Periodic Oscillations in Transmission Decay of Anderson Localized One-Dimensional Dielectric Systems

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It is well recognized that the transmittance of Anderson localized systems decays exponentially on average with sample size, showing large fluctuations brought up by extremely rare occurrences of necklaces of resonantly coupled states, possessing almost unity transmission. We show here that in a one-dimensional (1D) random photonic system with resonant layers these fluctuations appear to be very regular and have a period defined by the localization length ξ of the system. We stress that necklace states are the origin of these well-defined oscillations. We predict that in such a random system efficient transmission channels form regularly each time the increasing sample length fits so-called *optimal-order* necklaces and demonstrate the phenomenon through numerical experiments. Our results provide new insight into the physics of Anderson localization in random systems with resonant units.

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Anderson localization [1] is an interference phenomenon which rules the transport in a wide class of disordered materials [2–6]. It has the characteristic that in a disordered medium the system eigenfunctions localize at a length scale, called the localization length, ξ, when the sample size becomes larger than the latter. In the Anderson localization regime the transmittance T (conductance) of such a system is governed by tunneling events and generally decays exponentially on average with increasing the sample thickness L,

$$\langle T\rangle = \exp\left(-\frac{L}{\xi}\right). \tag{1}$$

The average transmittance, however, often shows huge fluctuations, which originally were attributed to states localized at the center of samples. Later, Pendry [7] and Tartakovskii *et al.* [8] independently argued that these fluctuations are caused by exceedingly rare occurrences, resonantly coupled and delocalized through the sample states, called *necklaces*, of almost unity transmission. The number of resonances in necklaces was predicted to grow as $\sqrt{L}$ with increasing sample thickness, while the probability of such events to occur should reduce exponentially as $\exp(-L^{1/2})$ [7].

Experimental evidences for necklaces have been reported recently in 1D random dielectric systems at both optical [4,9] and microwave frequencies [10]. Time-resolved light propagation studies revealed the multiresonance character of necklaces [4], while interferometric phase measurements allowed the quantification of the necklace order (number of coupled resonances) through phase jumps of π-multiples [9]. In Ref. [10], necklaces were observed through multipeaked extended field distributions of overlapped quasimodes of an open dissipative waveguide filled with random dielectric slabs.

Still Pendry predicted that a trade-off between the expected number of states hybridizing to form a necklace band in a sample length L and the increasingly low probability of their occurrences should lead to a reduced number of necklace resonances. This reduction then should result in the formation of *optimal-order necklaces* [7]. Very recently, the question of how optimal-order necklaces form in 1D random dielectric stacks of resonant layers was addressed [11]. It was shown through numerical studies that the optimal necklace order m shifts gradually towards higher orders with increasing the sample size. By assuming an optimal length scale $l_0 = \alpha\xi$ for the most efficient coupling between two distinct resonances, it follows that m should increase by one each time the sample length increases by l_0 (the significance of α will be addressed in the following). It suggests that each time the sample thickness fits integer multiples of l_0 (i.e., $m = L/\alpha\xi$), the sample transmittance $\langle T\rangle$, averaged over many realizations, will be high. Such an assumption naturally brings up several questions: what is the system transmittance for sample lengths $ml_0 < L < (m+1)l_0$? Should $\langle T\rangle$ show a minimum? What is more intriguing, it follows immediately: should the transmission show periodic fluctuations?

In this Letter we predict and demonstrate through numerical calculations the existence of such oscillations in a 1D random stack of resonant dielectric layers. We state that these oscillations are due to the existence of necklaces, which periodically appear to be efficiently transmitting. Our theoretical model, developed for an oscillating transmittance decay reproduces very well the numerical results.

First, we address two characteristic parameters α and β, which define the optimal distance l_0 for the most efficient coupling between spatially distinct and resonant states. In a periodic 1D superlattice of coupled resonators (known as CROW structures) a flat and almost unity transmission band forms around the resonant energy, when the admittance of the unit cell (from resonance to resonance) matches that of the environment and its closest resonance (actually, the product of admittances on both sample interfaces) [12]. These admittances are functions of refractive index ratio of dielectric layers in the power of the number of half-wavelength periods. For chosen indices, these period numbers are scalable: e.g., choosing $(\beta/2)N$ periods between the environment and the 1st resonance and $\alpha N(=\beta N)$ periods for the inter-resonance region, for the unity transmission in a CROW miniband one obtains always α: $2(\beta/2) = 1{:}1$. This relation can be expressed in wave-function decay length units ξ', obtaining $l_0 = \alpha\xi'$ for intercavity spacing and $(\beta/2)\xi'$ for environment-1st-resonance distance.

In periodic superlattices with broken symmetry [13], the efficient resonant transmission channel forms when $\alpha/\beta = 2.5$. This is explained by the fact that other resonances (originally absent in a CROW), spectrally close but out of the resonant energy, decrease the intercavity barrier through a contribution from the tails of the spectral lines; therefore, they impose a larger separation of resonances for an efficient coupling. With this analogy, through this study we introduce similar parameters for a disordered system with resonant layers.

Let us consider a finite length random sample which has the "right thickness" to fit optimally two resonances at some frequency. Assuming two characteristic distances $(\beta/2)\xi$ (for an efficient mode coupling to the environment) and $\alpha\xi$ for the optimal coupling between two resonances, the right thickness condition for a second order necklace reads as $\frac{L-\beta\xi}{\alpha\xi} = m - 1 = 1$ [11]. The sample transmission will be high due to an efficient tunneling-through channel in the sample. Upon increasing the thickness, the efficiency of second order necklaces will diminish. While it will become more probable for third order necklaces to appear, they will not be optimal ones unless the sample thickness is not enough to fit three resonances, i.e., $\frac{L-\beta\xi}{\alpha\xi} < 2$. Thus, in the range $1 < \frac{L-\beta\xi}{\alpha\xi} < 2$, the transmittance will stay low and will increase again when the sample thickness fits an optimal third order necklace. This increasing-decreasing scenario will continue periodically with increasing sample length.

Locating necklaces spectrally is an effortful task and requires many realizations of disorder. Knowing at which frequency the necklace should appear could facilitate such study. One way to overcome the problem is to study positional random multilayer systems composed of quarter-wave thick dielectric materials. The advantage of such systems is that their transmittance spectra almost always possesses at least one state at the resonant frequency; hence, necklaces become more probable to form. Thus, we choose to study 1D binary random structures built by resonant nonabsorbing dielectric layers of A and B type (refractive indices n_A and n_B, respectively). The positional randomness is introduced by giving each layer a 0.5 probability to be of type A or B. Layer thicknesses d_A and d_B were chosen by imposing a quarter-wave condition, $n_A d_A = n_B d_B = \lambda_0/4$ ($\lambda_0 = 1.5$ μm in this study).

The transmission spectra of random structures were calculated numerically through the transfer matrix method [14]. For each sample thickness 10^3 realizations of randomness were performed and the average of integrated transmittance in a narrow frequency range around c/λ_0 was recorded. In Fig. 1 we plot the results of numerical calculations (open circles) for three different pairs of refractive indices, n_A/n_B. In particular, we chose 1.5/3 [Fig. 1(a)], 1.5/3.5 [Fig. 1(b)], and 1.1/3.5 [Fig. 1(c)]. The localization length for each case was independently obtained from the negative slope of $\ln\langle T\rangle = -L/\xi$, calculated (not reported) for three random multistacks with nonresonant layers but similar refractive index contrasts [15].

We indeed observe periodic fluctuations in transmission. From Fig. 1 we appreciate immediately the regular oscillations appearing in the mean transmission with increasing sample thickness. In all cases the transmission drops exponentially while a periodically oscillating envelope is superimposed to this decay.

Next, we try to model this interesting behavior in order to reproduce the periodic oscillations. As a starting point, we assume that the transmission, given by Eq. (1), is modulated by some periodic function of sample length

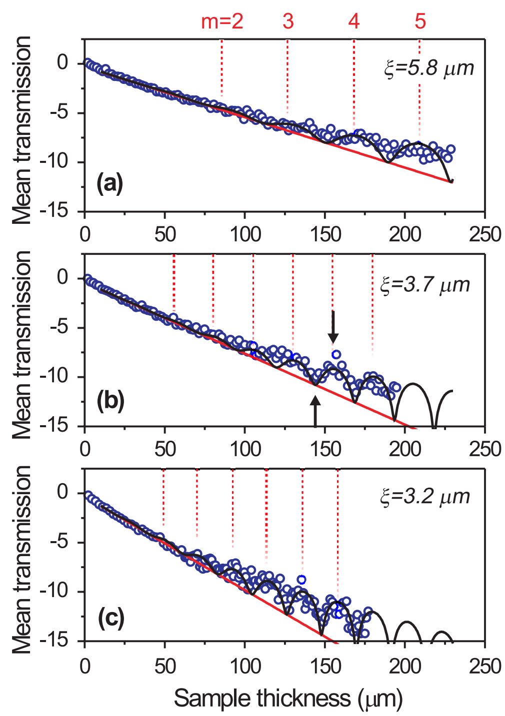

FIG. 1 (color online). The logarithm of average transmission decay as a function of sample thickness: (a) refractive indices $n_A/n_B = 1.5/3$, (b) 1.5/3.5, and (c) 1.1/3.5. The results of numerical calculations (open circles) show regular oscillations for thicker samples and are well reproduced by theoretical fits through Eq. (4) (oscillating line). The straight lines show the classical exponential decays. Dashed vertical lines indicate the optimal sample thicknesses that fit efficiently transmitting mth order necklaces, with the leftmost one for $m = 2$ in all plots.

$$\langle T' \rangle = \langle T \rangle (\text{some periodic function}). \quad (2)$$

For the modulating term we choose a cosine function with an argument $\phi(L, \xi) = 2\pi \frac{L-\beta\xi}{\alpha\xi}$. This way $\cos\phi$ reaches maxima each time the sample length fits an optimal-order necklace. With this, Eq. (2) now reads

$$\langle T' \rangle = \langle T \rangle \left[1 + f(L, \xi) \left(1 + \cos\left(2\pi \frac{L - \beta\xi}{\alpha\xi} \right) \right) \right], \quad (3)$$

where $f(L, \xi)$ is a fitting function to reproduce the increase in the oscillations amplitude with increasing sample length. The modulating term in Eq. (3) provides that $T' = T$ when the cosine approaches its minima; thus, it considers the case without the contribution from necklaces. The logarithm of the average transmission, after small simplification, looks as

$$\ln\langle T' \rangle = -\frac{L}{\xi} + \ln\left[1 + f(L, \xi)\cos^2\left(\pi \frac{L - \beta\xi}{\alpha\xi} \right) \right]. \quad (4)$$

We have plotted in Fig. 1 for all three cases the logarithm of the classical transmission T (red straight lines) and $\ln\langle T' \rangle$ (black oscillating lines). The slopes were fitted using corresponding ξ values, obtained from samples with non-resonant layers. The parameters $\alpha = 6.8$ and $\beta = 2$ were then independently adjusted to fit, respectively, the oscillations period and their peak positions on the L axis (similar to those reported in Ref. [11]). As mentioned in the beginning, we observe an even more increased α/β ratio, which is 3.4 now. We explain this further increase by the fact that contrary to a superlattice with broken symmetry, when both spatial and energetic positions of contributing resonances are known exactly, here many more randomly distributed resonances contribute to the reduction of inter-resonance barriers, resulting to higher α values for similar environment coupling β.

The numerical data confirm that in all three cases the transmission drops exponentially with increasing sample size. However, over about $L \approx 20\xi$ a clear deviation from a pure exponential starts. We observe an oscillatory behavior in the transmission decay, with a continuous increase in the oscillations amplitude [16]. The latter is explained by the fact that in thick samples the role of necklaces becomes much important in supporting a higher-on average transmission. We note, that using Eq. (4) for the same (α, β)-set and considering only the slope values ξ for calculated data in the range $L < 20\xi$, the oscillations period and their minima-maxima positions on the L axis were reproduced quite nicely for all three cases [17]. Data fitting procedures have verified that in all cases the oscillating function given by Eq. (4) provides better fits than the pure exponential one as for the whole thickness range, as well as only over $L > 20\xi$.

As a general result, following from Eq. (4), we show in Fig. 2 a three-dimensional plot of the average transmission as a function of localization length ξ and the sample thickness L. The oscillating surface plot is periodic in L, while not periodic in ξ, as it follows from the definition

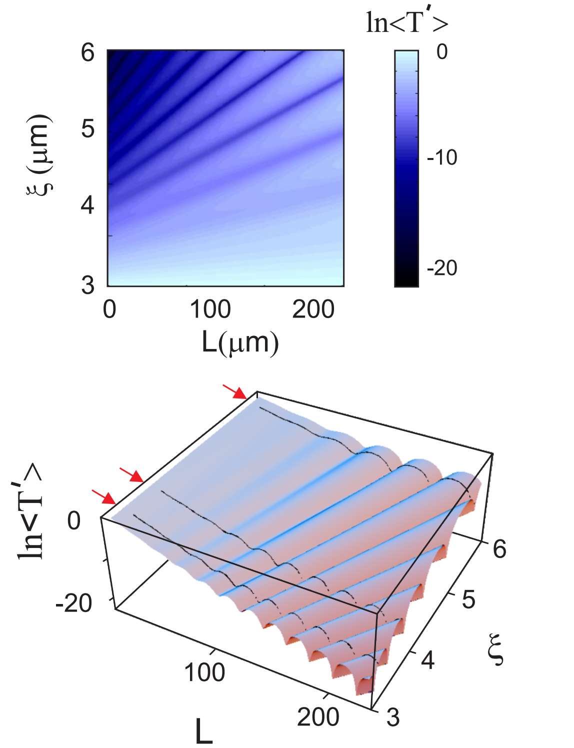

FIG. 2 (color online). A three-dimensional plot of the average transmission as a function of the localization length (refractive index contrast) and sample thickness, calculated through Eq. (4) for $\alpha = 6.8$ and $\beta = 2$. The oscillating lines over the surface plot correspond to $\ln\langle T'(L) \rangle$ curves for $\xi = 3.2$ μm, 3.7 μm, and 5.8 μm. The top graph plots the transmission intensity map.

of the cosine argument ϕ [Eq. (3)]. Three $\ln\langle T'(L) \rangle$ curves, corresponding to ξ values of 3.2 μm, 3.7 μm, and 5.8 μm, are plotted over the surface. In the top graph of Fig. 2 the transmission intensity map, related to the continuous surface plot, is shown as well.

Inspired by these results, we decided to look in more detail at an arbitrarily chosen minimum of the mean transmission and the next maximum. For this, we chose the case $n_A/n_B = 1.5/3.5$ and performed 100 other realizations of disorder at sample thicknesses of 144 μm and 157 μm [arrow marks in Fig. 1(b)]. These, respectively, correspond to the transmission minimum for $m = 5.5$ and the next maximum for $m = 6$. We look at the number of resonances, hybridizing to form a necklace, and plot their counts in histograms of Fig. 3. For a more complete analysis, we plot in the top panels of Fig. 3 the average transmission for all observed necklaces.

First of all, we observe that the histograms have different maxima, namely, at the oscillation minimum ($m = 5.5$, left panel) still fifth order necklaces dominate, while at the transmission maximum ($m = 6$, right panel) sixth and seventh orders become much frequent. Moreover, from the top panels we see that for $m = 5.5$ case, the average transmission has a maximum of $\langle T \rangle \approx 7 \times 10^{-3}$ both for the fifth and sixth order necklaces. This is consistent with our initial prediction: necklaces built up by five or six

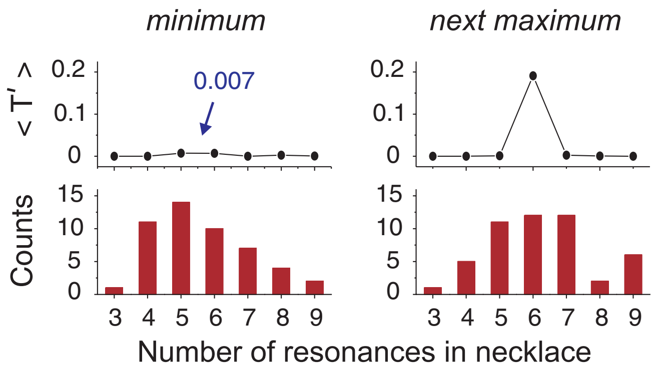

FIG. 3 (color online). Calculated histograms of optical necklaces and their corresponding transmissions (top panels) in the random sample with $n_A/n_B = 1.5/3.5$. The average transmission at an oscillation maximum (right panels) is almost 30 times higher than that of the preceding minimum (left panels), and is supported by higher order necklaces.

resonances contribute similarly to the system transmission, since the sample length is neither optimal for $m = 5$, nor for $m = 6$. On the contrary, from the top-right panel in Fig. 3 we see that at the oscillation maximum sixth order necklaces have an average transmission of ~ 0.2, which is about 30 times higher than in the previous case. This is again to confirm that the sample is thick enough now to fit optimally sixth order necklaces, or in other words, when necklaces with $m = 6$ occur, they are the most efficiently transmitting ones.

In conclusion, we predict and demonstrate in numerical experiments periodic oscillations in the average transmission of an Anderson localized 1D random optical system with resonant units. Our theoretical model reproduces well the numerically calculated periodic oscillations of transmission and shows that necklaces of resonantly coupled states are responsible for these. Necklaces can appear efficiently transmitting each time the sample size fits an optimal number degenerate in energy resonances. These results provide a new insight into the physics of Anderson localization.

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[1] P. W. Anderson, Phys. Rev. **109**, 1492 (1958).

[2] See, e.g., Ping Sheng, *Introduction to Wave Scattering, Localization, and Mesoscopic Phenomena* (Academic, New York, 1995); *Wave Scattering in Complex Media: From Theory to Applications*, edited by B. A. van Tiggelen and S. E. Skipetrov (Kluwer, Dordrecht, 2003); *Photonic Crystals and Light Localization in the 21st Century*, edited by C. M. Soukoulis (Kluwer, Dordrecht, 2001).

[3] S. John, Phys. Rev. Lett. **53**, 2169 (1984); P. W. Anderson, Philos. Mag. B **52**, 505 (1985); K. Arya, Z. B. Su, and J. L. Birman, Phys. Rev. Lett. **57**, 2725 (1986); A. Lagendijk, M. P. v. Albada, and M. B. v.d. Mark, Physica (Amsterdam) **140A**, 183 (1986); E. Abrahams *et al.*, Phys. Rev. Lett. **42**, 673 (1979); P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).

[4] J. Bertolotti, S. Gottardo, D. S. Wiersma, M. Ghulinyan, and L. Pavesi, Phys. Rev. Lett. **94**, 113903 (2005).

[5] V. Milner and A. Z. Genack, Phys. Rev. Lett. **94**, 073901 (2005).

[6] H. Gimperlein, S. Wessel, J. Schmiedmayer, and L. Santos, Phys. Rev. Lett. **95**, 170401 (2005); C. Fort *et al.*, *ibid.* **95**, 170410 (2005).

[7] J. B. Pendry, J. Phys. C **20**, 733 (1987).

[8] A. V. Tartakovskii *et al.*, Sov. Phys. Semicond. **21**, 370 (1987).

[9] J. Bertolotti, M. Galli, R. Sapienza, M. Ghulinyan, S. Gottardo, L. C. Andreani, L. Pavesi, and D. S. Wiersma, Phys. Rev. E **74**, 035602(R) (2006).

[10] P. Sebbah, B. Hu, J. M. Klosner, and A. Z. Genack, Phys. Rev. Lett. **96**, 183902 (2006).

[11] M. Ghulinyan, Phys. Rev. A **76**, 013822 (2007).

[12] A. Thelen, J. Opt. Soc. Am. **56**, 1533 (1966); Yong-Hong Ye, J. Ding, D.-Y. Jeong, I. C. Khoo, and Q. M. Zhang, Phys. Rev. E **69**, 056604 (2004); M. Ghulinyan *et al.*, Appl. Phys. Lett. **88**, 241103 (2006).

[13] M. Ghulinyan, C. J. Oton, Z. Gaburro, L. Pavesi, C. Toninelli, and D. S. Wiersma, Phys. Rev. Lett. **94**, 127401 (2005).

[14] J. B. Pendry, Adv. Phys. **43**, 461 (1994).

[15] In samples with nonresonant layers, the fluctuations in the transmittance are much less pronounced because of less frequent appearances of necklaces; therefore, the relative error in calculating the localization length in this conditions is small (see also Ref. [11]).

[16] The continuous increase in the oscillations amplitude, observed in numerical calculations and described by $f(L, \xi)$ in Eq. (3), can be understood in the following way: the average transmission at each L results from the contribution of exponentially narrowing localized states and almost unity transmitting necklaces for different realizations. We consider a simple model, in which over N realizations necklaces of unity transmission occur with a probability $\wp$ and in the rest of the cases $(1 - \wp)$ localized states contribute by $T \sim \exp(-L/\xi)$. Then, for the fitting function one can use the ratio between the contributions from necklaces and localized states as $f(L, \xi) = \frac{\wp N}{(1-\wp)N \exp(-L/\xi)} = \frac{\wp}{1-\wp} \exp(L/\xi)$. While the model can be developed for a more detailed mathematical description, at the current stage it allows a satisfactory explanation and theoretical fit. We note, that when $\wp = 0$ (no necklaces), Eq. (4) describes the classical exponential decay.

[17] In all cases, for slope fitting, the first term in Eq. (4) was modified to $-L/a\xi$, with $a = 3.45$. This is explained by the fact that due to resolution requirements we chose to look at a small frequency window around c/λ_0, which almost always fitted with the necklace bandwidth. This resulted in a larger effective localization length equal to $a\xi$.