particle wave functions become very nonuniformly distributed around the ring. Here we note that the below-band-gap spectum does not change with Φ as much as the above-bandgap one. This is explained by strong localization of the lowest states, which contribute to the below-band-gap absorption, and thus their insensitivity to the magnetic field.[18]

Finally we comment on the possibility of observing the excitonic AB effect. Our predictions are based on what is essentially a one-dimensional model. For a typical NR width of 100–200 Å the effect of $F \lesssim 5$ kV/cm field on the lowest subband is expected to be rather small.[19] Thus keeping only one subband is well justified for NR of Ref. 6 but may be not so good an approximation for donut-shaped NR of Ref. 4, although the model likely captures essential physics. The relevant magnetic-field strength, ~ 1 T, is relatively small and will not lead to appreciable sub-band mixing. The approximation of true Coulomb potential by a cusp-type one becomes better as the NR width decreases; this corresponds to smaller values of the parameter a_0 in Eq. (5). However, quasi-one-dimensional structures are known to have extremely well localized exciton states with the size approaching zero in the true one dimensional limit.[13] We performed numerical calculations using real Coulomb potential (4) for very narrow (<100 Å) NR's. The results indicated similar trends as those presented here but correct quantitative description required prohibitively high spatial resolution in the numerical integration. In general, the F-induced AB effect can be observed in very narrow NR's but this requires stronger electric fields.

To conclude, we have shown that a lateral electric-field applied to a semiconductor NR gives rise to a substantial periodic modulation of the optical absorption spectrum as a function of the threading magnetic flux. This is a direct consequence of field-induced delocalization of the e-h motion caused by the electric field. The enhancement of the modulation, as compared to the case without the electric field, may allow observation of the excitonic AB effect in semiconductor NR's.

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*Present address: Eloret Corporation, NASA Ames Research Center, MS 229-1, Moffett Field, CA 94035.

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