Quantum nondemolition measurements: the route from toys to tools

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The history of the theory of quantum nondemolition (QND) measurements from the 1920s until today is reviewed. The definition and main principles of QND measurements are outlined. Achievements in the experimental realization of QND measurements and several new promising schemes of QND measurements are described. A list of the most important problems (from the authors' point of view) in the area of QND measurements is presented. The problem of measurement of a quantum oscillator phase is considered. A new method of phase measurement is proposed. Examples of possible solutions of fundamental physical problems using QND methods are given.

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I. HISTORICAL INTRODUCTION

Fragmentary notes concerning the problem of quantum nondemolition (QND) measurements may be found dating back to as early as the 1930s in the publications by the founders of the quantum theory. For example, a footnote in the paper by Landau and Peierls (1931) contains the following statement (translated from the original German): if there existed an interaction Hamiltonian depending on velocity only, one would be able to measure the velocity of a free particle with the arbitrary high precision. Further, in the same footnote, an incorrect conclusion was given: such a Hamiltonian does not exist, and therefore such a measurement is not possible.

In the outstanding monograph by von Neumann (1932) published one year later, one can find an analysis of the measurement of free mass velocity using the Doppler effect. Such a meter must have a resolution different from the coordinate meter in Heisenberg's microscope. Von Neumann did not complete this analysis. It was made several decades later.

In the fundamental monograph by Bohm (1952), published 20 years after von Neumann's book, one can find the following condition of a QND measurement: diagonality of the meter-object interaction Hamiltonian in the representation of the observable to be measured. Many years later, special analysis showed this condition to be excessive.

The lack of interest in quantum measurements before the 1960s could probably be explained by the fact that in the overwhelming majority of experimental methods of that time, physicists dealt only with serial tests (ensemble measurements). In this type of measurement, arbitrary desirable precision can be achieved by an increase in the number of tests. The interest in quantum measurements with single objects grew again with the emergence of quantum electronics and nonlinear optics. Simultaneously, the interest of physicists was drawn to nonclassical states of electromagnetic (e.m.) fields—the squeezed states [first described by Schrödinger (1927); the term itself appeared later]. At the same time, substantial progress was being made in the development of the mathematical apparatus of the measurement theory (Stratonovich, 1973; Helstrom, 1976; Kholevo, 1982).

The main impetus for a detailed analysis of ultimate sensitivity in quantum measurement with a single object was given by the problem of detection of gravitational waves. A burst of gravitational radiation caused by astrophysical catastrophe (see, e.g., Thorne, 1987) produces very weak a.c. tidal forces (acceleration gradients) which may be detected either by the relative displacement of two separated masses or by the occurrence of a.c. strain in one spatially extended mass. Simple analysis, made in 1967 (Braginsky, 1967), has shown that by improving the isolation of such macroscopic test masses from the heat bath (by reducing friction), one can bring them into the domain of substantially quantum behavior, even if the temperature T is high. In this case, the sensitivity in measurement of displacement or acceleration is limited by the (quantum) back action of the meter. The corresponding characteristic limits of sensitivity in coordinate measurements, as suggested by Thorne, were named the standard quantum limits (SQL). In 1974 (Braginsky and Vorontsov, 1974), it was proposed that a sensitivity better than SQL can be achieved if the meter "extracts" information only on one specially chosen observable. This article contained, however, an incorrect example; the first correct example—in essence, a scheme of gedanken QND experiment—was published three years later (Braginsky, Vorontsov, and Khalili, 1977; Unruh, 1978). In this example, it was proposed that the energy in e.m. resonators be measured by registering the ponderomotive pressure which acts on the resonator's walls. In this procedure the energy is not absorbed, and it may be repeated many times in the absence of dissipation. One year later, Thorne, with colleagues (1978) showed that there exists another type of QND meter for an oscillator which registers one of the two quadrature amplitudes.

Since then, more than 200 papers have been published on the subject of QND measurement, describing different aspects of this problem and suggesting and analyzing different measurement schemes, for both mechanical and e.m. systems. It has recently become evident that the area of probable applications of QND meters is much larger than the solution of the problem of sensitivity in gravitational-wave antennas. In the 1980s, the problem of QND measurements attracted quantum opticians, and the first practical success was achieved; QND measurements were realized experimentally, however, in the form of feasibility demonstrations.

From a theoretical point of view, as shown by special analysis after the first publications, QND measurements are the most fundamental type of quantum measurement, free of any nonfundamental uncertainties. On the other hand, successful development of QND methods on the engineering level undoubtedly promises a qualitative improvement of sensitivity in many experiments. As a result, intense studies in this field are now carried out in relatively large numbers of laboratories in several countries. The goals of this review are (1) to familiarize the reader with the principles of QND measurements; (2) to describe the present state of experimental programs and their prospects; (3) to outline unsolved problems in the area of QND measurements; and (4) to give a few examples of possible solutions of fundamental physical problems on the basis of QND methods.

II. DEFINITION AND MAIN PROPERTIES OF QUANTUM NONDEMOLITION METHODS

This section is intended to familiarize readers with QND measurement schemes without their needing to refer to the original papers. Let us first consider a simple example demonstrating the origin of the standard quantum limit. Suppose that the coordinate $x(t)$ of the mass m of an oscillator with eigenfrequency ω is continuously monitored. The value $x(t)$ may be expressed by two quadrature amplitudes X_1, X_2,

$$x(t)=X_1\cos(\omega t)+X_2\sin(\omega t), \tag{1}$$

which satisfy the uncertainty relation

$$\Delta X_1\Delta X_2\geqslant\frac{\hbar}{2m\omega}. \tag{2}$$

The continuous monitoring of the coordinate with the time-independent accuracy is evidently equivalent to the simultaneous symmetrical measurement of quadrature amplitudes:

$$\Delta X_1=\Delta X_2, \tag{3}$$

where ΔX_1 and ΔX_2 are the measurement errors. Substituting this condition in the uncertainty relation (2), we obtain the standard quantum limit for the coordinate of the oscillator:

$$\Delta X_{\rm SQL}=\Delta X_1=\Delta X_2=\sqrt{\frac{\hbar}{2m\omega}}. \tag{4}$$

If the coordinate is continuously monitored with the accuracy $\Delta X_{\rm SQL}$, the amplitude of oscillations A will be known with the same accuracy:

$$\Delta A=\Delta X_{\rm SQL}\geqslant\sqrt{\frac{\hbar}{2m\omega}}. \tag{5}$$

It follows that, for the energy of oscillations

$$\mathscr{E}=\frac{m\omega^2A^2}{2}+\hbar\omega/2=\hbar\omega(N+1/2) \tag{6}$$

with N quanta in the mode, the standard quantum limit is equal to

$$\Delta\mathscr{E}_{\rm SQL}=m\omega^2A\Delta A=\hbar\omega\sqrt{N}. \tag{7}$$

For an electromagnetic oscillator, the analog of formula (4) is the standard quantum limit for the electrical field stress

$$E_{\rm SQL}=\sqrt{\frac{\hbar\omega}{4\pi V}}, \tag{8}$$

where V is the effective volume occupied by the field. Analogous simple analysis gives the following value for the standard quantum limit for the coordinate of a free mass:

$$\Delta X_{\rm SQL}=\xi\sqrt{\frac{\hbar\tau}{2m}}, \tag{9}$$

where τ is the duration of measurement and ξ is the factor of the order of unity depending on the form of the signal. For example, for sinusoidal 2π pulse,

$$\Delta X_{\rm SQL}=\frac{1}{2\pi}\sqrt{\frac{\hbar\tau}{m}}. \tag{10}$$

Apparently, to overcome the standard quantum limit, the meter must extract information *only* on the single specified observable. The meter, designed in accordance with this principle, does not disturb the value to be measured, and the others (noncommuting with it) are disturbed precisely to the extent that provides satisfaction of the uncertainty principle. This type of measurement is called the QND measurement.

Main properties of the ideal QND measurement precisely reproduce the properties of the abstract quantum measurement determined by von Neumann's postulate of reduction (1932).

If the object is initially in an arbitrary state with the density operator $\hat\rho$ and if the value q is measured, then the QND measurement will yield one of the eigenvalues q of the operator $\hat q$ with probability $\langle q|\hat\rho|q\rangle$ (where $|q\rangle$ is the corresponding eigenstate). After the QND measurement, the object will be in the state $|q\rangle$. If a quantum object is in the state with a certain defined value of the measured observable, then the same value will be obtained as the result of measurement. After the measurement, the object will remain in the same state. The measurement may be repeated many times, each time giving the same result. (It is assumed here that the evolution of the measured value can be neglected: either it is small or the measured value is an integral of motion; see below.) That is why the ideal QND measurement is an exact one: the meter does not add any perturbation, and possible variance is the consequence of the *a priori* uncertainty of the value to be measured.

Some quantum observables may have their own additional uncertainties which will limit the accuracy of measurement, e.g., $\hbar/\tau$ for the energy or $1/N$ for the phase of oscillator. The problem of existence of such limitations is not yet solved.

The origin of the term "quantum nondemolition" translates from the intention to emphasize the following basic property: if, before a measurement, an object is not in one of the eigenstates of the measured value, the QND measurement destroys this state but does not demolish it. For example, if an oscillator is initially in the coherent quantum state, the QND measurement of energy will destroy this state and create N-state, although this measurement does not include demolition, as in classical photodetectors.

On the basis of the above consideration, one can formulate a general necessary and sufficient condition that the QND meter must satisfy (see Braginsky and Khalili, 1992):

$$[\hat{q},\hat{U}]|\psi\rangle=0. \tag{11}$$

Here, $\hat{q}$ is the operator of the value to be measured; $|\psi\rangle$ is the initial state of the quantum meter; and $\hat{U}$ is the operator of the joint evolution of the quantum meter and the object under study. Condition (11) is usually replaced by the simpler sufficient (not necessary) condition

$$[\hat{q},\hat{U}]=0 \tag{12}$$

(see details in Sec. IV).

To test implementation of conditions (11) and (12), one has to know the operator $\hat{U}$, i.e., to solve the problem of evolution of the coupled system "quantum meter + object under study." In most cases this is a rather complicated problem, and therefore usually another sufficient (not necessary) condition is used: the measured value should be an integral of motion for the coupled system. From the general form of a dynamic equation in a Heisenberg evolution approach, it can be shown that the latter condition is equivalent to the following equation:

$$i\hbar\frac{\partial\hat{q}}{\partial t}+[\hat{q},\hat{H}]=0, \tag{13}$$

where $\hat{H}$ is the Hamiltonian of the coupled system. If the operator of the measured value does not depend directly on time,

$$\frac{\partial\hat{q}}{\partial t}=0, \tag{14}$$

then condition (13) is reduced to the requirement that $\hat{q}$ commute with the Hamiltonian:

$$[\hat{q},\hat{H}]=0. \tag{15}$$

Apparently, this condition is more rigid than condition (12). If it is secured, the measurement is nonperturbing independently of the interaction time of the meter with the object; i.e., equality (12) turns to identity independent of the measurement time.

At about the same time that the term "QND measurement" was introduced, another term—"QND observable"—began to be used. The values of the operator of QND observable $\hat{q}$ commute with those taken at a different time:

$$[\hat{q}(t),\hat{q}(t')]=0, \tag{16}$$

in the Heisenberg picture of evolution. This feature permits continuous monitoring of such an observable with the error less than SQL, and, as a result of this procedure, it is possible to detect very weak external action on the probe quantum object.

For simplest objects, (1) free mass and (2) oscillator, QND observables are, correspondingly, (1) momentum and energy, and (2) quadrature amplitude, energy, and phase. The latter is an example of a QND observable which is not an integral of motion

$$\hat{\varphi}(t)=\hat{\varphi}+\omega t. \tag{17}$$

For the observables-integrals of motion, condition (13) can be simplified. The Hamiltonian $\hat{H}$ in most cases can be presented in the form of a sum,

$$\hat{H}=\hat{H}_0+\hat{H}_M+\hat{H}_I, \tag{18}$$

where $\hat{H}_0$ is the Hamiltonian of the object under study; $\hat{H}_M$ is the Hamiltonian of the meter; and $\hat{H}_I$ is the interaction Hamiltonian. If the measured variable q is an integral of motion for the object under study, then the following equality is valid:

$$i\hbar\frac{\partial\hat{q}}{\partial t}+[\hat{q},\hat{H}_0]=0. \tag{19}$$

From formula (19) and from the evident fact that

$$[\hat{q},\hat{H}_M]=0, \tag{20}$$

we further derive that, for integrals of motion, condition (13) is reduced to the commutation of the measured value with the interaction Hamiltonian:

$$[\hat{q},\hat{H}_I]=0. \tag{21}$$

As an example of QND meter, one can consider a slightly modified scheme of the ponderomotive meter of e.m. energy, proposed in Braginsky, Vorontsov, and Khalili (1977).

Let one wall in an e.m. resonator be flexible or movable (as a piston in a cylinder). By measuring the pressure of an e.m. field imposed on the movable wall, one can evidently calculate the energy. If the inertia of the wall is large enough, then the phase of e.m. oscillations will not influence its motion. The motion of the heavy wall is slow compared to the frequency of electromagnetic oscillations (i.e., it is adiabatic). It is known, on the other hand, that the number of quanta in a resonator does not change during its adiabatic deformation. The ponderomotive force acting on the resonator wall is equal to

$$F=\frac{\mathscr{E}}{d}, \tag{22}$$

where $\mathscr{E}$ is the energy in the resonator, and d is the value of the order of resonator dimensions, depending on the chosen e.m. mode. Under the action of this force, the momentum of the wall will change by the value

$$\delta p=\frac{\mathscr{E}\tau}{d}, \tag{23}$$

where τ is the duration of measurement. It is evident, therefore, that the better the initial momentum of the wall is defined, the higher will be the precision of measurement of the energy:

$$\Delta\mathscr{E}_{\rm meas}=\frac{d}{\tau}\Delta p, \tag{24}$$

where Δp is the initial uncertainty of the momentum.

On the other hand, in accordance with the uncertainty principle, the less Δp is, the greater will be the initial uncertainty of the wall coordinate Δx. The presence of the uncertainty of coordinate translates into the uncertainty of the resonator frequency $\Delta\omega$ during measurement time τ and therefore to the random-phase shift

$$\Delta\varphi_{\rm pert}=\Delta\omega\tau=\omega\tau\frac{\Delta x}{d}. \tag{25}$$

From relations (24) and (25), taking into account the inequality

$$\Delta x\Delta p\geqslant\hbar/2, \tag{26}$$

we obtain that

$$\Delta\mathscr{E}_{\rm meas}\Delta\varphi_{\rm pert}\geqslant\frac{\hbar\omega}{2}. \tag{27}$$

It is interesting to note that the interaction Hamiltonian in the considered scheme is proportional to the square of the generalized coordinate in the resonator and not to the energy; i.e., condition (21) is not fulfilled. However, nondiagonal matrix elements of the Hamiltonian (in presentation of the measured value) oscillate with the frequency 2ω. If the measurement time is large enough,

$$\omega\tau\gg 1, \tag{28}$$

then their contribution to the operator of evolution is small, and the more general condition (12) is secured. Thus the considered measurement is actually a nonperturbing one. A more detailed and rigorous analysis shows that the ultimate precision of measurement in the considered scheme depends substantially on the *a priori* information on the value of energy in the resonator. In particular, if $\Delta\mathscr{E}_{a\ priori}\simeq\mathscr{E}$, the minimal measurement error is equal to

$$\Delta\mathscr{E}_1=\hbar\omega\sqrt{N}\frac{1}{\sqrt{\omega\tau}}\ll\Delta\mathscr{E}_{\rm SQL}, \tag{29}$$

where N is the mean number of quanta. In the meantime, by repeating the measurement several times with the increasing precision of *a priori* information, one can obtain in the ultimate limit the following precision of measurement:

$$\Delta\mathscr{E}_{\rm QND}=\frac{\hbar}{\tau}\ll\Delta\mathscr{E}_1. \tag{30}$$

Derivation of formulas (29) and (30), as well as a more detailed analysis of different aspects of QND measurement theory, can be found in the monograph by Braginsky and Khalili (1992).

III. STATE OF THE ART IN QUANTUM NONDEMOLITION MEASUREMENTS

Before reviewing the realized QND measurement schemes, it is worth touching briefly on one rather important condition of their feasibility: the degree of isolation of the specified quantum object from the heat bath. The criterion of sufficient isolation can be easily obtained by comparing the random change of the chosen variable under the action of the heat bath (during chosen averaging time $\tau\ll\tau^*$, τ^* is the relaxation time) with the *a priori* defined accuracy of measurement. For example, if one has to achieve a resolution slightly better than SQL for a free mass or oscillator in a heat bath with high temperature T, then the following inequalities must be fulfilled:

$$\frac{2kT\tau^2}{\tau^*}\leqslant\hbar, \tag{31}$$

or

$$\frac{2kT\tau}{Q}\leqslant\hbar, \tag{32}$$

where k is the Boltzmann's constant, and Q is the quality factor of the oscillator (Braginsky, 1967).

It is obvious that inequalities (31) and (32) are, in fact, the conditions of quantum behavior of the chosen objects. If, further, one has to measure, for example, the energy of the oscillator with the accuracy of one quantum, then the condition will be more rigid:

$$\frac{kT\tau(2N+1)}{Q}\leqslant\hbar, \tag{33}$$

where N is the initial (premeasurement) number of quanta in the oscillator.

It is worth noting that the above conditions have long been familiar to physicists involved in the development of gravitational-wave antennas. [Condition (32) was obtained as early as 1967.] Relatively recently, they were derived anew on the basis of a rigorous analysis of the decoherence process of a quantum object in a heat bath (see Caldeira and Leggett, 1983; Zurek, Habib, and Paz, 1993; Zurek, 1993, and references therein).

When the value of T is small enough so that $kT\ll\hbar\omega$ for an oscillator or $kT\ll\hbar/\tau$ for a free mass, conditions (31)–(33) must be replaced by a single, more general relation:

$$\Delta_{\rm QND}\simeq\Delta_{\rm SQL}\sqrt{\frac{\tau}{\tau*}}, \tag{34}$$

where $\Delta_{\rm QND}$ is the measurement error of the chosen QND observable; $\Delta_{\rm SQL}$ is the corresponding standard quantum limit for this observable; and τ^* is the relaxation time of the object. As the above consideration shows, values τ^* and Q determine the possibility of realization of QND measurement and the value of measurement error. The history of experimental physics demonstrates the following clear-cut trend: experimentalists invent new technologies or modify old ones to increase the values τ^* and Q, and then it translates in the improvement of resolution in experiments. For example, 20 years ago the finesse of the best optical mirrors was at the level of $F\simeq 10^3$; recently, Kimble and his colleagues demonstrated $F\simeq 2\times 10^6$ (Rempe, Thompson, and Kimble, 1992). (The quality factor of optical resonator $Q=2FL/\lambda$, where L is the mirror separation and λ is the wavelength.) If mirrors with such a finesse are used in Fabry-Pérot resonators of the advanced versions of laser gravitational antennas (projects LIGO-VIRGO-GEO; see, for example, Abramovichi *et al.*, 1992), then their quality factor will exceed $Q\simeq 10^{16}$, and the relaxation time $\tau^*\simeq 10$ s. Relatively recently, in the optical range of frequencies, the so-called whispering-gallery microresonators were demonstrated, possessing a quality factor up to $Q\simeq 3\times 10^9$ in combination with a very small volume of e.m. field localization (Braginsky, Gorodetsky, and Ilchenko, 1989, 1993; Collot *et al.*, 1993), attractive from the point of view of cavity QED experiments and QND measurements (see below). In the microwave band, the quality factors in excess of $Q\geqslant 10^9$ were demonstrated in dielectric resonators of single-crystal sapphire at $T>4$ K (Braginsky, Ilchenko, and Bagdasarov, 1987; Luiten, Mann, and Blair, 1993). It is important to mention that with a further reduction of impurity concentration in sapphire, one can expect to reach $Q_e\simeq 10^{15}$ at 4 K in resonators of this type—the level defined by weak low-temperature lattice absorption, which corresponds to relaxation time $\tau_e^*\simeq 10^4$ s.

In the low-frequency mechanical domain, pendulum suspension of high-purity fused silica with $\tau_m^*\geqslant 4.4\times 10^7$ s at $T=300$ K has been demonstrated (Braginsky, Mitrofanov, and Okhrimenko, 1993). With this value of τ_m^*, condition (31) is fulfilled for $\tau\leqslant 5\times 10^{-4}$ s. The experimental know-how in the development of these suspensions, and the established empirical rules, allow one to expect the achievement of $\tau_m^*\simeq 10^{12}$–10^{13} s (Braginsky, Mitrofanov, and Vyatchanin, 1994).

It is important to emphasize that the demonstrated values of Q and τ^* to date, and the projected improvement expectations, are based on concrete dissipation mechanisms for any given particular case. These concrete mechanisms may, in principle, be either strongly suppressed or removed completely. At present, only a single, fundamental (principally nonremovable) limit for dissipation has been found (Braginsky and Khalili, 1991), which is associated with zero-point fluctuations. This limit is many orders of magnitude smaller than the most optimistic projections for existent systems. Therefore there exists a substantial potential reserve to increase the values of Q and τ^*, and its practical implementation depends only on the ingenuity and devotion of experimentalists.

Let us now consider the achievements in concrete schemes of QND measurements obtained during recent years.

After a gedanken experiment scheme based on the ponderomotive effect (Braginsky, Vorontsov, and Khalili, 1977; Sec. II), a more realistic QND energy measurement procedure was proposed on the basis of cubic nonlinearity $\chi^{(3)}$ (Braginsky and Vyatchanin, 1981). In the simplest variant of this scheme, two e.m. resonators have an overlapping volume filled with a nonlinear dielectric. In the presence of energy $\mathscr{E}_1$ in the first resonator (signal mode), the phase of oscillations φ_2 in the second resonator (probe mode) during the time τ will be shifted by the value

$$\delta\varphi_2\simeq\frac{12\pi^2\chi^{(3)}\mathscr{E}_1\omega_2\tau}{V}, \tag{35}$$

where V is of the order of the resonator volume, and ω_2 is the frequency of the second resonator. It is evident that high sensitivity of measurement can be obtained only under condition $\tau\ll Q_1/\omega_1$ (Q_1 is the quality factor of the first resonator with frequency ω_1). Therefore parameter $\chi^{(3)}Q/V$ is crucial in this scheme. This rule is general for all schemes of QND energy measurements: one has to combine high reactive nonlinearity with a low level of dissipation.

It is evident that this principle of QND energy measurement can also be used for measurement of the energy of e.m. wave packets in a nonlinear waveguide. If two wave packets are propagating in the waveguide—the signal one with mean frequency ω_1 and the probe one with frequency ω_2—and if during time τ these packets overlap, then, because of the nonlinearity $\chi^{(3)}$ of the waveguide dielectric, the phase shift of the probe packet will be proportional to the energy in the signal wave $\mathscr{E}_1$, interaction time τ, and to the value of $\chi^{(3)}$. In this case, too, the low dissipation requirement is tough: it is necessary that $\alpha c\tau/n\ll 1$ (where α is the attenuation in the waveguide, and c/n is the speed of e.m. wave propagation). The $\chi^{(3)}$ employing scheme was realized by several groups. First, Levenson and his colleagues (1986) reported that, according to their measurements in commercially available silica fibers at 2 K, nonlinearity and the level of fluctuations do not prevent realization of QND measurements with the error $\Delta\mathscr{E}_1$ smaller than $\hbar\omega_1\sqrt{N}$ (see also Imoto *et al.*, 1985). Later, by measuring the interaction of two solitons in a silica waveguide (at $T=300$ K), Friberg, Machida, and Yamamoto (1992) reported that they managed to obtain a resolution better than SQL for QND registration of spectral components of energy fluctuations (each soliton in this experiment contained $N \simeq 10^8$ photons, with 10^8 solitons per second). In 1992 Grangier and his colleagues reported that they managed to cross the level of SQL in QND energy measurement using resonant optical nonlinearity in sodium vapor, in dispersive limit. [More detailed descriptions of these experiments, besides the original papers, can be found in the review articles (Roch *et al.*, 1992).] So far, the best resolution in the described types of experiments, to the authors' knowledge, has been obtained by Kimble and his colleagues (Pereira, Ou, and Kimble, 1994) and by Poizat and Grangier (1993).

It is worth mentioning another type of QND measurement of quadrature amplitude, proposed by Shelby and his colleagues (1987). Later, LaPorta and his colleagues (1989) reported on the first experimental demonstration of this type of measurement.

Almost all experimental achievements in the field of QND measurements of optical field can be attributed to the "linear" area, according to Grangier's classification (Roch *et al.*, 1992), when the amplitude of oscillations of the signal wave is much larger than the quantum uncertainties of its quadrature amplitudes. In this field, measurement of the number of quanta is fully equivalent to the measurement of quadrature amplitude synchronous to the mean value of coordinate, with the error

$$\Delta X_1 = \frac{\Delta N}{\sqrt{N}} X_{\mathrm{SQL}}, \tag{36}$$

where X_{SQL} is the measurement error corresponding to SQL. The presently achieved values of the measurement error lie in the range $\Delta N < \sqrt{N}$, $\Delta X < X_{\mathrm{SQL}}$, but $\Delta N \gg 1$.

All experiments realized so far can be considered only as demonstrations. In other words, realized schemes of QND measurements are only "toys" in the hands of experimentalists, and not instruments that can be used for the solution of concrete experimental problems.

IV. PROSPECTS AND UNSOLVED PROBLEMS OF QUANTUM NONDEMOLITION MEASUREMENTS

The achieved level of sensitivity in the above-described experiments prompted the invention of several new schemes promising a substantial further improvement of resolution.

The principle of the QND measurement scheme employing deflection of atomic or molecular beams can be explained by the following simple example (Braginsky and Vyatchanin, 1988). If an electron moves during a certain time in the nonhomogeneous evanescent field of a dielectric waveguide with removed cladding, parallel to the waveguide axis, it will be subject to a repulsive force perpendicular to the axis and proportional to the square of electrical field stress E:

$$F_\perp \simeq \frac{e^2 E^2}{2m\omega_e^2 D}, \tag{37}$$

where e and m are electron charge and mass; D is a characteristic value describing the scale of the evanescent field; and ω_e is the frequency of electron oscillations in the field of the e.m. wave. This latter value can be much smaller than the actual frequency of the wave, if the velocity of the electron is close to the velocity of the wave. Calculations have shown (Vyatchanin, 1990) that only a few electrons suffice for the detection of a monophotonic state (a single well-localized photon in the waveguide) by detecting transverse momentum $F_\perp \tau$ on the background of diffractional uncertainty $\hbar/2D$. The necessity of using a specially profiled waveguide with close phase and group velocities is a disadvantage of this scheme.

The logical continuation of this scheme was the proposal to use a quadratic deflection of atoms in a nonhomogeneous electromagnetic field of a resonator (Herkommer *et al.*, 1992; Averbukh *et al.*, 1994). For example, the idea of using the evanescent field of optical whispering-gallery microresonators looks very attractive (see Sec. III). A single photon in a mode of such a resonator will cause a substantial deflection of atomic beam because of ponderomotive atom-field interaction, if the frequency of atomic transition is close to the resonator mode frequency. (The sign of the deflection angle will depend on the sign of detuning.) Independent calculations performed by three research groups (Collot *et al.*, 1993; Matsko *et al.*, 1994) have shown that this scheme allows one to obtain the following resolution in energy measurement:

$$\Delta \mathscr{E} \simeq 0.1 \hbar \omega_{\mathrm{optic}}. \tag{38}$$

The above-described deflection schemes of QND energy measurement refer to the optical range of frequencies. The analogous principle can be used in the microwave band, with the addition of another effect predicted and discovered more than 40 years ago by Kopfermann and his colleagues (Friedburg 1951). This effect—selective deflection of atoms or molecules in different quantum states by static inhomogeneous electrical or magnetic field—was successfully used by Townes, Basov, Prokhorov, and Ramsey during the creation of ammonia and hydrogen masers. The scheme of QND measurement of the energy of a single photon using Kopfermann's effect consists of three stages: (a) an atom (or molecule), with a transition frequency close to that of the resonator mode, passes through part of the resonator volume during one half-period of Rabi frequency and absorbs one field photon from the resonator; (b) after that, the atom (molecule) passes the area of nonhomogeneous electrical or magnetic field, where it obtains a transverse momentum depending on its quantum state; (c) finally, during the pass through another part of the resonator volume, again equal to one half-period of Rabi frequency, the atom (molecule) "gives back" the photon to the resonator mode. It is obvious that the acquired transverse momentum leads to a deflection of the particle and can be registered as the readout of measurement. Calculations have shown that with Rydberg atoms as probe particles, this method can be used for QND detection of single microwave photons in sapphire disk resonators. Since the transit time through all three stages is approximately 2×10^{-5} s, the relaxation time of sapphire resonators $\tau_e^*\simeq3\times10^{-3}$ s (corresponding to the quality factor 10^9) gives enough room for repetitive measurements (Braginsky and Khalili, 1994).

It is worth noting that this type of measurement satisfies neither the simple condition of QND (21) nor the more general condition (12), for the complete set of quantum states of an e.m. field. However, it satisfies condition (12) for the subset consisting of ground and one-photon states. Therefore it can be regarded as *QND detection* of single photons.

A similar principle of QND measurement of the energy of microwave quanta in a cavity was proposed by Haroche and his colleagues (Brune *et al.*, 1990). The basic difference in their elegant scheme is the employment of the quadratic Stark effect instead of Kopfermann's and a Ramsey-separated oscillatory fields technique instead of a deflection of atoms. Preliminary testing of this scheme allowed them to register the Stark shifts of Rydberg-atom energy levels by zero-point oscillations in a mode of microwave cavity with high accuracy (Brune *et al.*, 1994). Historically, the first achievement in this area was the experiment by Walther and his colleagues (see the review article by Walther, 1992), where by they demonstrated that during nonlinear interaction of a solitary Rydberg atom with a high-Q superconducting cavity (the transit time of an atom in the cavity is of the order of one period of single-photon Rabi frequency), the values of nonlinearity and quality factor are sufficient for the realization of QND energy measurement with a small number of microwave photons.

The above-described schemes of QND energy measurement in optical and microwave bands present, in the opinion of the authors of this review, a variety of proposed (to the present moment) realistic methods, capable of becoming new instruments in experimental physics. Outlined is a certain sequence of stages (steps) characterizing the level of achievements in this field: (1) presently realized demonstration-type experiments with the resolution $\Delta\mathscr{E}\leqslant\hbar\omega_e\sqrt{N}$, with $N\gg1$; (2) the above-described new schemes, which promise resolution $\Delta\mathscr{E}\leqslant\hbar\omega_e$; (3) apparently the last stage in the development of QND energy measurement, which would allow $\Delta\mathscr{E}\ll\hbar\omega_e$ down to $\Delta\mathscr{E}\simeq\hbar/\tau$.

Let us note that the principal possibility of having the measurement error smaller than $\hbar/\tau$ has for a long time been a subject of discussion. Vorontsov (1981) proposed a scheme of gedanken experiment on the measurement of energy in an e.m. resonator, which was free of above the limitation. It is required, however, in this scheme, that meter provide perturbation of phase

$$\Delta\varphi_{\rm pert}\geqslant\frac{\hbar\omega}{2\Delta\mathscr{E}}\geqslant\omega\tau; \tag{39}$$

i.e., the uncertainty of the frequency during measurement should exceed the unperturbed value of the frequency before the measurement

$$\Delta\omega_{\rm pert}=\frac{\Delta\varphi}{\tau}\geqslant\omega, \tag{40}$$

and so it is not clear whether such a procedure can be realized experimentally.

In experiments with mechanical objects, the number of achievements concerning QND measurements is much smaller. Although the scheme of measurement of quadrature amplitude was analyzed in detail relatively long ago (see, for example, Braginsky, Vorontsov, and Thorne, 1980), it is not yet realized. Cinquegrana *et al.* (1993) report, however, on the results of the development and testing of a gravitational bar-antenna with a QND meter of quadrature amplitude. As for another QND observable—the energy of a mechanical oscillator—no experimental scheme has been proposed so far.

The proposed schemes for the measurement of speed, which is a QND observable for a free mass (Braginsky and Khalili, 1988, 1990), in the authors' opinion, present only ideas of gedanken experiments but no basis for experimentally realizable techniques. A basic idea of these schemes is that a flux of e.m. energy prepared in a special noncoherent state, upon reflection from a free mass, may bring out little information on its coordinate and much more information about its speed.

A special problem in QND measurements is that of the measurement of the phase of oscillator (or e.m. resonator) if the latter is in the phase-squeezed state. This problem is considered in detail in Sec. V of this review. To conclude this section, let us touch on several unsolved problems in the theory of QND measurements, including the definition of terms.

(1) In recent years, active discussion has revolved around this question which has methodological and fundamental importance: where, under the conditions of finite measurement accuracy, finite perturbation of the variable to be measured, and finite dissipation, lies the border between QND and non-QND measurements? For energy measurements, at least for the case $N\gg1$, a clear classification was proposed by Grangier and his colleagues (1992) and by Imoto and his colleagues (1989). In general, however, the problem is not yet solved. It can be shown, for example, that the statistics of the results of QND energy measurements in an oscillator with dissipation, under the condition $\Delta N\ll1$, is rather nontrivial and drastically different from what can be obtained from simple semiclassical consideration.

In the authors' opinion, a logical approach should be based on the following simple criterion: the measurement is approximately nonperturbative as long as it provides (possibly, being repeated several times) both a measurement error and the perturbation of the measured value smaller than SQL.

(2) Application of QND methods will undoubtedly improve the sensitivity of laser gravitational-wave antennas. Potential improvement of sensitivity here is based on the substantial reserve, because with achievable quality of optical mirrors $\tau/\tau^*\simeq10^{-4}$ in large-scale laser antennas. It is not yet clear, however, what should be the quantum state of e.m. field in the antenna, how it should be prepared, and what will be the registration procedure. It is obvious that there exists a much wider

spectrum of possibilities of solving this problem besides the "one-dimensional" set of "phase-squeezed–amplitude-squeezed" states.

(3) There exists a fundamental problem, not connected directly to the theory of QND measurements but playing the key role for their experimental realization: is there an intrinsic correlation between dissipation and nonlinearity in different physical systems? [A particular example of such an interrelation is given by fundamental losses in high-quality dielectric crystals (see Gurevich, 1981).] Almost 20 years ago, one of the authors (VBB) addressed this question to Richard Feynman. His answer was negative, although he had no rigorous proof. This proof has not yet been discovered.

(4) Most of the existing procedures of QND measurements, both practically realized and discussed in the literature, belong only to a certain subclass of the full set of all QND measurements. Indeed, fulfillment of the necessary and sufficient condition (11) can be obtained either by following the traditional criterion (12) or by choosing the initial state of the meter in such a way so that it will be an eigenstate of the operator $[\hat{q},\hat{U}]$.

A second way is totally nonelaborated, despite the fact that it contains some interesting possibilities. For example, there is no principal restriction for the existence of a measurement scheme allowing nonperturbative measurement of different noncommuting observables by varying only the initial state of quantum meter, without changing its physical structure and the character of its interaction with the quantum object.

V. QUANTUM PHASE MEASUREMENT

The theory of the oscillator phase has for a long time been, and partially still remains, an uncompleted part of quantum mechanics. The reason for this lies in the absence of a "good" phase operator defined on the entire space of oscillator states. An analysis of the mathematical aspects of this problem can be found in the paper by Carruthers and Nieto (1968). In recent years, considerable progress has been achieved in this field. The results of the latest studies on the subject may be found in the collection of articles edited by Schleich and Barnett (1993), and in the article by Kulaga and Khalili (1993).

The unsolved state of the quantum phase problem is also illustrated by the fact that, until now, no procedure has been proposed for "true phase" QND measurement, which would reduce the wave function of an oscillator into the state with a given phase without extracting any information about energy.

Present methods of phase measurement are based on the following simple principle. If the mean amplitude of oscillator A is much larger than the uncertainties of its quadrature amplitudes,

$$\Delta X_{1,2} \ll A, \tag{41}$$

then the measurement of phase is reduced to the measurement of quadrature amplitude X_2 (shifted by $\pi/2$ with respect to the oscillations of the mean coordinate):

$$\varphi = X_2/A. \tag{42}$$

One can easily show that the measurement of X_2 with an accuracy at the level of SQL corresponds to the measurement of phase with the accuracy $1/2\sqrt{N}$, where N is the mean number of quanta. Further increasing the accuracy of the X_2 measurement, one can improve the accuracy of the phase measurement, but only up to the limit providing the fulfillment of condition

$$(\Delta X_1)_{\text{pert}} = \frac{\hbar}{2m\omega(\Delta X_2)_{\text{meas}}} \ll A, \tag{43}$$

which translates from relation (41). It follows from inequality (43) that, in the considered method, always

$$\Delta\varphi \gg 1/N. \tag{44}$$

A substantial increase in the accuracy of phase measurement and realization of QND phase measurement promises a qualitative breakthrough in different areas of physics, e.g., in ultrahigh resolution spectroscopy and, possibly, in laser gravitational-wave antennas. At the same time, to the authors' knowledge, only a few publications in recent years were devoted to the elaboration of new, at least *gedanken*, phase measurement procedures fundamentally different from the above-described algorithm. Among these few articles is the work by Noh, Fougers, and Mandel (1993) proposing a method of destructive measurement in a traveling wave (not in a resonator), which allows one to extract, under some limitations, the values of the sine and cosine operators for the phase of the wave (see also Yuen *et al.*, 1980). In Braginsky and Khalili (1993), it is shown that a special sequence of coordinate measurements, with certain *a priori* information on the initial state of the oscillator, may allow one to closely approach in phase measurement the accuracy of $\Delta\varphi \simeq 1/N$.

Given below is a semiqualitative description of the principle of a new scheme of measurement, which probably allows one to come close to solving the problem of direct measurement of a quantum oscillator phase. Let us assume that a mechanical oscillator is initially in a coherent state with amplitude A and, correspondingly, with uncertainties

$$\Delta X_{\text{SQL}} = \sqrt{\frac{\hbar}{2m\omega_m}} \tag{45}$$

for coordinate and

$$\Delta\varphi_{\text{SQL}} = \frac{\Delta X_{\text{SQL}}}{A} = \frac{1}{2\sqrt{N}} \tag{46}$$

for phase. Let us also assume that we have at our disposal a coordinate meter with a very strong nonlinear response, detecting the position of mass m only if its center-of-mass coordinate $x(t)$ falls into the interval

$$-\xi\Delta X_{\text{SQL}} \leq X(t) \leq \xi\Delta X_{\text{SQL}}, \tag{47}$$

where $\xi \ll 1$. Outside this interval, the response of the meter is zero. It is evident that during the time interval equal to one half-period of oscillations after switching on such a null detector, the experimenter will observe a single response pulse with duration

$$\Delta\tau = \frac{\xi \Delta X_{\rm SQL}}{\omega_m A} \ll \frac{2\pi \Delta \varphi_{\rm SQL}}{\omega_m}, \tag{48}$$

and the mass of the oscillator, as a result of measurement, will acquire a momentum with uncertainty

$$\Delta P = \frac{\hbar}{2\xi \Delta X_{\rm SQL}}. \tag{49}$$

This uncertainty of momentum will increase the uncertainty of the amplitude of oscillations up to the value

$$\Delta A_{\rm pert} \simeq \frac{\Delta X_{\rm SQL}}{\xi}, \tag{50}$$

and the oscillator will transit to the state with squeezed phase:

$$\Delta\varphi \simeq \xi \Delta \varphi_{\rm SQL} \tag{51}$$

with simultaneous fulfillment of the relation

$$\Delta\varphi m \omega_m^2 A \Delta A_{\rm pert} = \Delta\varphi \Delta \mathscr{E}_{\rm pert} \geqslant \frac{\hbar\omega}{2}. \tag{52}$$

Apparently, after approximately one half-period, the response pulse will be repeated. It is obvious that such a measurement can be repeated many times, if only $\xi \geqslant 1\sqrt{N}$. Therefore the null detector reduces the phase measurement to the registration, with high accuracy, of the time moment when the center of mass passes the origin of coordinate axis (equilibrium point of the oscillator).

The above-described procedure is not "purely" a gedanken experiment. The reader may imagine a mass $m = 10^{-9}$ g put into a potential well, providing $\omega_m = 10^3$ s^{-1}. Such an oscillator has $\Delta X_{\rm SQL} \simeq 2\times 10^{-11}$ cm. If we then assume that this mass is a small mirror forming a Fabry-Pérot resonator together with another (immobile) mirror, then such a resonator will be transparent for the flux of photons of specially chosen stabilized wavelength λ only in small interval of moving mirror positions:

$$\Delta X = \frac{\lambda(1-R)}{\pi} = 10^{-12} \quad {\rm cm} \times \left(\frac{\lambda}{6\times 10^{-5}\ {\rm cm}}\right) \times \left(\frac{1-R}{5\times 10^{-8}}\right). \tag{53}$$

Therefore, placing the immobile mirror at a distance equal to the integer number of half-wavelengths of the pump oscillator from the equilibrium point of mobile mass with the accuracy $\Delta X = 10^{-12}$ cm, one obtains the squeezing factor $\xi = 5\times 10^{-2}$ for the phase of the mechanical oscillator, if the mirror quality parameter can be improved until $(1-R) = 5\times 10^{-8}$ (20 times better than in the best existing mirrors).

In the authors' view, the principle of the null detector can be realized with microwave e.m. resonators (see details in Braginsky, Khalili, and Kulaga, 1995). The authors, however, have not succeeded in finding a realistic scheme of null detector for optical e.m. resonators.

VI. CONCLUSION

A realization of QND measurement techniques allowing multiple repetitions and having resolution much better than SQL will certainly mark a milestone in the art of experiment. An even more significant accomplishment would be the successful solution of a fundamental problem (or problems) with the help of QND methods. In our opinion, there is a variety of problems that can be solved only by using qualitatively new methods of quantum measurements. Let us consider two examples.

(a) In the first generation of laser interferometric gravitational-wave antennas, the projected sensitivity in units of the variation of the metric has to be $h \simeq 4\times 10^{-21}$ (Abramovichi *et al.*, 1992). This value is approximately one and a half orders of magnitude larger than SQL. In the second (or third) generation, the resolvable value of h should be close to or smaller than SQL. One can expect that the achievement of this level of sensitivity will allow one to resolve the wave-form features of the bursts of gravitational waves from black-hole coalescence events. This, in turn, may facilitate the test of general relativity in the ultrarelativistic limit, when the ratio of gravitational potential over c^2 is close to unity.

First attempts to propose QND-type measurement procedures in laser gravitational antennas have already been made (Braginsky and Khalili, 1990; Jackel and Reynaud, 1990; Vyatchanin *et al.*, 1994; the schemes proposed in the last two papers can be considered examples of QND measurement of the spectral component of a coordinate). In the authors' opinion, the schemes presented, however, should be considered as gedanken experiments and not as ready variants of engineering solutions.

(b) More than 30 years ago, J.A.Wheeler mentioned possible topological multiple connectivity of space, i.e., the existence of two or more independent shortest intervals between two points. In describing this property of space, the term "wormholes" is usually used (see, for example, Misner, Thorne, and Wheeler, 1973). About 30 years later, Hawking (1988) proposed the hypothesis that these wormholes were formed by Planck-scale fluctuations: virtually existing micro black holes could serve as fluctuating bridges connecting this universe with others. The basic property of a black hole—to absorb information and therefore to act as classical observer—in this model should lead to the phenomenon of decoherence of all possible micro- and macro-objects. A number of papers devoted to the evaluation of decoherence time (Coleman, 1988; Lavrelaschvili *et al.*, 1988; Ellis *et al.*, 1989; Jackel and Reynaud, 1994) demonstrate a surprising variety of predictions (by many orders of magnitude). This ambiguity is in essence a reflection of the fact that, for now, no satisfactory theory for the quantization of a gravitational field has been proposed. On the other hand, observation of this new, possibly existing phenomenon is a challenge for experimentalists. One of the possible schemes of the experiment is obvious: one has to isolate a single object from the heat bath in the best possible way, and then, using the QND technique, to monitor the evolution of a specified QND observable (for example, speed of free mass). If, during the experiment, the recorded variation of the observable exceeds the expected value predicted by the residual effect of the heat bath and nonideality of the QND meter, then the experimentalist will have to conclude on the presence of the effect and appearance of "a new essence."

These two examples should not be regarded as the only two possible areas of fundamental application of QND measurement techniques. The principles underlying QND techniques may be useful for the creation of new generations of communication devices (Caves and Drummond, 1994) and for the development of a quantum computer.

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