Experimental study of critical exponents of electrical conductivity in a two-dimensional continuum percolation system

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In this paper an experimental study is presented for critical exponents of electrical conductivity in an inverse Swiss-cheese model. Filled circles are drawn on random positions of square paper in drawing ink with an X-Y plotter, and electrical resistance between both opposite sides is measured automatically by the use of general purpose interface bus system. Electrical conductivity is obtained from the inverse of the electrical resistance. Electrical conductivity in a bond process is also measured with the same system. It is confirmed that the critical exponent of electrical conductivity of a continuum two-dimensional inverse Swiss-cheese model is different from that of a discrete one.

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I. INTRODUCTION

The percolation problem [1–5] is an important model that indicates critical phenomena and retains various applicabilities that spread over various fields, such as disease propagation in orchards, oil recovery in oil wells, random networks in cities, and so on. Critical exponents of discrete percolation models on a lattice have been almost clarified, but those of continuum models are not clarified well. Halperin *et al.* [6] predicted that some of the critical exponents in the continuum percolation models might be different from those in the conventional discrete percolation models. We carried out a numerical simulation on a Swiss-cheese model [7] and indicated that the value of the critical exponent ν for the correlation length of a two-dimensional continuum percolation problem is different from the value of a discrete model. The Swiss-cheese model is a percolation model with randomly placed uniform spherical voids. The next problem is to identify the reason for the theoretical difference, but it seems to be difficult to do that. Therefore, it is the main purpose of this paper to confirm the difference of the critical exponents between the continuum model and the discrete one with the experiment, although the reason for the theoretical difference between the discrete model and the continuum one is not clear.

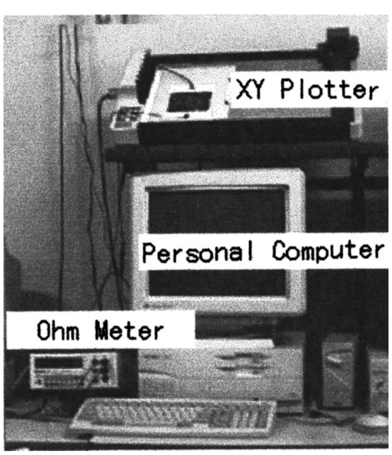

FIG. 1. Experimental apparatus.

In this paper, experiments on both continuum and discrete inverse Swiss-cheese models are carried out based on the single set of an experimental apparatus. As a result, it is confirmed that the critical exponent of electrical conductivity of a continuum inverse Swiss-cheese model is different from that of a discrete system, which is obtained within the same experimental precision.

II. EXPERIMENTAL APPARATUS

In the case of an experiment of a Swiss-cheese model (SCM), holes are punched randomly in a sheet of conductive material [8]. On the other hand, an inverse Swiss-cheese model (ISCM) can be obtained by putting conductive material on a nonconductive sheet. We take notice of the fact that

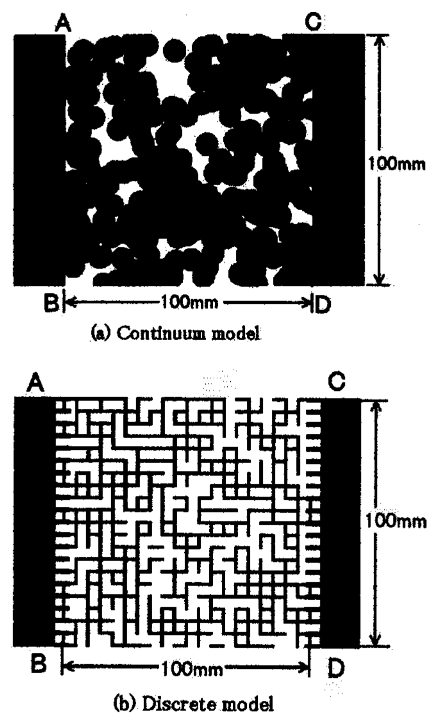

FIG. 2. Experimental samples. (a) Continuum model; (b) discrete model.

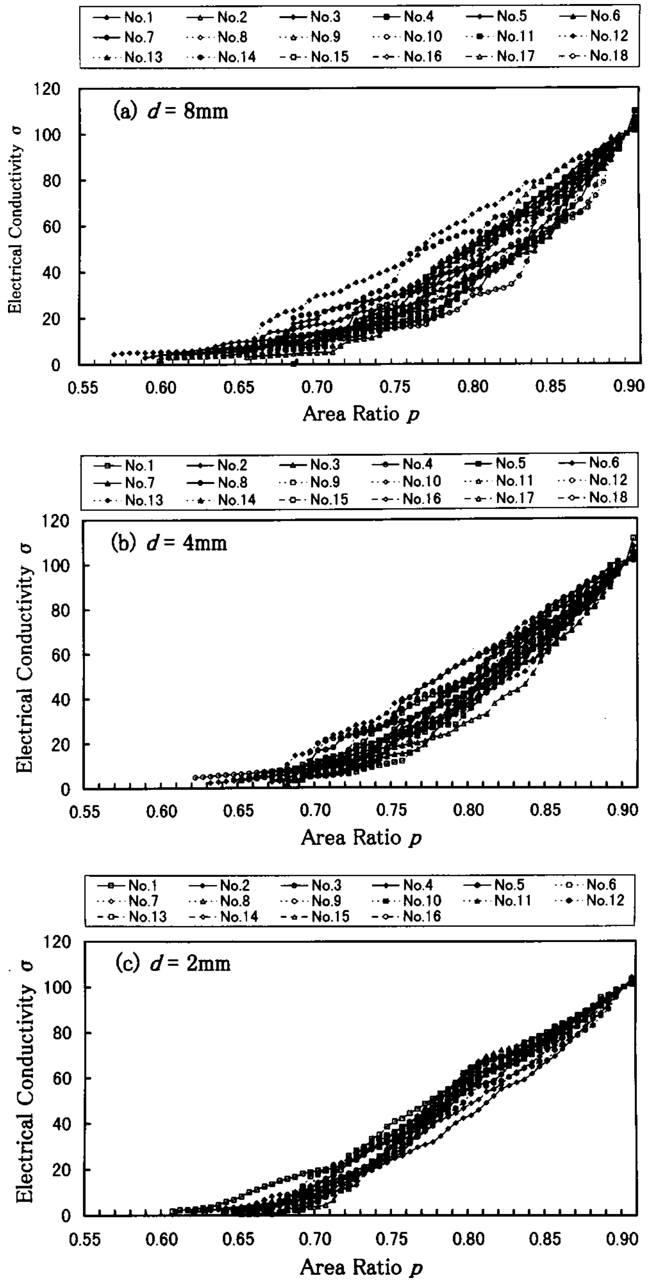

FIG. 3. The variation of the electrical conductivity of continuum percolation models σ with the area ratio p. (a) $d=8$ mm; (b) $d=4$ mm; (c) $d=2$ mm.

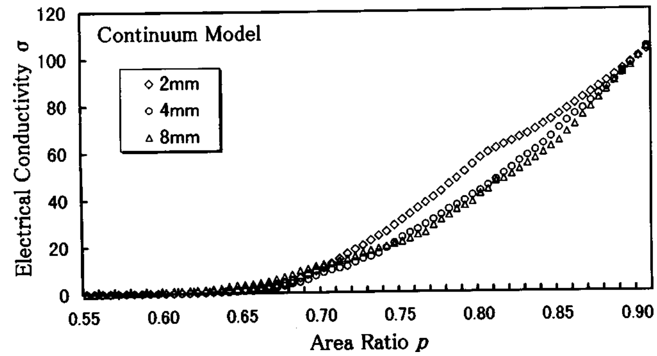

FIG. 4. The variation of the electrical conductivity σ of continuum percolation models with the area ratio p. The average values of σ over 20 experimental trials for $d=2$, 4, and 8 mm are depicted.

most drawing ink transmits electricity, and that an experiment of ISCM can be carried out by drawing filled circles on tracing paper with an X-Y plotter.

Figure 1 shows an experimental apparatus which is made up of a personal computer, an X-Y plotter, and an ohm meter. Sample patterns of continuum and discrete models are shown in Figs. 2(a) and 2(b), respectively. Filled circles and bonds are drawn in a rectangular plane $ABCD$. The filled rectangles at the both ends are the terminals for measurement. The procedure of measurement for a continuum model is as follows: (1) A seed of random numbers is determined. (2) Coordinates of the center of a circle are determined by random numbers. (3) A filled circle in drawing ink is drawn on white tracing paper with the X-Y plotter. (4) The same figure is also shown on the graphic display of the computer, and the area ratio p, which is the ratio of the black parts to the plane $ABCD$, is calculated. (5) Electrical resistance R between both opposite sides AB and CD is measured by the use of the ohm meter, which is connected to the computer with GP-IB (general purpose interface bus) system. The measurement is repeated ten times during 0.5 seconds, and the average value is obtained as R. (6) The area ratio p and the electrical resistance R are written in a file of the computer. (7) The steps from (2) to (6) are repeated until the plane $ABCD$ is covered with filled circles.

In the case of a bond model, location of an occupied bond is determined by a random number, and the occupied ratio p, which is the ratio of the number of occupied bonds to that of the whole bonds, is calculated by the computer.

III. EXPERIMENTAL RESULTS

Figure 3 shows the variations of the electrical conductivity σ with the area ratio p for continuum percolation models. Electrical conductivity is calculated from the inverse of the electrical resistance R. Owing to the use of drawing ink, the temperature and the humidity in the laboratory affect measured values of the electrical resistance a little, and values of the electrical conductivity are different from each other even if the values of the seeds are the same. Therefore we normalized the value of the electrical conductivity. In the case of a continuum model, it takes too much time until the plane $ABCD$ is covered with small filled circles. We ceased the measurement after $p=0.905$, and selected the value of σ for $p=0.9$ as a standard of normalization. The value of electrical resistance R for $p=0.9$ was several MΩ.

Figures 3(a), 3(b), and 3(c) show the results of about 20 samples for the diameter of filled circles $d=8$ mm, d

TABLE I. The average value and the standard deviation of critical exponent μ and the threshold value p_c for the continuum percolation models are given.

	Continuum model		
d	Critical exponent μ	Threshold value p_c	Sample
2 mm	1.76 ± 0.09	0.63 ± 0.03	16
4 mm	1.78 ± 0.04	0.63 ± 0.04	18
8 mm	1.80 ± 0.05	0.55 ± 0.05	18

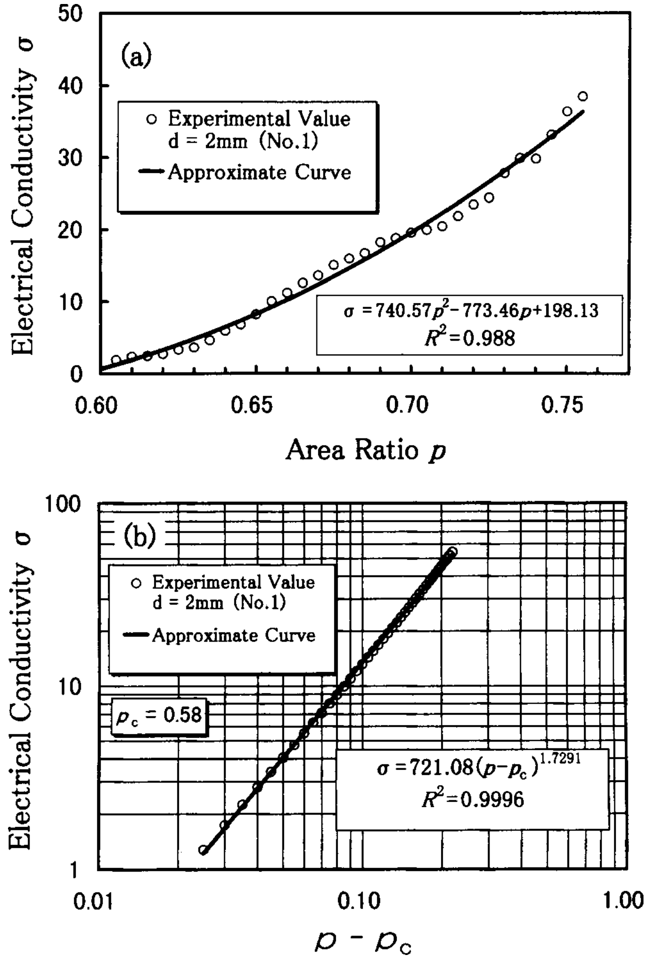

FIG. 5. An example of curve fitting of experimental data and a log-log plot diagram. The vertical axis is the conductivity and the horizontal one is p-p_c in the logarithmic scale.

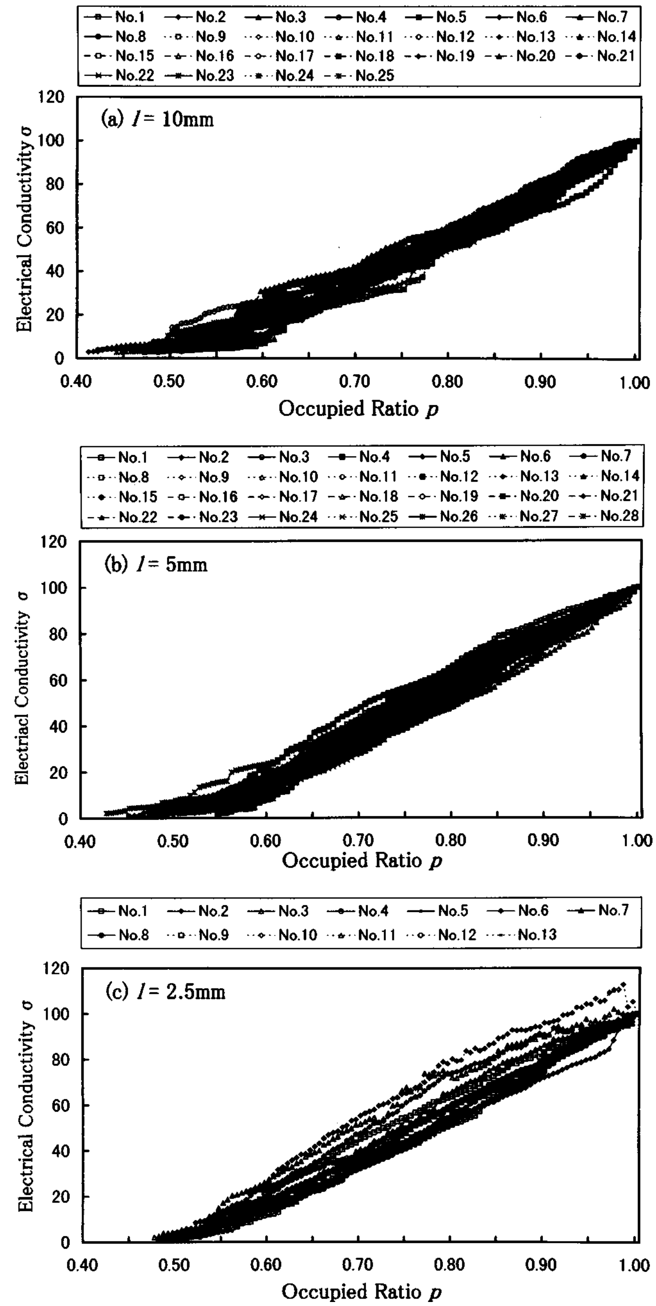

FIG. 6. The variation of the electrical conductivity of bond percolation models σ with the occupied ratio p. (a) $l=10$ mm (a 10×10 square lattice); (b) $l=5$ mm (a 20×20 square lattice); (c) $l=2.5$ mm (a 40×40 square lattice).

$=4$ mm, and $d=2$ mm, respectively. We defined the ratio of the area of the plane $ABCD$ to that of a circle as the system size. The decrease of the diameter d corresponds the increase of the system size. The electrical conductivity σ increases with the increase of the area ratio p. Since the paths from the side AB to CD are different in each sample, the values of σ vary with the samples even if the values of p are the same. However, the difference of the length of the path is short near the threshold value, so the dispersion of σ is small. Figure 4 shows the average values of σ for $d=2$, 4, and 8 mm. The threshold value p_c increases with the increase of the system size. Referring to our numerical simulation for a continuum Swiss-cheese model [7], the threshold value for the infinite system is 0.685.

In order to obtain the critical exponent μ of the electrical conductivity, data shown in Fig. 3 are plotted in a log-log plot diagram against $(p-p_c)$, where p_c is determined so as to obtain a straight fitting line by the use of the method of least squares. The slope of the straight line gives the critical exponent μ. This procedure is illustrated in Figs. 5(a) and 5(b). The variation of σ with p is not smooth as in Fig. 5(a). Also, σ sometimes increases rapidly because of the joining of two percolated clusters. In order to avoid that the critical exponent be affected by a few special data, the experimental data is approximated by a polynomial or an exponential function. Figure 5(b) shows a log-log plot diagram that is drawn with the values of σ calculated from the above-mentioned function. The values of R in Figs. 5(a) and 5(b) are correlation indices.

Table I shows the averages and the standard deviations of the critical exponent μ and the threshold value p_c of the electrical conductivity of continuum models. In the pioneering research, where the difference between the disrete and the continuum percolation model was clearly pointed out, it is reported that the critical exponents of electrical conductivity in a two-dimensional continuum system is the same as those in a discrete system and that the value is 4/3 [9]. However, the values of μ shown in Table I are larger than 4/3.

In order to prove that the difference of μ is not induced

TABLE II. The average value and the standard deviation of critical exponent μ and the threshold value p_c for the bond percolation models are given.

l	Discrete model Critical exponent μ	Threshold value p_c	Sample
2.5 mm	1.28 ± 0.07	0.48 ± 0.01	12
5 mm	1.32 ± 0.08	0.47 ± 0.04	28
10 mm	1.41 ± 0.04	0.41 ± 0.05	25

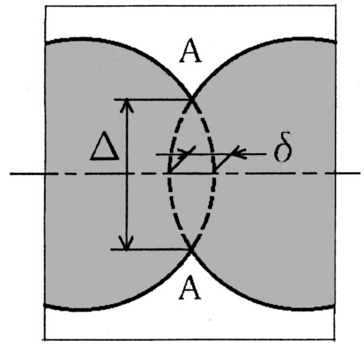

FIG. 7. Neck geometry in the inverse Swiss-cheese model.

from the characteristics of the experimental apparatus itself, we carried out the same experiment on a discrete system and confirmed the appropriateness of the results on a continuum model. Figures 6(a), 6(b), and 6(c) show the variation of the electrical conductivity σ of bond percolation models with the occupied ratio p for $l=10$ mm, 5 mm, and 2.5 mm, where l is the length of the bond and p is the ratio of the number of bonds to the total of all lattices. In the case of $l=2.5$ mm, the number of lattices is 1600 $(=40\times 40)$. In these figures, the value of σ for $p=1.0$ is selected as the standard of normalization.

Table II shows the averages and the standard deviations of the critical exponent μ and the threshold value p_c for the bond percolation models. These values are also obtained by the use of the above-mentioned method. As is well known, the threshold value of bond percolation on a square lattice is $p_c=0.5$ and the critical exponent is $\mu=4/3$ [10]. The value of the critical exponent μ for $l=10$ mm, 5 mm, and 2.5 mm are 1.41, 1.32, and 1.28, respectively. These values are less than those for the continuum models shown in Table I and are also close to 4/3. Therefore, it is considered that the critical exponent of electrical conductivity of the inverse Swiss-cheese model is different from that of the discrete system.

Although it is quite difficult to give a theoretical interpretation for the difference of the exponents in detail here, and identifying the reason is our next problem, we can make a rough prediction as follows. Figure 7 shows the neck geometry in the ISCM. δ is the width of the overlapped part, Δ is the length of the constricted region, and A is a projection. Δ is related to δ by $\Delta=2(\delta a)^{1/2}$, where a is the radius of a circle. Feng *et al.* have shown that the strength of a bond of overlap δ could be described by $\Delta\times\Delta$, and they have replaced the neck with a square bond whose length and width are Δ [9]. As a result, they have predicted that the exponent of the electrical conductivity for a two-dimensional continuum inverse Swiss-cheese model lay in the same universality class as bond percolation. In their discussion the influence of the projection A was not taken into consideration. This assumption may be correct as long as the value of Δ closes to that of the diameter $2a$. But the area of percolated clusters is small near the percolation threshold, so the values of δ and Δ are small in comparison with the diameter $2a$. Since the influence of the projection A on electrical conduction seems to increase with the decrease of Δ, contrary to the prediction made by Feng *et al.*, the exponent of the electrical conductivity for the continuum inverse Swiss-cheese percolation model may be different from that for the discrete model.

IV. CONCLUSION

It is shown that electrical conductivity in an inverse Swiss-cheese model can be measured easily with an X-Y plotter and GP-IB system. The critical exponents of the electrical conductivity μ, which we obtained experimentally, are $\mu=1.76-1.80$ for continuum models and $\mu=1.28-1.41$ for discrete models. Comparing these experimental results, it is considered that the critical exponent of electrical conductivity of a continuum inverse Swiss-cheese model is different from that of a discrete system.

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[1] S.R. Broadbent and J.M. Hammersley, Proc. Cambridge Philos. Soc. **53**, 629 (1957).
[2] V.K.S. Shante and S. Kirkpatrick, Adv. Phys. **20**, 325 (1971).
[3] D. Stauffer, *Introduction to Percolation Theory* (Taylor & Francis, London, 1985).
[4] T. Odagaki, *Introduction to Percolation Physics* (Shokabo, Tokyo, 1993).
[5] S. Miyazima, K. Maruyama, and K. Okumura, J. Phys. Soc. Jpn. **60**, 2805 (1991).
[6] B.I. Halperin, S. Feng, and P.N. Sen, Phys. Rev. Lett. **54**, 2391 (1985).
[7] A. Okazaki, K. Maruyama, K. Okumura, Y. Hasegawa, and S. Miyazima, Phys. Rev. E **54**, 3389 (1996).
[8] L. Benguigui, Phys. Rev. Lett. **53**, 2028 (1984).
[9] S. Feng, B.I. Halperin, and P.N. Sen, Phys. Rev. B **35**, 197 (1987).
[10] A. Bunde and S. Havlin, *Fractals and Disordered Systems* (Springer-Verlag, Berlin, 1991).