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Physics Letters A 342 (2005) 497–503

PHYSICS LETTERS A

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Modification of thermal radiation by periodical structures containing negative refractive index metamaterials

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Received 14 April 2005; received in revised form 23 May 2005; accepted 28 May 2005

Available online 8 June 2005

Communicated by R. Wu

Abstract

We investigated thermal radiation power spectrum in 1D ordered structures containing negative refractive index materials. We utilized an approach based on the Kirchoff's second law and applied the transfer matrix method to calculate emittance and to obtain the power spectrum of the multilayer. We analyzed both on-axis and off-axis radiation.

PACS: 78.20.Ci; 42.70.Qs; 41.20.Jb; 44.40.+a

Keywords: Negative refractive index materials; Left-handed materials; Metamaterials; Thermal radiation; Emittance; Photonic bandgap

1. Introduction

Negative refractive index metamaterials (NRM) are artificial composite subwavelength structures with effective electromagnetic response functions (permittivity and permeability) artificially tuned to achieve negative values of their real part [1]. These materials were theoretically predicted by Veselago [2], rediscovered by Pendry [3], experimentaly confirmed by Smith [4] and both experimentaly and theoretically investigated by many different teams (see, e.g., [1] and references cited therein).

The underlying principle in constructing NRM relies on the appearance of effective permittivity and permeability both lower than zero in the same well defined frequency band. The analyticity of refractive index regarded as a complex function and the causality principle require that the real part of the refractive index also be negative [5]. A consequence is that the product of the electric and magnetic field vectors is antiparallel with the wave vector, i.e., we deal with backward waves whose phase velocity is antiparallel with Poynting vector, while electric, magnetic field and wave vector form a left-oriented set. This is the

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0375-9601/$ – see front matter
doi:10.1016/j.physleta.2005.05.076

reason why such structures are sometimes called "left-handed materials" (LHM).

Many interesting phenomena not appearing in natural media were predicted and observed in double negative materials. These include negative refraction (the reversal of the Snell's law), perfect lensing [6], the appearance of subwavelength resonant cavities [7], reversal of Cherenkov radiation, etc.

An interesting topic of investigation is the distribution of electromagnetic modes in layered structures incorporating negative index materials. The use of conventional photonic crystal structures to modify thermal radiation was investigated by Cornelius and Dowling [8]. Subsequent theoretical and experimental results on the same topic include [9–11]. In the case of wave propagation through a structure consisting of a both positive refractive index material (PRM) and NRM layers a very important phenomenon of phase compensation occurs, which may be described as a partial or complete removal of phase shift of an electromagnetic wave propagating through a PRM–NRM structure [12].

Until today, no comprehensive analysis of the influence of the NRM media to the thermal radiation distribution appeared in literature. In [13] a modification of Planck law in NRM was derived relying on a simple quantized field description. There are few other papers dealing with the quantum field description of NRM-related phenomena and some interesting phenomena that arise from it, like the modification of spontaneous emission [14] and super-radiance effect [15].

In this Letter we investigate the modification of thermal radiation power spectrum by one-dimensional structures incorporating both NRM and conventional materials, emphasizing influence of phase compensation. We analyze periodic 1D structures and include both normal and oblique wave incidence. We use an indirect method based on the second Kirchoff's law for thermal radiation to investigate the emittance of blackbody when a multilayer structure incorporating NRM is used as a filter on a thick blackbody. In our calculations we use the well-known transfer matrix technique [16,17].

2. The Planck law in NRM

Since metamaterials are structured on a subwavelength scale, it is assumed that their magnetic and electric response can be described by effective permeability μ and permittivity ε. A practice often met in literature when investigating NRM is to analyze the cases with frequency independent ε and μ [18–21], but a realizable metamaterial must be dispersive and lossy in order to preserve causality principle [5].

For the sake of simplicity we assume that both effective permittivity and permeability are of the same form

$$\varepsilon_{\mathrm{eff}}(\omega) = 1 - \frac{\omega_{pe}^2}{\omega(\omega + i\Gamma_e)},$$
$$\mu_{\mathrm{eff}}(\omega) = 1 - \frac{\omega_{pm}^2}{\omega(\omega + i\Gamma_m)} \tag{1}$$

and that plasma frequencies and dumping constants are equal $\omega_{pe} = \omega_{pm} = \omega_p$, $\Gamma_{pe} = \Gamma_{pm} = \Gamma_p$. This choice simplifies the form of ε and μ without a loss of generality. The refractive index $n_{\mathrm{eff}}(\omega) = \sqrt{\varepsilon_{\mathrm{eff}}(\omega)\mu_{\mathrm{eff}}(\omega)}$ is thus

$$n_{\mathrm{eff}}(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\Gamma)}. \tag{2}$$

In our calculations we assume that dumping is negligible, which is an acceptable approximation even from the experimental point of view [4]. When lossy metamaterial is considered, a common assumption in literature is that the dumping factor is given as a fraction of plasma frequency [22].

An expression for the Planck radiation law in NRM media was obtained in [13] by following a simple quantized-field description for radiation in negative-index material, which was assumed to be isotropic, dispersive and absorptionless at frequencies of interest. The approach was based on modified Einstein coefficients of spontaneous emission and absorption in the light of a simple electric dipole transition picture. Similar results were obtained in [14,15].

The same result can be obtained by a simple textbook approach (e.g., [23]), which, however, is valid for any medium described by dispersive refractive index and thus does not make a distinction between NRM and ordinary media. We apply it in the following manner. The density of states per volume of a photon set occupying a range of impulses $(\vec{p}, \vec{p} + d\vec{p})$ is $dG(p) = 2(4\pi p^2\, dp/h^3)$, where $\vec{p} = \hbar\vec{k}$ and $\vec{k}$ is the photon wavenumber, the multiplier 2 is due to the number of polarizations and h^3 stems from Heisenberg relations. We assume the standard free-space boundary conditions. The mean number of photons is described by the Bose–Einstein distribution, thus the density of photons is

$$dn(p) = \frac{2}{h^3}\frac{4\pi p^2\,dp}{\exp(cp/kT)-1}. \tag{3}$$

Further $p = \hbar\omega n(\omega)/c$, $dp = \hbar\omega\gamma(\omega)n(\omega)/c$, where $n(\omega)$ is the refractive index and $\gamma(\omega) = n(\omega) + \frac{d(\omega n(\omega))}{d\omega}$. The connection between impulse and frequency leads to the density of photons per unit frequency $\frac{dn(\omega)}{d\omega}$. As the last step we derive the spectral energy density as $\rho(\omega) = \frac{dn(\omega)}{d\omega}\hbar\omega$, thus arriving at the expression

$$\rho(\omega) = \frac{\hbar\omega^3}{\pi^2 c^3}\frac{n^2(\omega)\gamma(\omega)}{\exp(\hbar\omega/kT)-1}. \tag{4}$$

As we can see from (4) the Planck's law in NRM media differs from that in the free-space by a factor describing the dispersive properties of the media

$$\rho^{\mathrm{DNG}}(\omega)/\rho^{\mathrm{F.S.}}(\omega) = n^2(\omega)\gamma(\omega). \tag{5}$$

For the dispersion relations we use the simple form (2) to establish the modification factor of the power spectral density of equilibrium radiation within NRM versus that in vacuum as

$$n^2(\omega)\gamma(\omega) = \bigl(1-(\omega_p/\omega)^2\bigr)^2\bigl(1+(\omega_p/\omega)^2\bigr). \tag{6}$$

3. Thermal radiation and multilayers containing negative index media

We consider a system in thermal equilibrium at a given temperature, its radiation having a Planck's blackbody (BB) spectrum. From the point of view of prospective practical applications, the blackbody radiation may be modified using a photonic crystal filter and thus altering the spectral emissivity of the BB radiator and/or changing the angular distribution of the radiation. This was done in [8] for the case of purely positive-media structures.

In a most general case the photonic crystal filter may have a full 3D periodicity (or even be quasi-periodic). According to the John and Wang model [8], under certain conditions this can be reduced to a 1D case without a loss of generality.

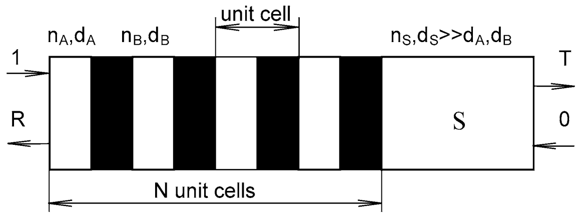

Fig. 1. Basic 1D multilayer structure for emissivity control.

To calculate the modification of thermal radiation, it is necessary to determine the thermal emittance E of the photonic crystal. This is done by an "indirect" method based on the Kirchoff's law of detailed balance. According to it a material's emittance in thermal equilibrium is proportional to its absorptance, and for a blackbody they are equal [8]. The absorptance is defined by the reflection and transmission coefficient, $A = 1 - R - T$.

Once the emittance is obtained, its multiplication by the Planck power spectrum gives the power spectrum of the PBG emitter $\rho^{\mathrm{PBG}}(\omega)$ in terms of its emittance $E^{\mathrm{PBG}}(\omega)$ and the blackbody spectrum $\rho^{\mathrm{BB}}(\omega)$

$$\rho^{\mathrm{PBG}}(\omega) = E^{\mathrm{PBG}}(\omega)\rho^{\mathrm{BB}}(\omega). \tag{7}$$

Fig. 1 represents a photonic band gap (PBG) structure selected to modify the mode density of radiation of the emitting substrate S. In a general case it contains both positive index materials and NRM. The structure inhibits thermal emittance from the substrate at frequencies within the PBG, but enhances it at the band-edge. This result is confirmed for the case of all-dielectric and metal–dielectric positive index PBG materials both theoretically [11] and experimentally [10].

The structure is composed from two media with refractive indices (n_A, n_B) and a geometrical thickness (d_A, d_B), A and B denoting the conventional and the NRM slabs, respectively. The structure is deposited on a thick substrate $(d \gg \lambda)$ with an index n_S. We chose layers with a quarter-wavelength optical thickness $n_A d_A = n_B d_B = \lambda_0/4$. Hence, the phase shifts in the corresponding layers are $\delta_A = (\pi/2)\Omega$ and $\delta_{\mathrm{B}} = (\pi/2)\Omega$ where $\Omega = \lambda_0/\lambda = \omega/\omega_0$ is normalized frequency. The entire structure is surrounded by a medium n_0 (air or vacuum).

We chose a quarter-wavelength optical thickness for our layers to establish a connection with prior experimental and theoretical work and to directly compare our results to those previously published. More specifically, one encounters the same choice of individual layers thickness in literature on absorptance and emittance tailoring by PBG structures [8,10,11] but also in papers on NRM-caused phase compensation and its application for antireflective coatings, high-reflective coatings, transmission filters, etc. [12, 18–20]. While there is no fundamental reason not to use a different optical thickness, quarter wavelength appears to be the most frequently used approach for different optical applications and one gets a clear physical picture without a loss of generality. Only a similar spectral behavior could be obtained by different choice of optical thickness [1,18], but it remains a question of practical implementation for a concrete design.

The transfer matrix technique which includes material dispersion and absorptive losses [16,17] can be used to compute transmission and reflection coefficients and the power spectrum. We apply it using interface matrices $M_{\alpha\beta}$ (α, β denote a or b at the interface) and propagation matrices M_γ (γ denotes a or b of the given layer)

$$M_{\alpha\beta} = 1/2[1 + n_\beta/n_\alpha 1 - n_\beta/n_\alpha; 1 - n_\beta/n_\alpha 1 + n_\beta/n_\alpha] \tag{8}$$

and

$$M_\gamma = \left[\exp(-i\delta_\gamma)0; 0\exp(i\delta_\gamma)\right], \quad \delta_\gamma = 2\pi n_\gamma L_\gamma/\lambda. \tag{9}$$

The product of $M_{\alpha\beta}$ and M_γ uniquely describes wave propagation through the multilayer. In the case of oblique incidence the matrices (8), (9) retain the same form, but the n_α is replaced by $n_\alpha\cos(\theta_\alpha)$ in the propagation matrices for both s- and p-polarization, $n_\alpha \to n_\alpha\cos(\theta_\alpha)$ for s-polarization and $n_\alpha \to \frac{n_\alpha}{\cos(\theta_\alpha)}$ for p-polarization in the interface matrices. θ_A, θ_B are the incident angles for the case of oblique incidence. The overall transfer matrix for a chosen structure is the product of the interface and the propagation matrices. The transmittance and the reflectance stem from the overall transfer matrix which has the form $M = [m_{11}m_{12}; m_{21}m_{22}]$. For a periodic structure composed from two media A and B, surrounded by a medium C the overall transfer matrix is given by $M = M_{CA}T_1^{(N-1)}(M_A M_{AB} M_B M_{BS} M_S M_{SC})$, where $T_1 = M_A M_{AB} M_B M_{BA}$ is the unit cell transfer matrix and N denoting the number of unit cells.

$$T = |t|^2 R = |r|^2,$$
$$\text{where } t = 1/m_{11} \text{ and } r = m_{21}/m_{11}. \tag{10}$$

In our investigation we consider isotropic media for both non-dispersive and dispersive cases.

One should note here that no plasmon modes interacting with the incident plane wave and changing emittance spectra were taken into account in this Letter. The reason is that a propagating plane wave incident on a flat surface cannot excite surface plasmons regardless of its incidence angle, since the plasmon modes must have a larger momentum at the same frequency for all energies considered [1]. Thus no influence of the plasmon mode to the emittance characteristics can be expected. To change this situation, one would have to provide an additional momentum to disturb the plasmon mode, e.g., to use surface roughness, a grating structure or similar. This is valid both in the case of positive and negative index materials [1,24].

4. Results and discussion

Fig. 2 shows the calculated emissivity of 1D PBG structures versus frequency and angle of incidence for different polarizations. Fig. 2a shows the emissivity of an all-dielectric PBG material for unpolarized case. We obtained similar dependencies for the all-dielectric structure for s- and p-polarizations (not shown here). Figs. 2b–d show the calculated emissivity of NRM-containing 1D PBG structures for different polarizations.

The dependence in Fig. 2a illustrates a problem pertinent to all positive-material PBG filters: such structures can either have an optimum performance in a very narrow wavelength range for all incident angles, or for a larger wavelength range, but for a very limited spatial angle.

This is not the case with the NRM-containing filters. Figs. 2b–d show that the angular dependence in emittance spectrum is much less prominent than in ordinary PBG structures. This points out to the possibility of designing efficient NRM filters almost insensitive on the radiation propagation angle. Structures containing NRM influence differently the thermal radiation spectrum in comparison to ordinary media PBG structure. The suppressed region of thermal radiation is wider, and the spectral characteristics more flat, i.e., without sharp oscillation typical for positive index materials. The influence of the angle of incidence is less noticeable than for the corresponding ordinary struc-

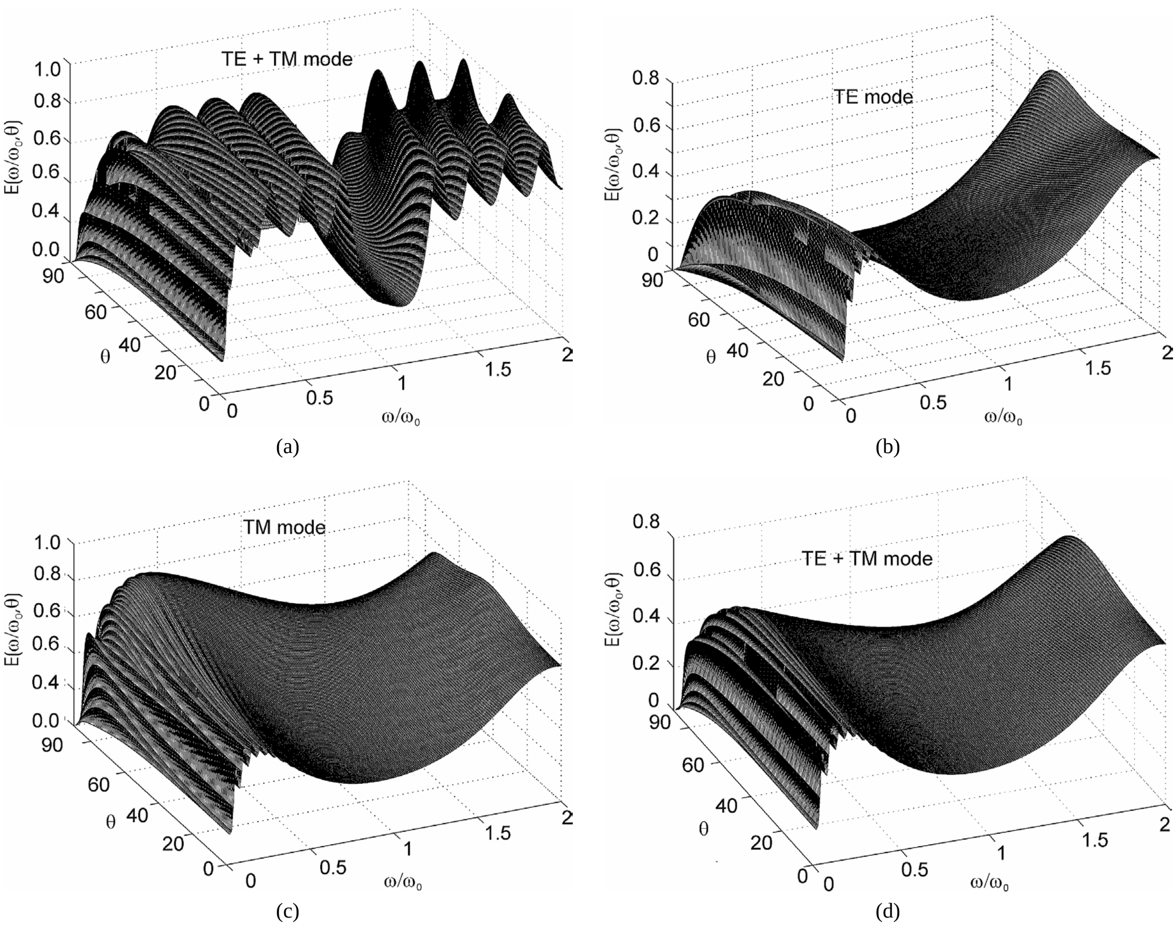

Fig. 2. Emittance as a function of incident angle θ and normalized frequency ω/ω_0 for NRM-containing 1D PBG, 5 periods, $n_A = 1.41$, $n_B = -2$; (a) unpolarized case, positive index material, $n_A = 1.41$, $n_B = 2$. (b) TE-mode polarized emittance E_{TE}; (c) TM-mode polarized emittance E_{TM}; (d) unpolarized $E = 1/2(E_{\mathrm{TE}} + E_{\mathrm{TM}})$; In all cases $n_S = 3 + i0.3$ for the substrate.

tures. The emittance shows no ripples and no sharp frequency shifts between polarizations. Such behavior is a result of phase compensation [12,25].

Further we considered a more realistic case with dispersion taken into acount. We used the form (2), assuming that $\Gamma = 0$ [1]. Fig. 3 depicts the BB emittance versus incident angle θ and normalized frequency ω/ω_0 modified by dispersive NRM for unpolarized case. We used the Drude model which is commonly accepted for the description of metamaterials [1]. The dispersion we used was $n_B(\omega/\omega_0) = 1 - 3(\omega/\omega_0)^2$ where $\omega_0 = 2\pi c/\lambda_0$, quarter-wavelength frequency. We chose $\omega_{pe} = \omega_{pm} = \omega_p = \frac{\omega_0}{\sqrt{3}}$ in Eq. (2), thus obtaining $n_B = -2$ at $\omega/\omega_0 = 1$, the same refractive index value as that used to calculate the dispersionless case in Fig. 2. We further used $n_A = 2$, $L_A = 0.25\lambda_0/n_A$, $L_B = 0.25\lambda_0/n_B$ and $L_S = 10\lambda_0/\operatorname{Re}(n_S)$. The calculated emittance in Fig. 3 clearly shows the existence of full phase compensation in the emittance spectrum (the peak values at the frequency $\omega/\omega_0 = 1$ in the $E(\omega, \theta)$ dependence).

Similar to zero-n photonic band gap [19], also obtained by stacking alternating layers of positive index materials and NRM but furnishing transmission minimum, this phase compensated situation arises when the averaged effective refractive index of the structure equals zero [26]. The resulting narow transmission peak is invariant with respect to a length scale change and insensitive to angular dependence. There is a shift toward higher frequencies in spectral emittance

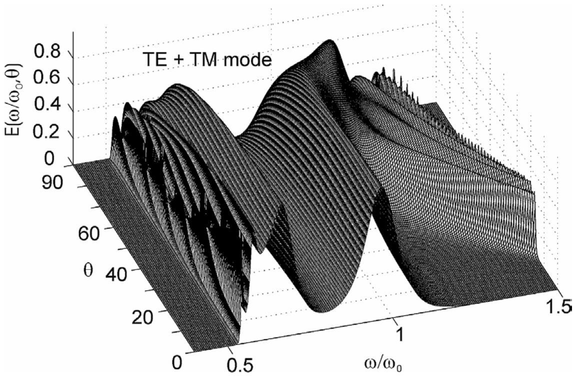

Fig. 3. Emittance $E(\omega, \theta)$ as a function of incident angle θ and normalized frequency ω/ω_0 for unpolarized case with dispersion taken into account, $n_B(\omega/\omega_0) = 1 - 3(\omega_0/\omega)^2$ and $\omega_0 = 2\pi c/\lambda_0$-quarter-wavelength frequency (at this frequency $n_B = -2$) $n_A = 2$, $L_A = 0.25\lambda_0/n_A$, $L_B = 0.25\lambda_0/n_B$, $L_S = 10\lambda_0/\mathrm{Re}(n_S)$.

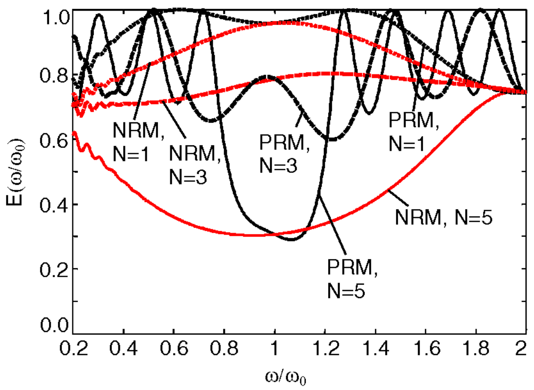

Fig. 4. Comparison of emittance for conventional multilayers and NRM for different numbers of layer pairs N, $n_A = 1.41$, $n_S = 3 + i0.3$. $n_B = -2$ for NRM case and $n_B = 2$ for conventional multilayer case.

for phase compensated situation in Fig. 3 for larger angles. The same feature can be observed both in periodic and in quasi-periodic spectra.

Fig. 4 analyses the evolution of spectral emittance with an increase of layer pairs number for dispersionless NRM-containing multilayers with absorptive substrate for the case of normal incidence. A comparison to the case of positive index multilayer material is given. While the spectrum of the positive material becomes progressively more complex with a layer number increase, the NRM-containing multilayers' dependence remains more flat, while the suppressed range is wider. The observed spectral behavior of the NRM-containing structures is a consequence of marked phase compensation. It suppresses the influence of multiple reflections between individual layers and even completely removes it at a single frequency corresponding to the fully compensated structure (zero phase shift). This results in much less pronounced side ripples in spectral characteristics.

An increase of the number of layers spreads the band of suppressed emittance instead of magnifying and multiplying the ripples like in positive index material case. Generally, the interference spectra which are characteristic for the all-PRM structures are flattened and spread due to phase compensation effects. This behavior is observed in NRM-containing multilayers regardless of the fact if dispersion is taken into account or not, since phase compensation occurs in both cases.

Another situation of interest for thermal radiation modification are quasi-periodic filter geometries [27]. Their band structure are more complex compared to periodic ones, which results in appearance of sharp resonance peaks [20,25]. Similar to periodic structures, quasi-periodic NRM multilayers also exhibit a strong influence of phase compensation to transmission [20,25]. Spectral self-similarity and narrow resonance spectral peaks occur, which has an applicative potential itself.

5. Conclusions

We analyzed modification of Planck's blackbody spectra by periodic structures incorporating NRM. Similar to positive-index photonic crystals to which such structures are related, they can be used to enhance, suppress or attenuate spontaneous emission in all or certain directions by changing the density of modes. The Letter handles the case of finite structures and does not consider the less realistic case of infinite crystals. Our results show that structures containing NRM show larger influence to the thermal radiation spectrum than all-dielectric PBGs. The suppressed region of thermal radiation is wider, and the spectral characteristics more flat, i.e., without sharp oscillation typical for the all-dielectric case. It can be also seen that the NRM-containing 1D structures are less dependent on the angle of incident radiation. The procedure presented here is of interest in designing special types of quasi-periodic NRM layers for emittance tailoring and will be published elsewhere. The described approach can be generalized to 2D and 3D PBG materials incorporating NRM media.

Acknowledgements

This work was partially funded by the Serbian Ministry of Science, Technologies and Development, within the framework of the contract TR-6151B.

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