ԵՐԵՎԱՆԻ ՊԵՏԱԿԱՆ ՀԱՄԱԼՍԱՐԱՆ

1919

YEREVAN STATE UNIVERSITY

ԵՐԵՎԱՆԻ ՊԵՏԱԿԱՆ ՀԱՄԱԼՍԱՐԱՆ

ԳՐԻԳՈՐՅԱՆ Ա.
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ՄԱԹԵՄԱՏԻԿԱՅԻ ԱՇԽԱՐՀՈՒՄ

Դասագիրք Մաթեմատիկայի և մեխանիկայի, Ինֆորմատիկայի և կիրառական մաթեմատիկայի ֆակուլտետների ուսանողների համար

ԵՐԵՎԱՆ
ԵՊՀ ՀՐԱՏԱՐԱԿՉՈՒԹՅՈՒՆ
2019

YEREVAN STATE UNIVERSITY

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DARBINYAN A.
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DEEPDIVE INTO MATHEMATICS

Textbook for the students of the Faculty of Mathematics and Mechanics, Informatics and Applied Mathematics

YEREVAN
YSU PRESS
2019

ՀՏԴ 51(075.8)
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*ՀՀ ԿԳ նախարարության կողմից հաստատվել է
որպես բուհական դասագիրք:*

*Հրապարակության է երաշխավորել
ԵՊՀ եվրոպական լեզուների և հաղորդակցության
ֆակուլտետի գիտական խորհուրդը:*

Դասագիրքը կազմել են՝

Գրիգորյան Ա. – Lesson 2, 8, 14, 20, 21, Miscellaneous, Proper Names,
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Դասագիրքը նախատեսված է բուհերի Մաթեմատիկայի և մեխանիկայի, Ինֆորմատիկայի և կիրառական մաթեմատիկայի ֆակուլտետների բակալավրիատի ուսանողների, ինչպես նաև ճշգրիտ գիտություններով զբաղվող մասնագետների համար:

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ԱՌԱՋԱԲԱՆ

“Deepdive into Mathematics” անգլերեն լեզվի սույն դասագիրքը բաղկացած է 22 դասից: Յուրաքանչյուր դաս սկսվում է բնաբանով, որը նպատակաուղղված է տվյալ նյութի նկատմամբ հետաքրքրություն առաջացնելուն: Դասագրքի նպատակն է ուսուցանել մասնագիտական բառապաշար, ամրապնդել մի շարք կարևոր քերականական դրույթներ, զարգացնել ուսանողների բանավոր խոսքը մասնագիտության բնագավառում, օժանդակել գիտական զեկույցներ, հոդվածներ գրելուն և հետազոտություններ կատարելուն:

Դասում ներկայացված հիմնական նյութն ընտրված է բարձրագույն մաթեմատիկայի տարբեր բնագավառներից: Կան նաև տեքստեր մաթեմատիկոս-գիտնականների կյանքի և մասնագիտական գործունեության վերաբերյալ:

Տեքստի բովանդակությունն ընկալելու և ուսանողի բառապաշարը հարստացնելու համար յուրաքանչյուր դասի տեքստին հաջորդում է մասնագիտական բառապաշարից բառացանկ, որտեղ տրվում է բառի հնչյունական տառադարձումը, խոսքիմասային պատկանելությունը և թարգմանությունը:

Հաջորդիվ տեղ են գտել ավանդական տարաբնույթ վարժություններ, որոնց նպատակն է զարգացնել սովորողների գրավոր և բանավոր խոսքը, խորացնել գիտական բառապաշարը, կատարելագործել քերականական հմտությունները:

Դասագրքում ընդգրկված վարժությունները միտված են կանխարգելել համընդհանուր բնույթի սխալները, ապահովել լեզվական նյութի ընկալումը: Հեղինակները դասագիրքը կազմելիս հիմնվել են նաև իրենց բազմամյա դասավանդման փորձի վրա՝ այն համապատասխանեցնելով դասավանդման ժամանակակից պահանջներին:

Տրված են նաև մաթեմատիկական խնդիրներ, որոնք հնարավորություն կտան ուսանողներին անգլերենով կատարել քննարկումներ իրենց մասնագիտության շուրջ:

Դասերում հանդիպող հատուկ անունները, իրենց տառադարձումներով, ամփոփված են տեղեկատու-հավելվածում՝ տվյալ գիտնականի կամ նշված վայրի վերաբերյալ համառոտ տեղեկությամբ:

Հավելվածում առանձին բաժնով ներկայացված է մաթեմատիկական սիմվոլները, նշանները, կոտորակներն ու բանաձևերն անգլերենով արտահայտելու ուղեցույցը:

Դասագրքում տեղ է գտել նաև լրացուցիչ ընթերցանության և ինքնուրույն աշխատանքի համար նախատեսված “Miscellaneous” խորագրով բաժինը, որը բաղկացած է մաթեմատիկային առնչվող տարբեր թեմաներով չափածո գրվածքներից, հանրահայտ գիտնականների կյանքի զվարճալի պատմություններից և հումորից:

Վերոհիշյալ բաժինը ամփոփվում է գիտնականների ասույթներով:

Սույն աշխատանքը խորհուրդ է տրվում օգտագործել անգլերեն լեզվի ուսուցանման Մաթեմատիկայի և մեխանիկայի, Ինֆորմատիկայի և կիրառական մաթեմատիկայի ֆակուլտետների բակալավրիատի ուսանողների կրթության համար:

Դասագրքի համահեղինակներն իրենց խորին երախտագիտությունն են հայտնում գրախոսներ ԵՊՀ եվրոպական լեզուների և հաղորդակցության ֆակուլտետի անգլերեն լեզվի No 1 ամբիոնի վարիչ, բ.գ.թ., պրոֆեսոր Մ. Հ. Ապրեսյանին, ԵՊՀ միջազգային հարաբերությունների ֆակուլտետի դիվանագիտական ծառայության և մասնագիտական հաղորդակցման ամբիոնի բ.գ.թ., դոցենտ Ա. Մ. Բաբայանին, ՀԱՊՀ օտար լեզուների ամբիոնի վարիչ, բ.գ.թ. դոցենտ Հ. Ձ. Ղազարյանին, ԵՊՀ մաթեմատիկայի և մեխանիկայի ֆակուլտետի ֆինանսական մաթեմատիկայի ամբիոնի վարիչ, ֆ.մ.գ.թ., դոցենտ Մ. Պ. Պողոսյանին՝ օգտակար դիտարկումների, ինչպես նաև գրքի կայացմանը նպաստելու համար:

PREFACE

The creation of the textbook “Deepdive into Mathematics” stemmed from the need to teach English on Bachelor’s courses and develop students’ study skills necessary to function effectively in a university context. Materials of the textbook are thoroughly selected to provide a good background of English for students-mathematicians and people who are interested in mathematics. The final objective of the textbook is to develop students’ confidence and competence, to make them autonomous learners in order to successfully carry out research and complete assignments, such as essays, reports, theses and oral presentations.

The structure of the textbook:

The textbook comprises 4 parts. The first part consists of 22 lessons. Each lesson begins with an epigraph, which is aimed at arousing students’ interest toward the contents of the lesson. The material of the texts is taken from different fields of higher mathematics. There are also texts about mathematicians and their contributions to the mathematical science, and the history of mathematics. The text to each lesson is provided with topical words, their transcriptions and translations into Armenian.

The texts are followed by various lexical and grammatical exercises to develop students’ reading, writing, speaking skills, improve their grammar knowledge and enrich professional vocabulary. In the final part of each lesson some mathematical problems and puzzles are given to advance students’ skills to read mathematical formulas, prove theorems, memorize definitions in English.

“Proper Names” contains valuable information about scientists and the geographical places mentioned in the textbook.

“Guide to Mathematical Symbols” possesses big tables of mathematical signs, symbols, formulas, equations and the way they are read in English.

The last part of the textbook entitled “Miscellaneous” is an additional reading material for students’ individual work. It consists of poems, jokes, humorous stories and quotes on mathematics, as well as interesting facts about prominent scientists and real stories from their lives.

CONTENTS

Lesson 1

WHAT IS MATHEMATICS?

Mathematics is the science that
draws necessary conclusions.
BENJAMIN PIERCE`[1]

The word *mathematics* comes from Greek. It means “something that must be learnt or understood”. The plural form of the word *mathematics* goes back to the Latin *mathematica* based on the Greek plural used by Aristotle meaning “all things mathematical”. Following the pattern of word formation, according to which the words physics, politics, economics, etc. were formed, the English word mathematics came to be used for mathematical study. It is often shortened to maths and takes singular verb forms.

Mathematics is the study of quantity, structure, space and change. As Karl Friedrich Gauss had said mathematics is the queen of all sciences. It helps the researchers to formulate new conjectures and establish new disciplines by rigorous deduction from axioms and definitions. Mathematics developed from counting, calculation, measurement and the study of the shapes and motions of physical objects by means of abstraction and logical reasoning.

The first uses of mathematics were seen in trading and commerce, land measurement, painting and weaving patterns, the recording of time, taxation, building and construction. The development of mathematics began as early as 3000 BC by the Babylonians and Egyptians, but the systematic study of it began between 600 and 300 BC by the ancient Greeks. Later mathematics continued to develop in China, India and Arabia until the Renaissance when mathematical innovations led to new scientific discoveries. Mathematics has been supplying methods and conclusions for science enabling scientists to predict results, describe phenomena and prepare their minds to new way of thinking.

One of the basic characteristics of mathematics is its symbolic language which is commonly acknowledged as “the language of science”. It consists mostly of signs and symbols which are universal and have remained almost unchanged throughout the time. Unlike the common language the language of mathematics is designed carefully and purposefully to be precise, unambiguous and concise which makes for the efficiency and clear representation of the thought.

Mathematics is also a good servant for other sciences. It is used throughout the world as an essential tool in many fields, including natural sciences, engineering, medicine and the social science. The application of mathematical knowledge to other fields leads to new mathematical discoveries. And as Geoffrey Hardy, the English mathematician of the 20th century states in “A Mathematician’s Apology,” mathematics is a study which “did not begin with Pythagoras and will not end with Einstein, but it is the oldest and the youngest of all”.[2]

🕮 TOPICAL VOCABULARY

draw conclusions			*եզրակացություններ անել*
conjecture	[kqn'GekCq]	n.	ենթադրություն, հիպոթեզ
definition	[defi'niSn]	n.	սահմանում
plural	['pluqrql]	a.	հոգնակի թիվ
pattern	['pxtqn]	n.	նմուշ, ձև, օրինակ
establish disciplines			*գիտության ճյուղ ստեղծել*
rigorous deduction	*հիմնավորված հետևություն, եզրակացություն*		

logical reasoning			*տրամաբանական դատողություն*
phenomenon	[fq'nOminqn]	n.	երևույթ
(pl. phenomena)	*[fq' nOminq]*		
weave	[wJv]	v.	հյուսել, գործել
(*wove, woven*)	*[wouv, wouvqn]*		
taxation	[txk'seiSqn]	n.	հարկում, հարկի գանձում
predict	[pri'dikt]	v.	կանխագուշակել
universal	['juni'vWsl]	a.	համընդհանուր
precise	[pri'sais]	a.	ճշգրիտ, ճիշտ
unambiguous	['Anxm'bigjuqs]	a.	հստակ, ոչ երկիմաստ
concise	[kqn'sais]	a.	համառոտ, հակիրճ
make for	['meik fL]	v.	նպաստել

I. What is the Armenian for?

follow the pattern, be based on, word formation, shorten, quantity, space, formulate, conjecture, a new discipline, rigorous deduction, definition, abstraction, logical reasoning, trading, land measurement, weaving patterns, innovations, supply methods, draw conclusions, unambiguous, clear representation of the thought, make for the efficiency, predict results

II. What is the English for?

ենթադրություն, հիմնված լինել, ձևակերպել, ճիշտ եզրակացություն, հաշվարկ, վերացարկման միջոցով, տրամաբանական եզրակացություն, առևտուր, հողաչափում, նախշանկարի նմուշ, ժամանակագրություն, հարկերի գանձում, մաթեմատիկական նորարարություններ, նոր մտածելակերպ

III. a) Arrange the following words in pairs similar in meaning:

1.	**apology**	a.	model
2.	**precise**	b.	scientist
3.	**conjecture**	c.	foretell
4.	**pattern**	d.	excuse
5.	**shorten**	e.	universal
6.	**researcher**	f.	effectiveness
7.	**reasoning**	g.	exact
8.	**weaving**	h.	certain
9.	**unambiguous**	i.	abbreviate
10.	**predict**	j.	assumption
11.	**efficiency**	k.	thinking
12.	**common**	l.	fabricating

b) Arrange the following words in pairs opposite in meaning:

1.	**ambiguous**	a.	expanded
2.	**plural**	b.	approximate
3.	**universal**	c.	singular
4.	**precise**	d.	unambiguous
5.	**concise**	e.	leave
6.	**systematic**	f.	king
7.	**remain**	g.	inaccurate
8.	**purposefully**	h.	limited
9.	**queen**	i.	disordered

IV. Match the following words with their definitions:

1.	**structure**	a.	accept or admit the existence or truth of something
2.	**symbolic**	b.	a branch of knowledge, typically one studied in higher education
3.	**axiom**	c.	form an opinion or supposition about (something) on the basis of incomplete information
4.	**commerce**	d.	the action of concluding or subtracting something
5.	**acknowledge**	e.	the arrangement of and relations between the parts or elements of something complex
6.	**discipline**	f.	serving as a symbol
7.	**conjecture**	g.	statement accepted as true without argument
8.	**deduction**	h.	the quality of dealing with ideas rather than events
9.	**abstraction**	i.	the activity of buying and selling, especially on a large scale
10.	**rigorous**	j.	the action or process of discovering or being discovered
11.	**discovery**	k.	extremely thorough and careful

V. Give the corresponding noun or adjective to the words given below:

Greek, China, Arabia, Babylonian, Egyptian

VI. Form new words using prefixes or suffixes:

mathematics, mean, structure, change, science, research, establish, measure, construct, conclude, ambiguous, efficiency, lead, apology

VII. Are the statements true or false? Contradict the statements which are not true:

1. Mathematics developed from counting by means of abstraction and logical reasoning.
2. The first uses of mathematics were still seen in prehistoric ages.
3. Signs and symbols are characteristic only to mathematical research.
4. New scientific conjectures are formulated from different axioms and definitions.
5. Mathematics is a distinct branch of science which leads mainly to mathematical discoveries.

VIII. Choose the suitable form of the word:

Algebra is a branch of mathematics. It … (introduces/ introducing) symbols (x, y, z, etc.) and a series of mathematical … (operations/operators) like factorization and expansions. It can be studied from an elementary level like … (adding/addition) and simplifications of algebraic fractions … (to solve/solving) simple simultaneous linear equations up to university level where one studies complex … (linear/line) systems, determinants, matrices, eigenvalues, vectors, spaces, fractals, etc.

IX. Match the beginnings with their appropriate endings:

1.	Mathematics is the study of …	a.	… counting, calculation and measurement.
2.	Mathematics developed from …	b.	… predict results and describe phenomena.
3.	The first uses of mathematics were seen …	c.	… quantity, structure, space and change.
4.	Mathematics enables scientists to …	d.	… precise, unambiguous and concise.
5.	Symbolic language is …	e.	… in trading and commerce, land measurement, etc.

X. Find a suitable title for the following passage and summarize it:

A philosophy professor stood before the class and had some items in front of him. When class began, wordlessly he picked up a large empty jar and proceeded to fill it with rocks right to the top, rocks about 2 inches in diameter. He then asked the students if the jar was full. They agreed that it was.

So the professor then picked up a box of pebbles and poured them into the jar. He shook the jar lightly. The pebbles, of course, rolled into the open areas between the rocks. He asked his students again if the jar was full. They agreed: it was.

The professor then picked up a box of sand and poured it into the jar. Of course, the sand filled up everything else. "Now," said the professor, "I want you to recognize that this is your life. The rocks are the important things - your family, your partner, your health and your children. If everything else was lost and only they remained, your life would still be full. The pebbles are the other things in life that matter, but on a smaller scale. The pebbles represent things like your job, your house, your car. The sand is everything else - the small stuff. If you put the sand or the pebbles into the jar first," he continued "there is no room for the pebbles or the rocks.

The same goes for your life. If you spend all your energy and time on the small stuff, material things, you will never have room for the things that are important to you. Pay attention to the things that are critical for your happiness. Play with your children. Take your partner out dancing. There will always be time to go to work, clean the house, give a dinner party and fix the disposal. Take care of the rocks first - the things that really matter. Set your priorities. The rest is just pebbles and sand".

XI. Render the following passage:

The Leading Mathematician of Antiquity

Great mathematicians of antiquity who had extraordinary talent for theoretical abstraction were quite eager to apply their theoretical knowledge and abilities to the practical problems which were so important for their civilization. They were trying to unite their theoretical studies with concrete scientific and engineering investigations.

Among the leading scientists of antiquity Archimedes is considered to be one of the most famous and the greatest of all times. Being primarily a mathematician he invented different machines, burning mirrors – which concentrated the sun's rays on Roman ships besieging his native city of Syracuse in the result of which the ships were burnt under the intense heat. His scientific discoveries were also based on hydrostatic principle – one of the first universal laws of science. Archimedes put two branches of mechanics – statics and hydrostatics on mathematical bases. He reached all the scientific conclusions intuitively and roughly long before their validity was possible to prove.

Archimedes is considered to be the first rational mathematician. His bright intellect, practical and theoretical interests, extraordinary mechanical skills and vivid imagination have been greatly admired and appreciated since his time.

XII. Discuss the following problems:

- Problem:

Find three numbers such that their sum is a square and the sum of any pair exceeds the third by a square.

Solution:

Let the sum of the three be $(x+1)^2$; let first + second = third + 1, so that the third $=\frac{1}{2}x^2+x$;

let second + third = first $+x^2$, so that first $=x+\frac{1}{2}$.

Therefore, second $=\frac{1}{2}x^2+\frac{1}{2}$.

It remains that first + third = second + a square.

Therefore $2x=$ square $=16$, say, and $x=8$.

The numbers are $8\frac{1}{2}$, $32\frac{1}{2}$, 40.

- Problem:

Divide unity into three parts such that, if three different given numbers be added to the parts respectively, the results are all squares.

Given numbers 2, 3, 4. Then divide 10 into three squares such that the first >2, the second >3, and the third >4.

Let us add $\frac{1}{2}$ of unity to each, and find three squares such that their sum is 10, while the first lies between $2,\ 2\frac{1}{2}$, the second between $3,\ 3\frac{1}{2}$, and the third between $4,\ 4\frac{1}{2}$.

It is necessary, first, to divide 10 (the sum of two squares) into two squares one of which lies between $2,\ 2\frac{1}{2}$, then if we subtract 2 from the latter square, we have one of the required parts of unity.

Next divide the other square into two squares, one of which lies between $3,\ 3\frac{1}{2}$; subtracting 3 from the latter square, we have the second of the required parts of unity.

Similarly we can find the third part.

XIII. Speak on the key points of the text: "What's Mathematics?"

Lesson 2

THE FIELDS OF MATHEMATICS

Mathematics has beauty and romance.
It's not a boring place to be, the mathematical world.
It's an extraordinary place; it's worth spending time there.
MARCUS DU SAUTOY[3]

There are so many different fields of Mathematics, from early number theory to the modern research areas of game theory, fractals, probability theory, spherical and spatial geometry, etc.

Mathematics may broadly be divided into the following fields:

1. ALGEBRA is a branch of Mathematics most people who have gone through High School would have studied at some stage: it introduces symbols (*x, y, z,* etc.) and a series of mathematical operations like factorization, expansions, etc. It can be studied from a very elementary level (like addition and simplifications of algebraic fractions, solving simple simultaneous linear equations involving 2 unknowns) up to college and university levels and beyond where one studies complex linear systems, determinants, matrices, eigenvalues, vectors spaces, algebraic structures like groups, rings, fields, etc.

2. In **GEOMETRY,** various theorems and lemmas regarding plane figures (straight lines, triangles, quadrilaterals, trapeziums, circles, ellipses etc.) are studied in detail. Geometry theorems are often associated with angles, graphing- with horizontal axis (*x-axis*) and the vertical axis (*y-axis*) with straight lines and methods of determining the slope of the straight line. This subdivision of Geometry is Cartesian Geometry or Coordinate Geometry, attributed to Rene Descartes. Again, the study of Geometry can progress from the very simple but can become highly complex as in Vector and Spherical Geometry, Topology, etc.

3. TRIGONOMETRY is the branch of Mathematics studying angles; in fact, it generally forms part of what used to be called Plane Geometry. In trigonometry the angles are associated with certain defined ratios and thus are born the trigonometric concepts of sine, cosine, tangent, secant, cosecant and cotangent associated with an angle of any magnitude. One studies the various trigonometric ratios and trigonometric identities and various operations involving these.

4. CALCULUS is probably one of the most important branches of Mathematics. It has many applications in other fields of knowledge – social science, physical sciences, biological sciences and all divisions of engineering. It introduces various important concepts and provides powerful mathematical tools that allow mathematicians to determine accurately and efficiently quantities like rates of flow of water from a tunnel, rate of decay of a radioactive chemical, etc.

5. STATISTICS usually studied together with Probability Theory is the Mathematics subject that examines the methods of collecting, representing, collating, comparing, analyzing and interpreting data. In probability theory, the concept of a probability of an event is defined, followed by discussions of various probability theorems and probability distributions like the Normal Distribution, Binomial Distribution etc. It introduces terms like mean or average, median, mode, and discusses various ways of representing data. There are also statistical tests (chi-squared tests, the t-tests) that are being used to correlate sets of data to determine if there is some significant relationship between them.

A comprehensive and exhaustive list of all the divisions of Mathematics are not provided here but the above 5 broad fields may be a rough guide as to how Mathematics has traditionally been compartmentalized. There are also other ways of classifying the various mathematical fields: some academics choose to divide them into two main parts – Pure and Applied Mathematics. Pure Mathematics refers to divisions which are "pure" or the more abstract fields of algebra, functional analysis, linear systems, logic etc. whereas Applied Mathematics groups those which have direct applications like Statistics.[4]

🕮 TOPICAL VOCABULARY

research	[ri'sq:C]	n.	հետազոտություն, հետախուզում, ուսումնասիրություն
area	['Fqriq]	n.	տարածություն, մակերես, հատված, չափս
fractal	['frxkt(q)l]	n.	կոտորակային չափականության բազմություն
probability	["prObq'biliti]	n.	հավանականություն
spherical	['sferikql]	a.	գնդանման, գնդաձև, գնդային
spatial	['speiSql]	a.	տարածական, տարածության
stage	[steiG]	n.	1. փուլ 2. շրջան, ժամանակաշրջան 3. էտապ
factorization	['fxktqraizeiSn] = ["fxktqrq'zeiSn]	n.	բազմապատկիչների վերլուծում
expansion	[iks'pxnSn]	n.	1. վերլուծում 2. ընդարձակում, ընդլայնում
simplification	["simplifi'keiSn]	n.	պարզեցում

fraction	['fr x kSn]	n.	1. կոտորակ 2. մաս, բաժին
simultaneous	["simql'teinjqs]	a.	միաժամանակյա, միաժամանակ
linear	['liniq]	a.	գծային
determinant	[di'tq:minqnt]	n.	դետերմինանտ, որոշիչ
analysis *(pl. analyses)*	[q'n x lqsis] *[q'n x lqsi:z]*	n.	վերլուծում, վերլուծություն, անալիզ
matrix *(pl. matrices)*	['meitriks] *['meitrisi:z]*	n.	մատրից
eigenvalue	['i:gin"v x lju]	n.	մատրիցի սեփական արժեք
lemma *(pl. lemmata = lemmas)*	['lemq] *['lemqtq]*	n.	լեմմա, օժանդակ պնդում
plane	[plein]	a.	հարթ
		n.	հարթություն
quadrilateral	["kw0dri'l x tqrql]	a.	քառակողմ, քառանկյուն
trapezium *(pl. trapezia = trapeziums)*	[trq'pi:ziqm] *[trq'pi:ziq]*	n.	սեղան
ellipse *(pl. ellipsis)*	[i'lips] *[i'lipsi:z]*	n.	էլիպս

associate	[q'souSieit]	v.	1. միացնել, միավորել 2. միանալ 3. զուգորդ(վ)ել
graphing	[grx'fiN]	v.	1. գծագրում 2. պատկերում
series	['siqri:z]	n.	(pl.) շարք
axis *(pl. axes)*	['xksis] *['xksi:z]*	n.	առանցք
slope	[sloup]	n.	թեքություն, թեքվածք, անկյունային գործակից
subdivision	['sAbdi"viZqn]	n.	ստորաբաժանում
attribute (to)	[q'tribju:t]	v.	վերագրել
ratio	['reiSiou]	n.	հարաբերություն, հարաբերակցություն
magnitude	['mxgnitju:d]	n.	մեծություն
rate of decay			*քայքայման, փրոհման արագություն*
collate	[kO'leit]	n.	համեմատել, ճշտել, ստուգել
interpret	[int'q:prit]	v.	մեկնաբանել, բացատրել
distribution	["distri'bju:Sn]	n.	բաշխում
binomial	[bai'noumiql]	n.	բինոմական, երկանդամային
mean	[mi:n]	n.	միջին
average	['xvqriG]	n.	միջին
median	['mi:diqn]	a.	միջին

mode	[moud]	n.	մոդ՝ թվերի բազմության մեջ ամենահաճախ հանդիպող թիվ
correlate	['k0rqleit]	n.	հարաբերակից
correlate (with)	['k0ri"leit]	v.	առնչվել, կոռելացված լինել
comprehensive	["k0mpri'hensiv]	a.	բազմակողմանի, համապարփակ, ընդգրկուն
exhaustive	[ig'z0:stiv]	a.	1. մանրամասն 2. սպառիչ
division	[di'viZ(q)n]	n.	բաժանում
compartmentalized	["k0mpa:t'ment(q)laizd]	a.	ճյուղերի բաժանված
Pure Mathematics			*մաքուր մաթեմատիկա*
Applied Mathematics			*կիրառական մաթեմատիկա*

I. What is the Armenian for?

Pure and Applied Mathematics, methods of collecting, collating, the slope of the straight line, comprehensive and exhaustive list of all the divisions of Mathematics, trigonometric ratios and trigonometric identities, attributed to, circle, analyzing and interpreting data, trapezium, representing, Probability Theory, Plane Geometry, ellipse, at some stage, fractals, up to college and university levels, a series of mathematical operations factorization, expansions, etc., plane figures, an angle of any magnitude, quadrilateral, lemma

II. What is the English for?

սինուսի եռանկյունաչափական հասկացություն, ռադիոակտիվ քիմիական նյութի տրոհման արագությունը, համարել (ընդունել) որպես հանրահաշվի ճյուղ, տվյալների վերլուծում և մեկնաբանում, վիճակագրական թեստեր, գծային համակարգ, հանրահաշվական կոտորակների պարզեցում, մանրազնին են ուսումնասիրվում, ուղիղ գծի անկյունային գործակից, Դեկարտյան երկրաչափություն, միջին

III. Write the plural forms of:

analysis, lemma, branch, matrix, identity, ellipse, quantity, axis, ratio

IV. a) Find the corresponding nouns from the text and translate them:

invent, important, deduce, apply, technological, commercial, social, know, build, true, define, historical

b) Find the corresponding verbs from the text and translate them:

division, solution, choice, introduction, expansion, definition, analysis, relationship, direction, exhaustive, significant

c) Find the corresponding adjectives from the text and translate them:

exhaust, to vary, statistics, trigonometry, significance, power, algebra, difference, line, sphere, vertex, space

V. Match the following words with their definitions:

1.	**plane**	a.	an examination of something to identify causes, key factors, relationships and possible results.
2.	**elementary**	b.	a number (such as1/2 or 3/4) which indicates that one number is being divided by another
3.	**decay**	c.	space between two lines or surfaces that meet
4.	**fraction**	d.	flat or level surface; surface that the straight line joining any points on it is touching it at every point
5.	**ellipse**	e.	to decrease usually gradually in size, quantity, activity or force
6.	**angle**	f.	1. size 2. (degree of) importance
7.	**analysis**	g.	not developed; simple; of, at or in the beginning stage(s)
8.	**magnitude**	h.	regular oval

VI. Choose the correct preposition:

1. The students of mathematics may wonder where the word "mathematics" comes … *(from / of)*.
2. Mathematics is a Greek word that means "knowledge acquirable … learning" or "general knowledge". *(through / by)*.
3. Mathematics arises … many different kinds … problems. *(from / in), (of / by)*.
4. Albert Einstein stated that "as far as the laws … mathematics refer … reality, they are not certain; and as far as they are certain, they do not refer … reality". *(in / of), (to, from), (to, from)*.
5. Mathematics is used … the world as an essential tool … many

fields including natural sciences, engineering, medicine, and the social sciences. *(throughout / for), (at / in).*

6. Applied mathematics is concerned … application … mathematical knowledge … other fields. *(with / in), (in / of), (from / to).*
7. Applied mathematics inspires and makes use … new mathematical discoveries and sometimes leads … the development … entirely new mathematical disciplines, such as statistics and game theory. *(of / in), (at / to), (in / of).*

VII. Match the beginnings with their appropriate endings:

1.	Algebra is a branch of Mathematics that introduces symbols (*x, y, z,* etc.) and a series of mathematical …	a.	… the methods of collecting, representing, collating, comparing, analyzing and interpreting data.
2.	In geometry, various theorems and lemmas regarding plane figures …	b.	… social science, physical sciences, biological sciences and all divisions of engineering.
3.	In trigonometry the angles are associated with certain defined ratios and thus are born the trigonometric concepts of …	c.	… operations like factorization, expansions, etc.
4.	Calculus has many applications in other fields of knowledge - …	d.	… are studied in detail.
5.	Statistics is the mathematical subject that examines …	e.	… sine, cosine, tangent, secant, cosecant and cotangent associated with an angle of any magnitude.

VIII. Choose the suitable word:

1. Mathematics is defined as the study of quantity, structure, space, change, and anything … to pattern and form. *(relating / related)*
2. Mathematics is a science that … be applied to just about anything and everything. *(can / must)*
3. At the same time, mathematics could be considered an art as …. *(good / well)*
4. There are many aspects of mathematics that are … to be intriguing pieces of art. *(considered / considerable)*
5. Mathematics … be applied to normal … life without having to resort to numbers that much. (has to / can), (every daily / everyday)

IX. Complete the following sentences using the words given below:

exceptional, defended, graduated, grader, rendered, talent, degree outstanding, information.

Sergey Mergelyan was an Armenian scientist, an … mathematician, who is the author of major contributions in Approximation Theory. The modern Complex Approximation Theory is based on Mergelyan's classical work. Mergelyan established the grounds for the development of … technology in Armenia.

He showed his precocious … in school years. He won the republican Olympics of mathematics and physics when an eighth grader at Mravian School. Afterwards, he … the exams for ninth and tenth … and entered the School of Physics and Mathematics of Yerevan State University in 1944, at the age of sixteen.

He passed the first year and most of the second year courses in one year, and started attending third year courses. Mergelyan … in three and a half years, instead of the normal five, and in 1947 he was sent to Moscow for graduate work at the Steklov Mathematics Institute of the

Academy of Sciences of the Soviet Union (now the Russian Academy of Sciences). Just two years later, on February 19, 1949, he … his Ph.D. dissertation on the approximation theory in mathematical functions. The scientific council of the institute assessed it as a study of … value, and unanimously awarded him both Ph.D. and Doctor of Science degrees. The acquisition of the highest … of Doctor of Science at the age of twenty-one became a record, unbeaten to this day, in the former USSR and present-day Russia.

X. Translate:

1. Մաթեմատիկա բառը գալիս է հունարենից և նշանակում է գիտելիք, գիտություն:
2. Մաթեմատիկայի զարգացումը տարբեր փուլեր է ունեցել:
3. Եռանկյունաչափությունը սկզբնավորվել է աստղագիտության պահանջներից:
4. Արդի մաթեմատիկայի դարաշրջանը սկսվում է 19-րդ դարի առաջին կեսից:
5. Ֆրանսիացի մաթեմատիկոս, մեխանիկ և ֆիզիկոս Դենի Պուասսոնը (1781-1840) ասել է. «Արժե ապրել հենց միայն նրա համար, որ զբաղվես մաթեմատիկայով»:
6. Մաթեմատիկան կարելի է սահմանել որպես գիտություն, որում մենք երբեք չգիտենք, թե ինչի մասին ենք խոսում, ոչ էլ գիտենք արդյոք մեր ասածը ճշմարիտ է:

   (Բ. Ռասսել)
7. Թվաբանությունն այն է, ուր թվերը աղավնիների պես ներս են վազում մեր ուղեղների մեջ և դուրս թռչում այնտեղից:

   (Կ. Սանդբերգ)

8. Եթե մարդիկ չեն հավատում, որ մաթեմատիկան պարզ է, ապա պատճառը միայն այն է, որ չեն գիտակցում, թե կյանքը որքան բարդ է:

(Ջոն ֆոն Նեյման)

9. Արքիմեդի վերջին խոսքերն էին. «Ձեռք մի՛ տուր իմ գծագրերին»:
10. Պատմությունը պահպանել է մաթեմատիկոս Լանգրանժի վերջին խոսքերը՝ «Մահից չպետք է վախենալ: Երբ այն վրա է հասնում առանց ցավի, ոչ մի տհաճ բան չկա նրանում: Ես երբեք ատելություն չեմ տածել որևէ մեկի հանդեպ: Ես ոչ մի վատ բան չեմ արել և ես հանգիստ եմ մահանում»:

XI. Translate the following quotes of famous people about mathematicians into Armenian:

1. “Relations between pure and applied mathematicians are based on trust and understanding. Namely, pure mathematicians do not trust applied mathematicians, and applied mathematicians do not understand pure mathematicians.”

(*Karl Weierstrass*) [5]

2. To think logically the logically thinkable – that is the mathematician’s aim.

(*C. J. Keyser*) [6]

3. It is impossible to be a mathematician without being a poet in soul.

(*Sofia Kovalevskaya*) [7]

4. Only professional mathematicians learn anything from proofs. Other people learn from explanations.

(*Ralph Boas*) [8]

5. It is easier to square a circle than to get round a mathematician.

(*de Morgan*). [9]

6. Some mathematician, I believe, has said that true pleasure lies not in the discovery of truth, but in the search for it.

(*Tolstoy*) [10]

7. A mathematician's reputation rests on the number of bad proofs he has given.

(*A.S. Besicovitch*) [11]

8. Since the mathematicians have invaded the theory of relativity, I do not understand it myself any more.

(*Albert Einstein*) [12]

XII. a) Show that the given sequence is geometric, and find the common ratio:

- $5, -\frac{5}{4}, \frac{5}{16}, \cdots, 5\left(-\frac{1}{4}\right)^{n-1}, \ldots$
- $\frac{1}{7}, \frac{3}{7}, \frac{9}{7}, \cdots, \frac{1}{7}(3)^{n-1}, \cdots$

b) Find the fifth term, the eight term, and the n^{th}, term of the geometric sequence:

- 8, 4, 2, 1, …
- 4, 1.2, 0.36, 0.108, …
- 300, – 30, 3, – 0.3, …
- $1, -\sqrt{3}, 3, -3\sqrt{3}, \ldots$
- 5, 25, 125, 625, …
- 2, 6, 18, 54, …
- 4, -6, 9, -13.5, …
- 162, -54, 18, -6, …
- $1, -x^2, x^4, -x^6, \ldots$

- $1,-\frac{x}{3},\frac{x^2}{9},-\frac{x^3}{27},\cdots$
- $2,2^{x+1},2^{2x+1},2^{3x+1},\cdots$
- $10,10^{2x-1},10^{4x-3},10^{6x-5},\cdots$

c) Find the sum:

- $\sum_{k=1}^{10} 3k$
- 16.
- $\sum_{k=0}^{9}\left(-\frac{1}{2}\right)^{k+1}$
- 17.
- $\sum_{k=1}^{9}\left(\sqrt{-5}\right)^{k}$
- 18.
- $\sum_{k=1}^{7}\left(3^{-k}\right)$

XIII. Render the text into English:

Ի ծնե մաթեմատիկոս՝ Սերգեյ Մերգելյան

Մոսկվայի գիտությունների ազգային ակադեմիայի Վ. Ա. Ստեկլովի անվան մաթեմատիկայի ինստիտուտում 1949թ.-ի փետրվարի 19-ին իր թեկնածուական թեզը պաշտպանող Սերգեյ Մերգելյանին գիտական խորհուրդը միաձայն քվեարկությամբ շնորհեց ֆիզիկա-մաթեմատիկական գիտությունների դոկտորի գիտական աստիճան: Այն ժամանակ Մերգելյանը 21 տարեկան էր: Նա Մոսկվա էր գործուղվել Երևանի պետական համալսարանի ֆիզիկամաթեմատիկական ֆակուլտետի մաթեմատիկական բաժինը երեք ու կես տարում ավարտելուց հետո:

Դպրոցում ուսանելիս պատանի Մերգելյանը իսկույն տարբերվեց մյուսներից և իր մտավոր հզոր կարողությունների շնորհիվ դպրոցն ավարտեց սահմանված տարիքից շուտ՝ հանձնելով էքստեռն քննություններ: Նրա անունը թնդաց նաև պետհամալսարանում, որտեղ մեկ տարվա ընթացքում հանձնեց առաջին կուրսի բոլոր ու երկրորդ կուրսի քննությունների մեծ մասը և սկսեց հաճախել երրորդ կուրս՝ ունկնդրելու հայ մաթեմատիկական դպրոցի հիմնադիր Արտաշես Շահինյանի դասախոսությունները: Շահինյանը իսկույն նկատում է լրջմիտ ուսանողին, որի մասին հետագայում գրում է. «Համակրանքը դեպի այդ համեստ ու լռակյաց պատանին, որ հայացքը վար, ուշադիր լսում էր և տալիս դիպուկ հարցեր, համակում է առաջին իսկ հանդիպողին: Զգացվում էր այդ ուսանողի բացառիկ սերը դեպի ստեղծագործական աշխատանքը և այն, որ նա բոլորովին չէր խուսափում դժվարին խնդիրներ ձեռնարկելուց»: Շահինյանը նաև ընդգծում է, որ Մերգելյանով վերջնականապես ձևավորվեց հայկական մաթեմատիկայի պատմության մի շրջանը:[13]

XIV. Speak on the key points of the text “The Fields of Mathematics”.

Lesson 3

FINANCIAL MATHEMATICS

Financial mathematics needs to tell not only what people ought to do, but also what people actually do. This gives rise to a whole new horizon for mathematical finance research: can we model and analyze the consistency and predictability in human flaws so that such flaws can be explained, avoided or even exploited for profit?

XUNYU ZHOU (OXFORD UNIVERSITY)[14]

Financial mathematics is a field of applied mathematics concerned with financial markets. It has a close relationship with financial economics. Financial mathematics derives and extends the mathematical and numerical models suggested by financial economics. For example, while a financial economist might study the structural reasons why a company may have a certain share price, a financial mathematician may take the share price as a given and attempt to use stochastic calculus to obtain the fair value of derivatives of the stock.

Financial mathematics also overlaps with the field of computational finance (also known as *financial engineering)*. These are largely synonymous although the latter focuses on application while the former focuses on modeling and derivation. The fundamental theorem of arbitrage-free pricing is one of the key theorems in financial mathematics.

The history of financial mathematics starts with the *Theory of Speculation* (1900) by Louis Bachelier which discussed the use of Brownian motion to evaluate stock options. The first influential work in this area is the theory of portfolio optimization by Harry Markowitz on using mean-variance estimates of portfolios to judge investment strategies, causing a shift away from the concept of trying to identify the best individual stock for investment. Using a linear regression strategy to understand and quantify the risk (i.e. variance) and return (i.e. mean) of an entire portfolio of stocks and bonds, an optimization strategy was used to choose a portfolio with largest mean return subject to acceptable levels of variance in the return. Simultaneously, William Sharpe developed mathematics of determining the correlation between each stock and the market. For their pioneering work Markowitz and Sharpe, along with Merton Miller, shared the 1990 Nobel Memorial Prize in Economic Sciences for the first time ever awarded for a work in finance.

Financial mathematics is interesting because it synthesizes a highly technical and abstract branch of mathematics: it measures theoretic probability with practical applications that affect people's everyday lives. Financial mathematics is exciting because by employing advanced mathematics it is developing the theoretical foundations of finance and economics. Just as physics has motivated new mathematics, financial mathematicians are now developing new mathematics to a model observed economically rather than physically. Aristotle tells us that Thales, the father of western science, became rich by applying his scientific knowledge to speculation, Galileo left the university of Padua to work for Cosimo II de Medici and wrote "On the Discoveries of Dice" becoming the first *quant.* Around a hundred years after Galileo left Padua, Sir Isaac Newton left Cambridge to become warden of the Royal Mint and lost the modern equivalent of £3.000.000 in the South Sea Bubble. Moreover, interesting things happen when mathematics meets finance: the concept of probability emerged out of the interface. Mathematics of the brain, the dynamics of networks, capturing and harnessing stochasticity in nature are all highly relevant to finance. According to Cardano, financial mathematics has been about understanding how humans make decisions in the face of uncertainty and then establishing how to make good decisions. One of the key objectives of financial mathematics is to understand how to construct the best strategies that minimize risks in the real world.

Making, or at least, not losing money is simply a by-product of this knowledge.[15]

🕮 TOPICAL VOCABULARY

consistency	[kqn'sistqnsi]	n.	հետևողականություն, կայունություն
predictability	[pridiktq'biliti]	n.	կանխագուշակելիություն
flaw	[flL]	n.	թերություն, սխալ
exploit	[iks'plOit]	v.	շահագործել, ցույց տալ, օգտագործել, բացահայտել
share price			*բաժնետոմսի գինը*
stochastic	[stO'kxstik]	a.	հավանական, ստոխաստիկ
stochastic calculus			*ստոխաստիկ հաշիվ*
fair value			*արդար գին*
derivation	["deri'veiSn]	n.	դուրս բերում, արտածում
bond	[bOnd]	n.	պարտատոմս
simultaneously	["simql'teiniqsli]	adv.	միևնույն ժամանակ
overlap	['ouvq'lxp]	v.	մասամբ համընկնել
arbitrage-free pricing			*արբիտրաժից զերծ գնագոյացում*
key theorems			*հիմնական թեորեմներ*
speculation	["spekju'leiSn]	n.	սպեկուլյացիա

stock options			*բաժնետոմսի օպցիաներ*
portfolio	[pO:t'fouliou]	n.	փաթեթ, պորտֆել
influential	["influ'enSql]	a.	ազդեցիկ
variance	['vFqriqns]	n.	շեղում, վարիացիա, դիսպերսիա
investment	[in'vestmqnt]	n.	ներդրում
cause a shift			*պատճառել փոփոխություն*
quantify	['kwOntifai]	v.	գնահատել
return	[ri'tWn]	n.	շահույթ, շահութաբերություն
correlation	["kOri'leiSn]	n.	հարաբերակցություն, կոռելյացիա
emerge	[i'mq:G]	v.	առաջանալ, ծագել
interface	["intq'feis]	n.	փոխգործակցություն
construct strategies			*կառուցել ռազմավարություն*
quant	['kwOnt]	n.	քվանտ (*ֆին.* քանակական տվյալների վերլուծող), վերլուծաբան
mint	[mint]	n.	դրամահատարան
warden	[wLdn]	n.	պատասխանատու, ղեկավար, պետ
capture	['kxptSq]	v.	գրավել, ուշադրության արժանացնել
objective	[Ob'Gektiv]	n.	խնդիր, նպատակ
harness	['ha:nis]	v.	օգտագործել, կիրառել
by-product			*կողմնակի արտադրանք*

I. What's the Armenian for?

give rise to a new horizon, human flaws, exploit for profit, obtain fair value of the derivatives of the stock, arbitrage-free pricing, portfolio optimization, mean-variance estimates, mean return, bond, subject to, advanced mathematics, dice, speculations, the pioneering work, a warden, capturing and harnessing, relevant to, in the face of, by-product, to construct the best strategies

II. What is the English for?

մարդկային կորուստ, գործածել ի շահ ինչ-որ բանի, մտահոգված լինել, արտածել և ընդհանրացնել, արդար գինը ստանալ, ազատ գնագոյացում, դրամահատարան, միջին դիսպերսիայով գնահատական, փոխգործակցություն, առաջանալ, կապված լինել, վերլուծաբան

III. a) Arrange the following words in pairs similar in meaning:

1.	**overlap**	a. assess
2.	**mean**	b. stock
3.	**return**	c. enlarge
4.	**flaw**	d. partly coincide
5.	**obtain**	e. profit
6.	**exploit**	f. average
7.	**extend**	g. fault
8.	**variance**	h. at the same time
9.	**judge**	i. deviation
10.	**bond**	j. use
11.	**simultaneously**	k. get

b) Arrange the following words in pairs opposite in meaning:

1.	**start**	a. disappear
2.	**regression**	b. unfair
3.	**pioneering**	c. latter
4.	**analyze**	d. theoretical
5.	**close**	e. stay
6.	**former**	f. maximize
7.	**fair**	g. irrelevant
8.	**practical**	h. last
9.	**emerge**	i. end
10.	**relevant**	j. remote
11.	**shift**	k. synthesize
12.	**minimize**	l. unimportant
13.	**influential**	m. progression

IV. Match the following words with their definitions:

1.	**optimization**	a.	fundamental
2.	**predict**	b.	place where coins are made
3.	**bond**	c.	average difference
4.	**overlap**	d.	profit, income
5.	**mint**	e.	use, apply
6.	**arbitrage-free pricing**	f.	buy and sell goods, stocks and shares with risk of loss and hope of profit through changes in their market value
7.	**return**	g.	voucher, stocks, assets
8.	**speculate**	h.	forecast, foretell
9.	**key (a)**	i.	flexible valuation
10.	**harness (v)**	j.	the fact of optimizing; making the best of anything
11.	**mean variance**	k.	partly cover or coincide
12.	**synthesize**	l.	to combine different ideas, styles, or systems into a single idea or system

V. Give derivatives of the following words:

investment, extend, value, speculation, quantify, correlation, consistency, profit, predictability, numerical, computational, application, variance, mean, regression, exciting, objective, uncertainty

VI. Insert prepositions: *in, to, of, with.*

The idea … attaching a number … a set or … an object is familiar … everybody. The length … a segment, the area … a triangle, the volume … a ball or the mass … a physical body are considered here. All these facts can be expressed … numbers. Predictability is also expressed … terms … numbers attached … events. The way … doing this is very much analogous … that … length, area and volume.

VII. Are the given statements true or false?

1. Financial mathematics is concerned with financial markets and has a close relationship with the discipline of financial economics.
2. A financial mathematician must study the structural reasons why a company may have a certain share price.
3. The fundamental theorem of arbitrage-free pricing is one of the key theorems in financial mathematics.
4. Financial mathematics synthesizes a highly technical and abstract branch of mathematics, measures theoretic probability with practical applications that affect people's everyday lives.
5. Financial innovation currently has a poor reputation and mathematicians should think twice before becoming involved with it.

VIII. Match the beginnings with their appropriate endings:

1.	Financial mathematics has a close relationship …	a.	… with the *Theory of Speculation* by Louis Bachelier.
2.	Financial mathematics overlaps …	b.	… developed mathematics of determining the correlation between each stock and the market.
3.	The history of financial mathematics starts …	c.	… with financial economics.
4.	William Sharpe …	d.	… the theoretical foundations of finance and economics.
5.	Financial mathematics is developing…	e.	…with the field of computational finance.

IX. Render into English:

Հավանականության տեսությունը մաթեմատիկայի հիմնական տեսություններից է: Այն սերտորեն առնչվում է թվային վերլուծության հետ: Ֆինանսական առումով մեծ կարևորություն է տրվում նաև դիֆերենցիալների գնահատմանը, ռիսկի զարգացմանը և ֆինանսական պայուսակի կառավարմանը՝ ռիսկի և շահույթի գնահատման միջոցով: Այս պատճառով է, որ մաթեմատիկական հմտություններով օժտված մասնագետների պահանջարկը գնալով ավելի է մեծանում աշխատաշուկայում:

X. Read the following passage and discuss it:

MEASURING INTEREST RATES

Credit market instruments fall into four types: a simple loan, a fixed-payment loan, a coupon bond and a discount bond.

1. A **simple loan** provides the borrower with an amount of funds that must be repaid to the lender at the maturity date along with an additional amount known as an *interest* payment. For example, if a bank made you a simple loan of $100 for one year, you would have to repay the principal $100 in one year's time along with an additional interest payment of, say, $ 10. Commercial loans to business are often of this type.
2. A **fixed-payment loan** provides the borrower with an amount of funds that is to be repaid by making the same payment every month, consisting of part of the principal and interest for a set number of years. For example, if you borrowed $1000, a fixed-payment loan might require you to pay $126 every year for 25 years. Installment loans (such as auto loans) and mortgages are frequently of the fixed-payment type.
3. A **coupon bond** pays the owner of the bond a fixed interest payment every year until the maturity date, when a specified final amount is repaid. The coupon payment is so named because the bondholder used to obtain payment by clipping a coupon off the bond and sending it to the bond issuer, who then sent the payment to the holder. Treasury bonds and notes and corporate bonds are examples of coupon bonds.
4. A **discount bond** (also called a **zero-coupon bond**) is bought at a price below its face value (at a discount), and the face value is repaid at the maturity date. Unlike a coupon bond, a discount bond does not make any interest payments, it just pays off the face value. For example, a discount bond with a face value of $1000 might be bought for $900 and in a year's time the owner would be repaid the face value of $1000.

XI. Translate the following problem into Armenian:

COUPON BOND

To calculate the yield to maturity for a coupon bond, equate today's value of the bond with its present value. Because coupon bonds have more than one payment, the present value of the bond is calculated as the sum of the present values of all the coupon payments plus the present value of the final payment of the face value of the bond.

The present value of a \$1000-face-value bond with ten years to maturity and yearly coupon payments of \$100 (a 10% coupon rate) can be calculated as follows: At the end of one year, there is a \$100 coupon payment with a *PV* of \$100 (1+ i); at the end of the second year, there is another \$100 coupon payment with a *PV* of \$100 (1+ i), and so on until at maturity, there is a \$100 coupon payment with a *PV* of \$100 (1+ i) plus the repayment of the \$1000 face value with a *PV* of \$1000 (1+ i). Setting today's value of the bond (its current price, denoted by *P)* equal to the sum of the present values of all the payments for this bond gives

$$P = \frac{\$100}{1+i} + \frac{\$100}{(1+i)^2} + \frac{\$100}{(1+i)^3} + \ldots + \frac{\$100}{(1+i)^{10}} + \frac{\$1000}{(1+i)^{10}}.$$

More generally, for any coupon bond,

$$P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n},$$

where P = price of coupon bond
C = yearly coupon payment
F = face value of the bond
n = years to maturity date.

XII. Speak on the key points of the text: "Financial Mathematics".

Lesson 4

WHAT IS AN ACTUARY?

It is a truth very certain that when it is not in our power to determine what is true we ought to follow what is most probable.

RENE DESCARTES[16]

As the future is uncertain some of the events that can happen may be undesirable and "risk" is the possibility that an undesirable event will occur. Thus, an actuary is a business professional who deals with the financial impact of risk and uncertainty. Actuaries provide expert assessments of financial security systems with a focus on their complexity, mathematics and mechanisms. They are experts in:

- evaluating the likelihood of future events,
- designing creative ways to reduce the likelihood of undesirable events,
- decreasing the impact of undesirable events that do occur.

The impact of undesirable events can be both emotional and financial. Reducing these events and their financial impact is very important as it helps to relieve emotional pain. Actuaries mathematically evaluate this likelihood of events and quantify the contingent outcomes in order to minimize emotional and financial losses, associated with uncertain undesirable events such as death, sickness, injury, disability or loss of property. As sometimes, unfortunately, they cannot be avoided, it is helpful to take measures to minimize their financial impact when they occur. These risks can affect both sides of the balance sheet and require asset management, liability management and valuation skills. Actuaries are the leading professionals in finding ways to manage the risk. It takes a combination of strong analytical skills, business knowledge and understanding of human behavior to design and manage programs that control risk. So, they use skills in mathematics, economics, computer science, finance, probability and statistics, as well as business to help businesses assess the risk of certain events occurring and to formulate policies that minimize the cost of that risk. For this reason, actuaries are essential to the insurance and reinsurance industry either as staff employees or as consultants.

Historically, actuarial science became a formal mathematical discipline in the late 17th century with the increased demand for a long-term insurance coverage. This requires estimating future contingent events, such as the rates of mortality by age, as well as the development of mathematical techniques. In the early twentieth century, actuaries were developing the techniques that can be found in modern financial theory, but for various reasons these developments did not achieve much recognition. However, the science has gone through revolutionary changes and from the late 1980s to early 1990s there was a distinct effort for actuaries to combine financial theory and stochastic methods into their established models. Today the profession both in practice and in the educational syllabi of many actuarial organizations combines tables, loss models, stochastic methods and financial theory. The study which used five main criteria to rank jobs (environment, income, employment outlook, physical demands and stress) classify actuary as the #1 job in many developed countries.

Actuaries love what they do. Their work is intellectually challenging and they are very well-paid. They are key players in the management team of the companies that employ them. Most actuaries work in a pleasant environment alongside other professionals and enjoy the respect of their peers.

Actuaries are the analytical backbone of our society's financial security programs. So we can go about our daily lives without worrying too much about what the future may hold for us. These are the safeguards that protect us from life's catastrophes.

The work of actuaries benefits all of us.[17]

🕮 TOPICAL VOCABULARY

actuary	['xktjuqri]	n.	ապահովագրական գործակալ, վիճակագիր, ակտուար
undesirable	['Andi'zaiqrqbl]	a.	անցանկալի, տհաճ
assessment	[q'sesmqnt]	n.	գնահատում
security	[si'kjuəriti]	n.	1. անվտանգություն, ապահովություն 2. երաշխավորություն, երաշխավորում
complexity	[kqm'pleksiti]	n.	բարդություն
likelihood	['laiklihud]	n.	հավանականություն
impact	['impxkt]	n.	ազդեցություն
relieve	[ri'lJv]	v.	թեթևացնել, մեղմացնել, նվազեցնել
quantify	['kw0ntifai]	v.	որոշել քանակը
contingent	[kqn'tinGqnt]	a.	պատահական
outcome	['autkAm]	n.	արդյունք, հետևանք, ելք
disability	["disq'biliti]	n.	անաշխատունակություն, հաշմանդամություն
balance sheet			*հաշվեկշիռ, բալանս*

management	['mxnidʒm(ə)nt]		n. կառավարում
asset ~			*ակտիվների կառավարում*
liability ~			*պասիվների կառավարում*
staff employee			*հաստիքային ծառայող*
policy	['pOlisi]	n.	1. քաղաքականություն 2. գործունեության ծրագիր՝ կուրս
long-term	['lON'tWm]	a	երկարաժամկետ
insurance coverage		*ապահովագրական ծածկույթ*	
mortality	[mL'txliti]	n.	մահացություն
mortality by age		*մահացություն ըստ տարիքի*	
syllabus ***(pl. syllabi)***	['silqbqs] *['silqbai]*	n.	ծրագիր
criterion ***(pl. criteria)***	[krai'tiqriqn] *[krai'tiqriq]*	n.	չափանիշ
table	[teibl]	n.	աղյուսակ
challenging	['tSxlinGiN]	a.	հետաքրքիր, մարտահրավեր նետող
environment	[in'vaiqrqnmqnt]	n.	շրջապատ, միջավայր
peer	[piq]	n.	հավասարակից, գործընկեր
enjoy the respect of ones' peers		*վայելել գործընկերների հարգանքը*	
backbone	['bxkboun]	n.	1. ողնաշար 2. հիմք, էություն
catastrophe	[kə'txstrəfi]	n.	աղետ
benefit	['benefit]	n.	օգուտ

I. What is the Armenian for?

risk, manage the risk, control the risk, leading professional, provide, assessment, injury, contingent event, disability, property, balance sheet, liability management, valuation skills, behavior, for this reason, consultant, long-term insurance coverage, mortality by age, in the early twentieth century, develop techniques, achieve recognition, stochastic methods, combine, to rank jobs, criterion, employment, outlook, challenging, well-paid, management team

II. What is the English for?

արդյունք, չափանիշ, երկարաժամկետ ապահովագրական ծածկույթ, ակտիվների կառավարում, վայելել գործընկերների հարգանքը, հաշմանդամություն, օգուտ, տեղի ունենալ, բարձր վարձատրությամբ աշխատանք, աղյուսակ, ծրագիր, պատահական դեպք, ստոխաստիկ մեթոդ, զբաղվածության հեռանկար, պասիվների կառավարում

III. a) Arrange the following words in pairs similar in meaning:

1.	**undesirable**	a.	chance
2.	**impact**	b.	happen
3.	**probability**	c.	estimate
4.	**evaluate**	d.	unsteady
5.	**occur**	e.	unpleasant
6.	**uncertain**	f.	classify
7.	**reduce**	g.	advantage
8.	**employee**	h.	protect
9.	**benefit**	i.	worker
10.	**rank**	j.	influence
11.	**safeguard**	k.	decrease

b) Arrange the following words in pairs opposite in meaning:

1.	**uncertain**	a.	ability
2.	**desirable**	b.	certain
3.	**professional**	c.	regress
4.	**complexity**	d.	ancient
5.	**disability**	e.	liabilities
6.	**development**	f.	undesirable
7.	**modern**	g.	simplicity
8.	**mortality**	h.	amateur
9.	**assets**	i.	disgusting
10.	**pleasant**	j.	immortality

IV. Match the following words with their definitions:

1.	**likelihood**	a.	create or prepare methodically
2.	**sickness**	b.	the way a thing turns out; a consequence
3.	**estimate**	c.	an event causing great and usually sudden damage or suffering
4.	**formulate**	d.	pay particular attention to
5.	**security**	e.	the state of being likely or probable
6.	**catastrophe**	f.	the state of being ill
7.	**focus on**	g.	the most important part of something, providing support for everything else
8.	**outcome**	h.	the state of being free from danger or threat
9.	**insurance**	i.	roughly calculate or judge the value, number, quantity, or extent of
10.	**backbone**	j.	a course or principle of action adopted or proposed by an organization or individual
11.	**policy**	k.	the agreement in which you pay a company money and the company pays the cost if you have an accident injury or loss

V. Give derivatives corresponding to the following words:

evaluate, ability, develop, recognition, occur, employee, manage, uncertainty, science, reason, professional, establish

VI. Write the plural forms of the following nouns:

employee, theory, technique, criterion, syllabus, catastrophe, calculus, cactus

And what about the plural of: *mathematics, economics, statistics*?

VII. Insert prepositions:

at/of, at/till, from, ago/about, in, from/before, down/to, up/about, to/for, at/of/with.

1. As soon as we hear … the suppliers we shall let you know. 2. I shall work late … the office this evening so I won't get home … ten. 3. … the end … the next week the agreement will be signed. 4. When he first arrived he couldn't speak a word … English. He had never studied …. 5. I read the book such a long time …that I've forgotten what it's…. 6. He gave … no hope … passing the examination though he had already failed twice. 7. Mr. Smith retired … the age … 70 having spent 40 years … the Company. 8. We shall get … … business as soon as they settle the agenda. 9. The scientist suddenly saw the answer … the problem that had occupied his mind … the last two months. 10. He anticipated getting a rise … salary soon.

VIII. Choose the suitable word:

1. People are the heart and soul of any… (*business/environment*).
2. The insurance … (*people/industry*) relies on the dedication, creativity and commitment of its people to thrive and grow.
3. Graduates who have studied human resources … (*criterion/ management*) may have an advantage, but most … (*psychology/ employers*) will accept applicants with any degree.

4. If you wish … (*to advance/to occur*) in the profession it will be necessary to study for further qualifications though the exact qualification for which you study will depend on your current educational … (*administration/background*).
5. Human resources … (*rely on/ deal with*) matters such as salaries, pensions and benefits, training and development, industrial and staff relations, welfare, health and safety.

IX. Match the beginnings with their appropriate endings:

1.	An actuary is a business professional …	a.	… in financial reporting of companies' assets and liabilities.
2.	The 17-th century was …	b.	… who specified that the chief official should be called an 'actuary'.
3.	The study of John Graunt became …	c.	… a period of extraordinary advances in mathematics and valuation of risk.
4.	Actuaries are also involved…	d.	… the basis for the original life table.
5.	It was Edward Rowe Mores …	e.	… who deals with the financial impact of risk.

X. Translate into Armenian:

The classical function of actuaries is to calculate premiums and reserves for insurance policies covering various risks. Premiums are the amount of money the insurer needs to collect from the policyholder in order to cover the expected losses, expenses and a provision for profit. Reserves are provisions for future liabilities and indicate how much money should be set aside now to reasonably provide for future payouts. If you inspect the balance sheet of an insurance company you will find that the liability side consists mainly of reserves. On the casualty side this analysis often involves quantifying the probability of a loss event, called the frequency, and the size of that loss event, called the severity. The amount of time that occurs before the loss event is also important as the insurer will not have to pay anything until after the event has occurred.

XI. Discuss the following problems. Mind that every interest problem involves four quantities:

- the principal originally invested,
- the accumulated value at the end of the period of investment,
- the period of investment, and
- the rate of interest.

➢ **Problem 1.** Eric deposits 8000 in an account on January 1.1995. On January 1.1997, he deposits an additional 6000 in the account. On January 1.2001, he withdraws 12.000 from the account. Assuming no further deposits or withdrawals are made, find the amount in Eric's account on January 1.2004, if $i = .05$.

Solution: In this example, withdrawals can be viewed as "negative deposits" in an equation of value.

8000	6000		– 12.000	
1995	1997	……	2001	2004

The resulting balance is

$$X = 8000(1.05)^9 + 6000(1.05)^7 - 12000(1.05)^3 = 6961.73.$$

➢ **Problem 2.** Find the rate of interest such that an amount of money will triple itself over 15 years.

Solution:

Let i be the required effective rate of interest.

We have $(1+i)^{15} = 3$, so that $i = 3^{1/15} - 1 = 0.07599$.

XII. Solve the following equations of value:

a) Boswell wishes to borrow a sum of money. In return, he is prepared to pay as follows: 200 after 1 year, 500 after 2 years, 500 after 3 years and 700 after 4 years. If $i = .13$, find the point at which a single payment of 21000 would be equivalent.

b) A vendor has two offers for a house: (i) 40.000 now and 400000 two years hence, or (ii) 28.750 now, 23.750 in one year, and 27.500 two years hence. He makes the remark that one offer is "just as good" as the other. Find the two possible rates of interest which would make his remark correct.

c) The present value of 2 payments of 1000 each, to be made at the end of n years and $n+4$ years, is 1250. If $i = 0.08$, find n.

XIII. Speak on the key points of the text: "What is an Actuary?"

Lesson 5

GEOMETRY THROUGH THE AGES

There is no royal road to geometry.
EUCLID[18]

The first considerations of geometry are very ancient. They originated from the human ability to perceive and recognize physical forms and compare their shapes and sizes. The notions of square, rectangle, triangle, curve, solid and surface are the first geometrical concepts. Later observations of concrete geometrical figures led to the recognition of certain general properties and relationships. In the result of everyday practical procedures general rules were introduced concerning areas, volumes and relationships of various objects (e.g. the rule of thumb).

Thus, the first stage of geometry which may also be called subconscious geometry started in ancient Egypt and Babylonia and raised to the status of science in ancient Greece. Thales of Miletus, who was considered to be one of the "seven wise men" of antiquity, insisted that empirical conclusions should be worked out by deductive or logical reasoning. It gave rise to establishing another stage of geometry about 600 BC, which was more developed and is known as systematic or demonstrative geometry.

Pythagoras, an outstanding Greek geometer of antiquity, continued the systematization of geometry. Other great scholars of the Golden Age of Greek geometry Euclid and Archimedes developed the main theories without which the development of geometry would have been impossible and which are still in use.

In the Middle Ages, known as Europe's Dark Ages (the period from the middle of the 5^{th} century until the 11^{th} century) schooling became almost non-existent, although the people of the East excelled in computation and developed our present numerical system.

In the early period of Renaissance in Europe, with the rebirth of art and learning new elements of geometrical theory were created, which concerned the way of representing and analyzing three-dimensional objects by means of their projection.

The seventeenth century was marked with the invention of the calculus by means of which the properties of curves and surfaces and their generalization were studied.

By the middle of the nineteenth century a number of different geometries came into existence the synthesis and classification of which were inevitable. Each geometry has its underlying transformation group to which the manifold of elements (such as point, line, circle, sphere, etc.) is to be subjected.

So, geometry received a further generalization in the 20th century through set theory. Each geometry has become a particular branch of mathematics and they are all unified through the employment of the geometrical language and imagery.[19]

TOPICAL VOCABULARY

consideration	[kqnsidq'reiSn]	n.	դիտարկում, քննարկում
perceive	[pq'sJv]	v.	ընկալել, հասկանալ, ըմբռնել
notion	['nouSqn]	n.	հասկացություն, պատկերացում
curve	[kWv]	n.	կոր
solid	['sOlid]	a.	պինդ մարմին, երկրաչափական մարմին
concept	['kOnsept]	n.	գաղափար, հասկացություն

subconscious	["sAb'kOnSqs]	a.	ենթագիտակցական
empirical	[im'pirikl]	a.	փորձով հաստատված, էմպիրիկ
reasoning	['rJzqniN]	n.	դատողություն, կշռադատում
extend	[ik'stend]	v.	ընդարձակել, տարածել
excel in	[ik'sel in]	v.	գերազանցել, աչքի ընկնել
rebirth	[rJ'bWT]	n.	վերածնունդ
three-dimensional	["TrJdi'menS(q)nql]	a.	եռաչափ
manifold	['mxnifould]	n	բազմաձևություն
		a.	բազմատեսակ
be subjected to	[bi sqb'Gektid tu]	v.	ենթարկվել (ազդեցության, գործողության)
unify	['ju:nifai]	v.	միավորել, միացնել
employment	[im'plOimqnt]	n.	կիրառություն, օգտագործում
imagery	['imiGqri]	n.	պատկեր, խորհրդապատկեր

I. What is the Armenian for?

ancient, considerations, recognize forms, square, rectangle, triangle, curve, solid, surface, area, volume, observation, to introduce, empirical, subconscious, logical reasoning, deductive, scholar, object, projection, to come into existence, by means of, generalization, transformation, three-dimensional

II. What is the English for?

հասկացություն, ճանաչել տարբեր ձևերը, ենթագիտակցական, գործնական գործառույթների արդյունքում, ծավալ, մակերես, փորձնական, գոյություն չունեցող, թվային համակարգ, ձևափոխություն, ընդհանրացում, համեմատել, փուլ, կրթություն, թվային համակարգ, անհնար

III. a) Arrange the following words in pairs similar in meaning:

1.	**originate**	a.	inference
2.	**notion**	b.	variety
3.	**observation**	c.	create
4.	**subconscious**	d.	various
5.	**conclusion**	e.	intuitive
6.	**reasoning**	f.	come from, derive
7.	**schooling**	g.	concept
8.	**invent**	h.	understand
9.	**difference**	i.	experimental
10.	**synthesis**	j.	thinking
11.	**perceive**	k.	research
12.	**manifold**	l.	unification
13.	**empirical**	m.	education

b) Arrange the following words in pairs opposite in meaning:

1.	**systematic**	a.	general
2.	**recognize**	b.	existent
3.	**practical**	c.	straight line
4.	**subconscious**	d.	fall behind
5.	**curve**	e.	disorganized
6.	**excel**	f.	deny
7.	**particular**	g.	sole
8.	**non-existent**	h.	theoretical
9.	**manifold**	i.	conscious

IV. Match the following words with their definitions:

1.	**circle**	a.	a plane figure with three straight sides and three angles
2.	**form**	b.	a bending without angles
3.	**square**	c.	made united, uniform, or whole
4.	**rectangle**	d.	the visible shape or configuration of something
5.	**triangle**	e.	total outside surface
6.	**curve**	f.	a four-sided figure with four right angles
7.	**surface**	g.	law, regulation
8.	**rule**	h.	external layer
9.	**area**	i.	measuring length, breadth and thickness
10.	**synthesis**	j.	a round plane figure whose boundary (the circumference) consists of points equidistant from a fixed point (the centre).
11.	**imagery**	k.	the measure of the amount of space inside of a solid figure, like a cube, cylinder, pyramid
12.	**unified**	l.	combining parts into whole
13.	**three-dimensional**	m.	a plane figure with four equal sides and four right angles

V. Change the following sentences from Active into Passive Voice or from Passive into Active Voice:

1. In the result of everyday practical procedures general rules were introduced concerning areas, volumes and relationships of various objects.
2. Thales of Miletus was considered to be one of the “seven wise men” of antiquity.
3. Pythagoras continued the systematization of geometry.

4. In the Middle Ages the people of the East developed our present numerical system.
5. In the early period of Renaissance in Europe new elements of geometrical theory were created.
6. The seventeenth century was marked with the invention of the calculus.
7. Geometry received a further generalization in the 20th century through set theory.

VI. Give the corresponding adjectives of the nouns given below:

rectangle, triangle, system, synthesis, geometry, science, cube

VII. Are the statements true or false? Contradict the statements which are not true:

1. The notions of square, rectangle, triangle, curve, solid and surface are the first geometrical concepts.
2. Demonstrative geometry was the first stage of geometry.
3. Euclid and Archimedes started subconscious geometry in ancient Egypt.
4. Curves and surfaces were studied by means of calculus.
5. Set theory was invented in the 20th century.

VIII. Choose the suitable preposition:

1. Plane geometry deals … (with/to) the analysis of sets … (in/of) numbers.
2. Greek geometry is concerned … (at/with) empirical conclusions derived (from/about) experience.
3. The Greeks changed the character … (of/in) mathematics (from/to) deductive and abstract system … (into/up) empirical science.

4. Euclid's "Elements" is a textbook … (up/on) geometry and his way … (about/of) reasoning is analogy.

IX. Match the beginnings with their appropriate endings:

1.	The first considerations of geometry originated …	a.	… may also be called subconscious geometry.
2.	The first stage of geometry …	b.	… by means of their projection.
3.	The empirical conclusions should be worked out …	c.	… from the human ability to recognize physical forms.
4.	The notions of square, rectangle, triangle and curve …	d.	… by logical or deductive reasoning.
5.	Three-dimensional objects are analyzed …	e.	… are the first geometrical concepts.

X. Read the following passage and divide it into four logical paragraphs:

An important aspect of geometry as a deductive system is that the conclusions which may be drawn are consequences of assumptions. The assumptions made for the geometry are essentially those made by Euclid. Euclid assumed that through a given point not on a given line there is no more than one parallel to the given line. In the nineteenth century the famous mathematicians Lobachevsky, Bolyai and Riemann developed non-Euclidean geometries. Lobachevsky and Bolyai assumed independently of one another that through a given point not on a given line there is more than one line parallel to the given line which led to the creation of non-Euclidean geometries. According to Euclidean geometry parallelograms and rectangles exist;

according to the geometries of Lobachevsky and Bolyai parallelograms exist but rectangles do not; according to the geometry of Riemann neither parallelograms nor rectangles exist. The conclusions of non-Euclidean geometry are just as valid as those of Euclidean geometry, even though the conclusions of non-Euclidean geometry contradict those of Euclidean geometry. This paradoxial situation becomes intuitively clear when one realizes that any deductive system begins with undefined terms. Although the mathematician forms intuitive images of the concepts to which the undefined terms refer, these images are not logical necessities, and, the reason for forming these intuitive images is only to help our reasoning within a certain deductive system. Thus, the intuitive images corresponding to the undefined terms straight line and plane are not the same for Euclidean and non-Euclidean geometries. For example, the plane of Euclid is a flat surface; the plane of Lobachevsky is a saddle-shaped or pseudo-spherical surface; the plane of Riemann is an ellipsoidal or spherical surface.

XI. Translate the following problems and solve them:

a) *A* line segment has the endpoints $B(-7,-14)$ and $C(19,3)$. Find the coordinates of its midpoint *M*.

b) Find the distance between the points $(6,2)$ and $(9,7)$. Write your answer as a whole number or a fully simplified radical expression. Do not round it.

XII. Speak on the key points of the text: "Geometry through the Ages".

Lesson 6

ALGEBRA AS A SCIENCE

Arithmetic! Algebra! Geometry!
Grandiose trinity! Luminous triangle!
Whoever has not known you is without sense.

COMTE DE LAUTREAMONT[20]

Algebra in the broad sense in which the term is used today, deals with operations upon symbolic forms. In this capacity, it not only penetrates the whole of mathematics but spreads over the domain of formal logic.

Algebra is a Latin variant of the Arabic word al-jabr as employed in the title of a book, “Hisab al-jabr w’al al-mugabalah”, written in Bagdad about A.D. 825 by the Arab mathematician Mohammed ibn-Musa al-Khowarismi. This treatise on algebra is commonly referred to, in shortened form, as al-jabr. The best translation of the book’s full title is “the science of equations”.

Just as the discovery of zero created the arithmetic of today, so did the literal notation usher in a new era in the history of algebra. Wherein lies the power of this symbolism?

First of all, the letter liberated algebra from the slavery of the word. This is important enough; but what is still more important is that the letter is free from the taboos which have attached to words through centuries of use. In the second place, the letter is susceptible of operations which enable one to transform literal expressions and thus to paraphrase any statement into a number of equivalent forms. But the most important contribution of symbolism is the role it has been playing in the formation of the generalized number concept.

Modern symbolism began to emerge around 1500. A banner year was 1545: in that year Girolamo Cardano, an Italian scholar, published his "Ars magna" ("The Great Art") containing the solution of the cubic and the quartic. These solutions represented the first really new material since antiquity, even though these essentially general solutions were achieved by "ingenious devices" rather than advances in insight and theory.

The watershed of algebraic thought separating the early shallow flow of "manipulative solution of equations" from the deeper modern stream which began with the theoretical properties of equations is personified in the Frenchman Francois Viete, who was the first, in his "Logistica speciosa" ("Calculation with Types") to introduce letters as general positive coefficients and to put some other finishing touches to symbolism, which was finally up-to-date by the time of Isaac Newton. Later Descartes used the first letters of the alphabet for given quantities and the last letters for the unknowns. The introduction and acceptance of negative, imaginary and complex numbers contributed to the development of modern algebraic notations. Niels Henrik Abel (1824) and especially Évariste Galois (1831) introduced the idea of a group in their independent proofs that a polynomial equation of degree greater than four has no general algebraic solution. It should be mentioned that the group concept didn't emerge suddenly with Abel and Gallois. In the works of the best mathematicians of the time an implicit grasp of the group concept was already to be found[21].

Mathematics is changing constantly, and algebra must reflect these changes if it wants to stay alive. This explains the fact that algebra is one of the most rapidly changing areas of mathematics: it is sensitive not only to what happens inside its own boundaries, but also to the trends which originate in all other branches of mathematics.

The most important new demands on algebra come from topology, analysis and algebraic geometry.[22]

TOPICAL VOCABULARY

personify	[pq'sOnifai]	v.	անձնավորել, մարմնավորել
attach	[q'txC]	v.	կապել, կցել
transform	[trxns'fLm]	v.	փոխակերպել, ձևափոխել
susceptible	[sq'septqbl]	a.	1. տպավորվող, զգայուն 2. ենթակա
literal	['litqrql]	a.	տառային
usher in	['ASqin]	v.	հայտարարել, ազդարարել
grasp	[gra:sp]	v.	հասկանալ, ըմբռնել, յուրացնել
slavery	['sleivqri]	n.	ստրկություն
ingenious	[in'GJnjqs]	a.	հնարագետ, ճարտարամիտ, սրամիտ

I. What is the Armenian for?

capacity, to penetrate, outset, to liberate, to paraphrase, literal notation, to generalize, a theory susceptible of proof, to put finishing touches, to be free from taboos

II. What is the English for?

ազատել, համարժեք, ազդարարել նոր դարաշրջան, ժամանակակից ուղղություն, անձնավորել, լայն իմաստով, թվային հասկացություն, կատարել վերջնական շտկումները, ապացուցման ենթակա տեսություն, մաթեմատիկայում առաջացած տենդենցներ

III. a) Arrange the words in pairs similar in meaning:

1.	**susceptible**	a.	meaning
2.	**boundary**	b.	understand
3.	**ingenious**	c.	range
4.	**attach**	d.	sign
5.	**domain**	e.	sensitive
6.	**grasp**	f.	connect
7.	**trend**	g.	limit
8.	**symbol**	h.	clever, creative
9.	**sense**	i.	tendency

b) Arrange the words in pairs opposite in meaning:

1.	**inside**	a.	dead
2.	**advantage**	b.	real
3.	**implicit**	c.	out of date
4.	**emerge**	d.	outside
5.	**logical**	e.	disadvantage
6.	**up-to-date**	f.	disappear
7.	**imaginary**	g.	explicit
8.	**alive**	h.	illogical

IV. Give derivatives of the following words:

symbol, to operate, slavery, solution, to reflect, to originate, to contribute, dependent, to attach, logic, to personify

V. Match the following words with their definitions:

1.	**logic**	a.	a set of symbols; symbolic meaning
2.	**analysis**	b.	the same in magnitude, meaning, effect etc.
3.	**domain**	c.	the science of correct reasoning
4.	**algebra**	d.	expression, containing powers and coefficients of x
5.	**introduction**	e.	the act of solving a problem, a question etc.
6.	**solution**	f.	that which introduces as a preface
7.	**polynomial**	g.	the branch of mathematics concerned with generalizing the arithmetic operations and analyzing equations
8.	**equivalent**	h.	determination of causes from results, induction
9.	**symbolism**	i.	sphere of action or knowledge

VI. Choose the suitable word:

1. Algebra is concerned with operations upon symbolic … (*forms / structures*).
2. The literal notation … a new era in the history of algebra. (*ushered / accomplished*).
3. The letter … algebra form the slavery of the word. (*liberated / restricted*).
4. Francois Viete put … touches to the symbolism. (*preliminary / finishing*).
5. Girolamo Cardano, an Iatalian scholar, gave the solution of … in his treatise “Ars magna”. (*the cubic and the quartic / the quintic and the sixtic*).
6. Mathematics is changing constantly and algebra must … these changes. (*reflect / distort*).

VII. Are these statements true or false? Correct the false statements:

1. Algebra deals with operations upon symbolic forms.
2. Algebra pervades the whole of mathematics.
3. The word “algebra” is derived from Greek.
4. The literal notation used in algebra hinders its progress.
5. Modern symbolism appeared in the 16^{th} century.
6. Francois Viete was the first to introduce letters as general positive coefficients.
7. Descartes never used algebraic symbols in his works.
8. The introduction of negative, imaginary and complex numbers contributed to the development of modern algebraic notation.
9. Niels Henrik Abel and Evariste Galois introduced the idea of the theory of games.
10. Algebra is developing separately never being influenced by any field of mathematics.
11. The most important new demands on algebra come from topology, analysis and algebraic geometry.

VIII. Insert prepositions: *with, of, to, into, for, from.*

Natural science is concerned … collecting facts and organizing these facts … a coherent body … knowledge so that one can understand nature. Originally much … science was concerned … observation, the collection … information and its classification. This classification gradually led … the formation … various “theories” that helped the investigators to remember the individual facts and to be able to explain and sometimes predict natural phenomena. The ultimate aim … most scientists is to be able to organize their science … a coherent collection … general principles and theories so that these principles will enable them both to understand nature and to make predictions … the outcome … future experiments. Thus they want to be able to develop a system … general principles (or axioms) … their science that will enable them to deduce the individual facts and consequences … these general laws.

IX. Speak on the key points of the text "Algebra as a Science".

X. Write questions to the text and answer the questions in writing:

THE PYTHAGOREAN THEOREM

Fundamental reforms in mathematics are universally ascribed to Pythagoras. Pythagoras was the first to construct geometry as a deductive science. The Pythagorean theorem is one of the most important propositions in the entire realm of geometry. It was taken as a principle in defining all matric spaces for a long period of time.

Despite the strong Greek tradition that associates the name of Pythagoras with the statement that "the square of the hypotenuse of the right-angled triangle is equal to the sum of the squares of the sides containing the right angle", there is no doubt that this result was known prior to the time of Pythagoras.

It is possible that Pythagoras gave the proof of the theorem based on the proportionality of similar figures. With the later realization that all segments are not necessarily commensurable, this proof became invalid. Thus, at the time of Euclid's "Elements" there was no need for a more adequate proof. Euclid's Proposition 1,47 is the Pythagorean theorem, with a proof universally credited to Euclid himself. Proclus's speculation was simply that Euclid rewrote the proof in order that he might put the proposition in his first book to complete it. There is also considerable evidence that the first book was written to lead to the climax of this theorem and its converse.

In 1907 L.S. Loomis published his book "The Pythagorean Proposition", a work that contained 370 proofs of this theorem. Probably no other theorem in mathematics can be demonstrated by such a wide variety of algebraic and geometric proofs. The Pythagorean theorem and the proof are so important in mathematics that Loomis writes in his book: "I noticed two or three American texts on Geometry in which Euclid's proof of the Pythagorean theorem does not appear. The leaving out of Euclid's proof is like the play of "Hamlet" with Hamlet left out".[23]

XI. Speak on Pythagoras's life and proofs of his famous theorem.

XII. Discuss the following mathematical statements:

A (2×2 real) matrix A is

$$A = \begin{bmatrix} a & c \\ b & d \end{bmatrix},$$

where $a,b,c,d \in \mathbb{R}$. If $$B = \begin{bmatrix} w & y \\ x & z \end{bmatrix},$$

then the ***product*** AB is defined by

$$AB = \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} w & y \\ x & z \end{bmatrix} = \begin{bmatrix} aw+cx & ay+cz \\ bw+dx & by+dz \end{bmatrix}.$$

The elements a, b, c, d are called the ***entries*** of A. Call (a, c) the first row of A and call (b, d) the second row; call (a, b) the first ***column*** of A and call the (c, d) second column. Thus, each entry of the product AB is a dot product of a row of A with a column of B. The ***determinant*** of A, denoted by $\det(A)$, is the number $ad - bc$, and a matrix A is called ***nonsingular*** if $\det(A) \neq 0$. The reader may calculate that

$$\det(AB) = \det(A)\det(B),$$

from which it follows that the product of nonsingular matrices is itself nonsingular. The set $GL(2, \mathbb{R})$, of all nonsingular matrices with operation matrix multiplication, is a (nonabelian) group, called the ***general linear group*** the identity is the identity matrix*.

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

and the inverse of a nonsingular matrix A is

$$A^{-1} = \begin{bmatrix} d/\Delta & -c/\Delta \\ -b/\Delta & a/\Delta \end{bmatrix}.$$

where $\Delta = ad - bc = \det(A)$.

* The word matrix (derived from the word meaning "mother") means "womb" in Latin; more generally, it means something that contains the essence of a thing.

Lesson 7

THE THEORY OF EQUATIONS

The most remarkable unsolved problem in algebra was the problem of solving equations of degree 5 $(ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0)$ and higher, which led to the new way of thinking.

Attempts to find solutions to this problem were made from the 16th century until early in the 19th century without success. The reason for this failure became evident in 1824 when N.H. Abel, a brilliant young Norwegian mathematician, proved, at the age of 22, that it is not possible to write the roots of the general equation of degree higher than 4 as algebraic expressions in terms of the coefficients. Although Abel succeeded in showing that for **n** greater than 4 the general polynomial equation could not be solved algebraically, he did not claim to have completely achieved the objective he set for himself:

1. To find all the equations of any given degree which are solvable algebraically.
2. To determine whether a given equation is or is not solvable algebraically.

It was fortunate, that Abel's proof, in which he used permutation groups to some extent, received early publication. This proof caught the imagination of Galois, who gave complete answers to the questions proposed by Abel. In 1831 Galois showed that a polynomial equation is solvable if and only if its group over the coefficient field is solvable. The concepts associated with this result are usually characterized as Galois's theory. In his work he used the idea of isomorphic groups, and was the first to demonstrate the importance of invariant (or normal) subgroups and factor groups.

The term "group" is due to Galois.

Although Galois's accomplishments were mathematical landmarks of the greatest significance and originality they did not immediately make their full impact on his contemporaries, because these men were slow to understand, appreciate and publish Galois's work. In fact, because of various circumstances he received no recognition for his work while he lived. He was killed in a duel in 1832 at the age of 21.

Ironically, his young contemporary Abel also had a promising career cut short. He died in poverty of tuberculosis at the age of 26, although his legacy lives on in the term "abelian" (usually written with a small "a"), which has since become commonplace in discussing concepts such as the abelian group, abelian category and abelian variety.

So Abel and Galois proved in entirely different ways that there cannot be any general formulas for solving polynomial equations of degree higher than 4. At least there can be no formulas which give the solutions in terms of the coefficients and which involve only addition, subtraction, multiplication, division and the extraction of roots.

After Galois's death the development of group theory was substantially advanced by Cauchy. In 1854 Arthur Cayley published an article entitled "On the theory of Groups as Depending on the Symbolic Equation, $0^n = 1^n$, which is noteworthy because it gives what is probably the earliest definition of a finite abstract group. It also gives the result now known as Cayley's theorem, that "every finite group is isomorphic to a regular permutation group".

In 1870 Camille Jordan published his "Traité des substitutions et des équations algebriques" ("Treatise on Substitutions and Algebraic Equations") – a masterly presentation of permutation groups – covering the results of Lagrange, Abel, Galois, Cauchy as well as his own contributions to the subject. In the same year Leopold Kronecker gave a set of axioms defining finite Abelian groups. Out of this complete axiomatic system for finite abelian groups Kronecker still working with completely arbitrary, abstract set of elements, derived the customary group properties such as the existence of the unity element for the set, inverses, etc.[24]

TOPICAL VOCABULARY

claim	[kleim]	v.	պահանջել
accomplishment	[q'kOmpliSmqnt]	n.	1. ավարտելը 2. նվաճում
accomplish	[q'kOmpliS]	v.	ավարտել
landmark	[lxndma:k]	n.	ուղենիշ, շրջադար-ձային կետ
substantial	[sqb'stxnSql]	a.	էական, կարևոր
noteworthy	['nout"wWDi]	a.	ուշադրության արժանի
impact	['impxkt]	n.	ազդեցություն
impart	[im'pa:t]	v.	հաղորդել (նորու-թյուն, գիտելիք)

I. What is the Armenian for?

to achieve the objective, a permutation group, a landmark, noteworthy, to claim, in terms of, inverse

II. What is the English for?

բնութագրել, արմատ հանել, կամայական, ներդրում, գնահատել գիտական աշխատությունը, կյանքի օրոք ճանաչում չգտնել, վեր-ջավոր, վերացական խումբ

III. a) Arrange the words in pairs similar in meaning:

1.	**objective**	a.	resolve
2.	**extent**	b.	appear
3.	**recognition**	c.	achievement
4.	**noteworthy**	d.	degree
5.	**accomplishment**	e.	goal, aim
6.	**original**	f.	appreciation
7.	**arise**	g.	turning point
8.	**solve**	h.	exceptional
9.	**landmark**	i.	creative

b) Arrange the words in pairs opposite in meaning:

1.	**prove**	a.	monomial
2.	**regular**	b.	infinite
3.	**failure**	c.	particular, limited
4.	**appreciate**	d.	unhappy
5.	**complete**	e.	unskillful
6.	**masterly**	f.	incomplete
7.	**fortunate**	g.	depreciate
8.	**general**	h.	prosperity
9.	**polynomial**	i.	irregular
10.	**finite**	j.	disprove

IV. Match the following words with their definitions:

1.	**succeed**	a.	creative, inventive
2.	**original**	b.	accomplish what is attempted; terminate usually well
3.	**group**	c.	an essential attribute or quality
4.	**axiom**	d.	of the same age
5.	**contemporary**	e.	a set of elements satisfying some axioms
6.	**property**	f.	a proposition deemed to be self-evident and assumed without proof

V. Give derivatives of the following words:

to achieve, master, axiom, to equate, similar, complex, finite, general, evident, solvable, complete, line, variant, group, to associate, recognition, algebra, substance, to accomplish

VI. Answer the questions:

1. What was the most remarkable unsolved problem in algebra?
2. How long did mathematicians attempt to solve the problem?

3. What did Abel prove?
4. What questions did Abel raise?
5. Did Galois give complete answers to the questions proposed by Abel?
6. How can you characterize the role of Galois in the development of algebra?
7. Was Galois appreciated by his contemporaries?
8. What term was coined by Galois?
9. How did Abel and Galois die?
10. What was their contribution to algebra?
11. How did mathematicians develop theories created by Abel and Galois?

VII. Make the following sentences passive:

1. Analytic geometry founded an algebraic approach to geometry.
2. The mathematicians of the leading Greek schools proved the statements from axioms already accepted.
3. Mathematicians created a new term to designate the study of calculating the rates of change of slope and curvature.
4. Kepler introduced effective methods of working with the conic sections in astronomy.
5. The coordinates of any point that lies on the curve will satisfy the equation.
6. They abstracted notions from physical objects.
7. The mathematicians of the leading Greek schools proved the statements from axioms already accepted.
8. The stretched string gives the concept of the straight line.
9. They have already used facts known from the lectures.
10. These relations determine the new properties of this figure.

VIII. Translate the following sentences with inverted word order:

1. Perhaps nowhere have been achieved better results as in this field of mathematics.
2. Not until Cantor created his famous set theory could mathematicians resolve Zeno's paradoxes.
3. Nor should we underestimate this contribution of the Greeks.
4. Nowhere can one see such rapid progress as in engineering.
5. Should the problem of this kind be so absurd, the men of science would hardly have taken so deep an interest in its solution.
6. Only at a rather advanced stage of intellectual development does the abstract character of the idea of number become clear.
7. One does not indeed see electricity. Nor can one hear it, taste it or smell it.
8. Incomplete though these figures are they give more information in several respects than has before been available
9. Never did mathematicians worry much over the foundations of algebra.

IX. Comment on the text: "The Theory of Equations".

X. What is implied in the following statements and quotations?[25]

1. In mathematics all roads lead back to Greece.
2. "Mathematics is the queen of the sciences and Arithmetic – the queen of mathematics".

   (*C.F. Gauss*)
3. "The four rules of arithmetic may be regarded as the complete equipment of the mathematician".

   (*J.C. Maxwell*)
4. "One must always invert in mathematics".

   (*Jacobi*)
5. The world of mathematics is far richer in irrational numbers than it is in rational ones.

6. “God used beautiful mathematics in creating the world”.
(*Paul Dirac*)
7. “Pure mathematics is, in its way, the poetry of logical ideas”.
(*Albert Einstein*)
8. “Don’t worry about your difficulties in mathematics. I can assure you mine are still greater”.
(*Albert Einstein*)
9. “Without mathematics, there’s nothing you can do. Everything around you is mathematics. Everything around you is numbers”.
(*Shakuntala Devi*)
10. “Mathematics is the most beautiful and most powerful creation of the human spirit”.
(*Stefan Banach*)
11. “As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality”.
(*Albert Einstein*)
12. “Mathematics knows no races or geographic boundaries; for mathematics the cultural world is one country”.
(*David Hilbert*)

XI. Contradict the false statements. Give your reasoning:

1. A ray is a subset of a point.
2. A straight line extends infinitely only in one direction.
3. A polygon with exactly four sides is a polyhedron.
4. We call decimals in which one digit or one group of digits repeats over and over nonterminating decimals.
5. We find the area directly by laying out little squares over the entire floor of the room.
6. There exists no formula to calculate the volume of a cube.
7. We underestimate the contribution of the ancient Greek mathematicians.

8. A logical system must define every concept it uses.
9. Plane geometry deals with the analysis of sets of numbers.
10. There exist no geometries, excepting that of Euclid's.

XII. Prove:

- Prove that the cube of any integer can be written as the difference of two squares

[Hint: Notice that

$$n^3 = \left(1^3 + 2^3 + \cdots + n^3\right) - \left(1^3 + 2^3 + \cdots + (n-1)^3\right).]$$

- Prove that $n! > n^2$ for every integer $n \geq 4$, whereas $n! > n^3$ for every integer $n \geq 6$.

- (a) Find the values of $n \leq 7$ for which $n!+1$ is a perfect square (it is unknown whether $n!+1$ is a square for any $n > 7$).

  (b) True or false? For positive integers m and n,

$$(mn)! = m!n! \text{ and } (m+n)! = m!+n!.$$

Lesson 8

AMALIE EMMY NOETHER

"Mathematicians are born, not made."
HENRI POINCARÉ[26]

Amalie Emmy Noether described by Albert Einstein, David Hilbert and others as "the most important woman in mathematical history, since the higher education of women began" was born on March 23, 1882 in Germany, Bavaria. She was born to a Jewish family in the Bavarian town of Erlangen. Her father was a mathematics professor at the University of Erlangen and her mother was from a wealthy family. As a girl, Emmy Noether was well-liked. She did not stand out academically although she was known for being clever and friendly. Emmy was near-sighted and talked with a minor lisp during childhood. A family friend recounted a story years later about young Emmy quickly solving a brain teaser at a children's party, showing logical acumen at that early age. Emmy was taught to cook and clean – as were most girls of the time – and she took piano lessons. She pursued none of these activities with passion, although she loved to dance. Emmy originally planned to teach French and English after passing the required examinations, but instead studied mathematics at the University of Erlangen. After completing her dissertation in 1907, she worked at the Mathematical Institute of Erlangen without payment for seven years. In 1915 she was invited by David Hilbert and Felix Klein to join the mathematics department at the University of Gòttingen, a world-renowned center of mathematical research. She pursued important mathematical work that confirmed key parts of the general theory of relativity. The philosophical faculty objected, however, and she spent four years lecturing under Hilbert's name. Hilbert continued to work to get Noether accepted as a faculty member at Gòttingen, but he was unsuccessful against the cultural and official biases against women scholars. He was able to allow her to lecture – in his own courses, and without salary. In 1919 she won the right to be a privatdozent – she could teach students, and they would pay her directly, but the university did not pay her anything. In 1922, the University gave her a position as an adjunct professor with a small salary and no tenure or benefits.

Emmy Noether was a popular teacher with the students. She was seen as warm and enthusiastic. Her lectures were participatory, demanding that students help work out the mathematics being studied.[27] Her habilitation was approved in 1919, allowing her to obtain the rank of privatdozent. Noether remained a leading member of the Gòttingen mathematics department until 1933; her students were sometimes called the "Noether boys". In 1924, Dutch mathematician B. L. van der Waerden joined her circle and soon became the leading expositor of Noether's ideas: her work was the foundation for the second volume of his influential 1931 textbook, Modern Algebra. By the time of her plenary address at the 1932 International Congress of Mathematicians in Zurich, her algebraic acumen was recognized around the world. The following year, Germany's Nazi government dismissed Jews from university positions, and Noether moved to the United States to take up a position at Bryn Mawr College in Pennsylvania. In 1935 she underwent surgery for an ovarian cyst and, despite the signs of recovery, died four days later at the age of 53.[28]

Noether's mathematical work has been divided into three "epochs". In the first epoch (1908–1919), she made significant contributions to the theories of algebraic invariants and number fields. Her work on differential invariants in the calculus of variations, Noether's theorem, has been called "one of the most important mathematical theorems ever proved in guiding the development of modern physics”. In the second epoch (1920–1926), she began work that “changed the face of algebra”. In her classic paper *Theory of Ideals in Ring Domains*, 1921 Noether developed the theory of ideals in commutative rings into a powerful tool with wide-ranging applications. She made elegant use of the ascending chain condition, and objects satisfying it are named Noetherian in her honor. In the third epoch (1927–1935), she published major works on noncommutative algebras and hypercomplex numbers and united the representation theory of groups with the theory of modules and ideals. She sometimes allowed her colleagues and students to receive credit for her ideas, helping them develop their careers at the expense of her own. Several of her colleagues attended her lectures, and she allowed some of her ideas, such as the crossed product of associative algebras, to be published by others. In addition to her own publications, Noether was generous with her ideas and is credited with several lines of research published by other mathematicians, even in fields far removed from her main work, such as algebraic topology.[29]

Amalie Emmy Noether was an influential mathematician known for her groundbreaking contributions to abstract algebra and theoretical physics. She revolutionized the theories of rings, fields, and algebras. In physics, Noether’s theorem explains the fundamental connection between symmetry and conservation laws.

She was one of the famous women mathematicians who proved her mathematical genius and undoubtedly, will always find a place in the pages of history. After World War II ended, the University of Erlangen honored her memory, and in that city a coed gymnasium specializing in math was named for her. Her ashes are buried near Bryn Mawr’s Library. At an exhibition at the 1964 World’s Fair devoted to Modern Mathematicians, Noether was the only woman represented among the notable mathematicians of the modern world.

The crater Nòther on the far side of the Moon is named after her. Also the 7001 Noether asteroid is named for Emmy Noether.[30][31]

🕮 TOPICAL VOCABULARY

near-sighted	[niq'saitid]	a.	կարճատես
lisp	[lisp]	n.	1. շվշվախոսություն, սվսվախոսություն 2.խշշոց, շրշյուն
recount	['rikaunt]	v.	1. պատմել 2. շարադրել
teaser	['ti:zq]	n.	դժվար հանելուկ, գլուխկոտրուկ
acumen	[q'kju:men]	n.	խորաթափանցություն
pursue	[pq'sju:]	v.	1. զբաղվել (մի բանով) 2. շարունակել քննարկումը 3. հավաքել
habilitation	[hq"bili'teiS(q)n]	n.	դոկտորական աշխատանքին համարժեք աշխատանք Եվրոպայում
bias	['baiqs]	n.	կողմնակալություն, սուբյեկտիվ մոտեցում, հակվածություն, բացասական տրամադրվածություն
scholar	['skOlq]	n.	գիտնական, գիտուն, սովորող
tenure	['tenjuq]	n.	մշտական աշխատանք, աշխատատեղ
adjunct	[' x GAnkt]	n.	օգնական
expositor	[eks'pOzitq]	n.	բացատրող
plenary address			*պլենար զեկույց, հիմնական զեկույց*
recovery	[ri'kAvqri]	n.	ապաքինում,

			առողջացում
epoch	['i:pOk]	n.	1. դարաշրջան 2. ժամանակաշրջան
invariant	[in'vFqriqnt]	n.	ինվարիանտ
domain	[d'Omein]	n.	1.տիրույթ 2. բնագավառ, ասպարեզ
commutative	[kq'mju:tqtiv]	a.	տեղափոխական, կոմուտատիվ
ascending	[q'sendiN]	a.	աճող
associative	[q"sousi'qtiv]	a.	զուգորդական, զուգակցվող
groundbreaking	['graund'breikiN]	a.	ցնցող, գյուտարարական, նորարական
exhibition	["eksi'biSn]	n.	ցուցահանդես, ցուցադրում
devoted (to)	[di'voutid]	a.	նվիրված
asteroid	['xstqrOid]	n.	փոքրիկ մոլորակ. աստերոիդ
coed	['kou'ed]	n.	երկսեռ գիմնազիա
crater	['kreitq]	n.	խառնարան (հրաբխի)

I. What's the Armenian for?

in the fields far removed from her main work, develop their careers at the expense of her own, hypercomplex numbers, despite the signs of recovery, in addition to her publications, notable mathematicians of the modern world, algebraic invariants, known for her groundbreaking contribution, the crossed product of associative algebras, Dutch mathematician

II. What’s the English for?

աշխատել անվճար, ենթարկվեց վիրահատության, մաթեմատիկա-կան հետազոտությունների աշխարհահռչակ կենտրոն, կարճատես, ազդեցիկ մաթեմատիկոս, կոչվում է նրա անունով, այդ վաղ տարիքում, արմատական կապ, նրա պատվին, տեսական ֆիզիկա, մոդուլների տեսություն, մաթեմատիկական հանճար, պաշտոն զբաղեցնել

III. a) Arrange the words in pairs similar in meaning:

1.	**in addition to**	a.	let, permit
2.	**receive**	b.	complete
3.	**allow**	c.	famous
4.	**important**	d.	dedicated to
5.	**finish**	e.	far
6.	**devoted to**	f.	operation
7.	**known**	g.	significant
8.	**remoted**	h.	besides
9.	**surgery**	i.	get

b) Arrange the words in pairs opposite in meaning:

1.	**major**	a.	low
2.	**far**	b.	learn, study
3.	**theoretical**	c.	minor
4.	**powerful**	d.	practical
5.	**near-sighted**	e.	disapprove
6.	**high**	f.	assymetry
7.	**ascending**	g.	near
8.	**teach**	h.	far-sighted
9.	**quickly**	i.	slowly
10.	**approve**	j.	powerless
11.	**symmetry**	k.	descending

IV. Match the following words with their definitions:

1.	**acumen**	a.	one of the very small planets that move around the sun between Mars and Jupiter
2.	**hypercomplex**	b.	be opposed to; make a protest against
3.	**symmetry**	c.	sharpness of judgment; ability to understand clearly
4.	**asteroid**	d.	standard or unit of measurement as used in building
5.	**dissertation**	e.	composed of a number of imaginaries or complex quantities
6.	**to object**	f.	unchanged by specified mathematical or physical operations or transformations
7.	**invariant**	g.	quality of harmony or balance between the parts
8.	**module**	h.	long written account submitted for a higher university degree

V. Find the corresponding adjectives from the text:

academy, logics, influence, fundament, continue, lead, associate, fame, ascend, theory

VI. Put in definite or indefinite articles where necessary:

In … spring of 1915, Noether was invited to return to … University of Gӧttingen by David Hilbert and Felix Klein. Their effort to recruit her, however, was blocked by … philologists and historians among … philosophical faculty: … women, they insisted, should not become *privatdozent*. One faculty member protested: “What will our soldiers think when they return to … university and find that they are required to learn at … feet of … woman?" Hilbert responded with indignation, stating, "I do not see that … sex of … candidate is … argument against her admission as *privatdozent*. After all, we are … university, not … bath house."

VII. Match the beginnings with their appropriate endings:

1.	As a girl, Emmy Noether …	a.	… objects satisfying it are named Noetherian in her honor.
2.	The crater Nȍther on the far side of the …	b.	… with a minor lisp during childhood.
3.	Emmy Noether made elegant use of the ascending chain condition, and …	c.	… was well-liked.
4.	Her habilitation was approved in 1919 …	d.	… Moon is named after her.
5.	Emmy was near-sighted and talked …	e.	… allowing her to obtain the rank of privatdozent.

VIII. Fill in the blanks with prepositions if necessary:

Although Noether's theorem had a profound effect … physics, … mathematicians she is best remembered … her seminal contributions … abstract algebra. As Nathan Jacobson says … his Introduction … Noether's Collected Papers, "The development … abstract algebra, which is one … the most distinctive innovations … twentieth century mathematics, is largely due … her – … published papers, … lectures, and … personal influence … her contemporaries."

… addition … her mathematical insight, Noether was respected … her consideration … others. Although she sometimes acted rudely … those who disagreed … her, she nevertheless gained a reputation …

constant helpfulness and patient guidance … new students. Her loyalty … mathematical precision caused one colleague to name her "a severe critic," but she combined this demand … accuracy … a nurturing attitude. A colleague later described her this way: "Completely unegotistical and free … vanity, she never claimed anything … herself, but promoted the works … her students above all."

IX. Complete the following sentences using the words given below:

devotion, results, local, published, clarify, state, basis, spontaneous, attended.

Emmy Noether did not follow a lesson plan for her lectures, she used her lectures as a … discussion time with her students, to think through and … important cutting-edge problems in mathematics. Some of her most important … were developed in these lectures, and the lecture notes of her students formed the … for several important textbooks. Several of her colleagues … her lectures, and she allowed some of her ideas, such as the crossed product of associative algebras, to be … by others.

Noether showed a … to her subject and her students that extended beyond the academic day. Once, when the building was closed for a … holiday, she gathered the class on the steps outside, led them through the woods, and lectured at a … coffee house.

X. Answer the following questions:

1. When and where was Emmy Noether born?
2. What was Einstein's and others' opinion of Noether?
3. What do we know about Emmy's childhood?
4. Why did she move to the United States?
5. What surgery did Noether undergo in 1935?
6. What was the result of the surgery?
7. When did Noether die?

8. What contribution did Amalie Noether make to mathematics and physics?

XI. Translate into Armenian:

Academician Mkhitar Jrbashian

Mkhitar Jrbashian was a notable Armenian mathematician, who made significant contributions to the constructive theory of functions and fundamental contribution to the classical theory of univalent analytic functions. Mkhitar Jrbashian created some well-known mathematical theories.

He was born in Yerevan on September 11, 1918 in a family of refugees from the town Van of Western Armenia escaping from the Armenian Genocide of 1915 in Turkey.

Being deprived of continuing his secondary education in Yerevan, he accomplished his school education in Tiflis in 1936, thanks to the assistance of his uncle's family. Back in Yerevan after Stalin's declaration that "children are not responsible for their parents", Mkhitar Jrbashian could enroll in Yerevan State University, where his scientific activities were encouraged by Prof. Artashes Shahinian, talented teacher who directed his pupils mainly to *Approximation Theory*.

He was greatly influenced by the results of Rolf Nevanlinna in *Complex Analysis*, which he studied attending lectures delivered by Mstislav Keldysh at Yerevan State University. This directed his scientific research to the field of *Complex Analysis.*

Under the supervision of Prof. Artashes Shahinian, he was the first to defend a Candidate of Sciences Thesis in Mathematics at YSU in 1945. In 1949 Mkhitar Jrbashian defended his Doctor of Science Thesis in Moscow State University.

Being the leading figure in the Mathematics of Armenia, a Full Member of Armenian Academy of Sciences from 1956, he did everything possible for the development of Armenian Mathematical

School to the the high international standards in many branches of mathematics. He was the founder and Director of Institute of Mathematics of National Academy of Sciences of Armenia (1971-1989), then the Honorary Director of the same institute up to his death on May 6, 1994 of a heart attack. He was the founder of Izvestiya Natsionalnoi Akademii Nauk Armenii, Matematika and its Editor in Chief (1971-1994), the Dean of the Physical-Mathematical and then Mechanical-Mathematical Department of YSU (1957-1960), and the Head of the Chair of Function Theory (1978-1986).[32]

XII.

- **a) Prove the following statement:**

Use mathematical induction to prove that for every positive integer *n*, the sum of the first *n* positive integers is $\frac{n(n+1)}{2}$.

- **b) Solve the following problems:**

1. For all $n \geq 1$, prove the following by mathematical induction:

(a) $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{n^2} \leq 2-\frac{1}{n}$.

(b) $\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\cdots+\frac{n}{2^n}=2-\frac{n+2}{2^n}$.

2. Show that the expression $\frac{(2n)!}{2^n n!}$ is an integer for all $n \geq 0$.

XIII. Speak on the key points of the text "Amalie Emmy Noether".

Lesson 9

THE DIGIT THAT MEANS NOTHING

Numbers rule the Universe.
THE PYTHAGOREANS[33]

The invention of zero and our number system is one of greatest achievements of the human race without which the progress of science, industry and commerce could be impossible.

An essential contribution to modern technological development was the introduction of the zero to the mathematics of the Western World. The concept of symbolically representing “nothing” in a numerical system is considered to be a revolutionary invention.

Various people throughout the world have used systems of counting without having the zero. The classical Greeks assigned a different letter of their alphabet, to represent each number from 1 to 10 and each of the multiples of 10. Any number not represented by a single letter symbol was expressed by the sum of the value of several symbols. For example, the number 238 was indicated by writing the letter symbols for 200, 30, and 8 adjacent to each other.

The Romans used fewer symbols which represented a more limited number of integers, such as 1, 5, 10, 50, 100, 500 and 1000, and employed the additive principle to a greater degree. Thus, in writing the number 238, nine individual symbols would be required.

The nature of such systems makes them unsuited to mathematical maneuvering, so for computations the ancients employed the abacus, and written numerals were used merely to record the results.

The zero of modern civilization had its origins in India about 500 A. D. By 800 A.D. its use had been introduced to Baghdad, from where it was spread throughout the Moslem world. The zero, along with the rest of our “Arabic” numbers was known in Europe by the year 1,000 A. D., but because of the strong tradition of Roman numerals, there was considerable resistance to its adoption. It was not until the late 14^{th} century that the zero was in general use in Western Europe.

Including the Hindu it appears that the concept of the zero, with its idea of positional value, was independently arrived at in three cultures which were widely separated in space and time.

A striking note about the Hindu zero is that, unlike the Babylonian and Mayan zero, the Hindu zero symbol came to be understood as meaning “nothing”. This is probably because of the use of number words that preceded the symbolic zero.

About 500 B.C., the Babylonians began to use a symbol to represent a vacant space in their positional value numbers. Babylonian mathematicians used the separator (effectively the first zero) in the middle position only. The person doing the calculation knew what order of magnitude he was working with and didn’t add any separators at the end of his notations. This allowed them to note fractional degrees and minutes of arc and made their computations more accurate. Despite the invention of zero as a placeholder, the Babylonians never quite discovered zero as a number. On an accounting tablet recording the distribution of grain there is a notation at the end of a column of numbers that reads “The grain is exhausted”. Another example from the same era is a description subtracting 20 from 20: “twenty minus twenty … you see”. However, before the idea could be disseminated to other areas, its use apparently died out about 2000 years ago along with the culture that gave it birth.

The Mayas of Central America began using a zero about the beginning of the Christian era. They were highly skilled mathematicians, astronomers, artists and architects. The Mayas had a very complex calendar system and needed a placeholder in their elaborate date system. Key to their invention of zero is the complicated Long Count calendar which measured time from the start of the Mayan civilization (August 12, 313 B.C.) and completes a full cycle on

December 21, 2012. Having been in possession of the zero for more than millennium longer than the Spaniards, in many aspects of mathematics the Mayas were further advanced than were their conquerors.

The benefits modern civilization derives from the use of the zero are incalculable and range from the practical to the theoretical.

Indeed, it might even be considered somewhat ironic that our culture, which has such a materialistic emphasis, should be so dependent on a symbol for nothingness.[34]

TOPICAL VOCABULARY

essential	[i'senSql]	a.	էական, հիմնական
contribution	["kOntri'bjHSn]	n.	ավանդ, ներդրում
achievement	[q'CJvmqnt]	n.	նվաճում
concept	['kOnsept]	n.	հասկացություն
introduce	["intrq'djHs]	v.	մտցնել, ներս տանել, ներմուծել
various	['vFqriqs]	a.	1. տարբեր 2. բազմազան
assign	[q'sain]	v.	1. նշանակել, որոշել (ժամկետ, սահման) 2.վերագրել
represent	["repri'zent]	v.	ներկայացնել, պատկերել
multiple	['mAltipl]	n.	բազմապատիկ
principle	['prinsqpl]	v.	սկզբունք
record	[ri'kLd]	v.	գրառել, արձանագրել, գրանցել

resistance	[ri'zistqns]	n.	դիմադրություն, դիմադրում
employ	[im'plOi]	v.	գործածել, օգտագործել, կիրառել
adoption	[q'dOpSn]	n.	1. որդեգրում 2. ընդունում, յուրացում
culture	['kAlCq]	n.	մշակույթ
disseminate	[di'semineit]	v.	տարածել (ասմունք, հայացքներ)
possession	[pq'zeSn]	n.	տիրակալություն, սեփականություն
elaborate	[i'lxb(q)rit]	a.	խնամքով մշակված
millennium	[mi'leniqm]	n.	հազարամյակ
Spaniard	['spxnjqd]	n.	իսպանացի
advanced	[qdva:nst]	a.	1. առաջ քաշված 2. առաջադեմ
benefit	['benifit]	n.	օգուտ, շահ
conqueror	['kONk(q)rq]	n.	հաղթող, նվաճող
incalculable	[in'kxlkjulqbl]	a.	անհաշվելի, անթիվ, անհամար
derive	[di'raiv]	v.	ծագել, սկիզբ առնել, սերվել

I. What is the Armenian for?

essential contribution, great intellectual achievement, limited number of integers, adjacent to each other, to employ additive principle, abacus, to spread throughout the Moslem world, considerable resistance, adoption of Roman numerals, to disseminate the idea to other areas, to be in possession of smth, to be further advanced, incalculable benefits

II. What is the English for?

ժամանակակից տեխնոլոգիական զարգացում, կարևոր ներդրում, մաթեմատիկական մանևրում, հազարամյակ, գործնական և տեսական կարևորություն, ծագել Հնդկաստանում, ներմուծվել Բաղդադ, օգուտ քաղել, արդյունքներ արձանագրել

III. a) Arrange the words in pairs similar in meaning:

1.	**complex**	a.	get
2.	**essential**	b.	divide
3.	**count**	c.	originate
4.	**advance**	d.	antique
5.	**benefit**	e.	spread
6.	**require**	f.	calculate
7.	**disseminate**	g.	important
8.	**derive**	h.	evident
9.	**employ**	i.	demand
10.	**acquire**	j.	progress
11.	**ancient**	k.	use
12.	**apparent**	l.	profit
13.	**separate**	m.	complicated

b) Arrange the words in pairs opposite in meaning:

1.	**general**	a.	uncertain
2.	**different**	b.	whole
3.	**easy**	c.	concrete
4.	**certain**	d.	difference
5.	**fractional**	e.	unreal
6.	**abstract**	f.	late
7.	**real**	g.	unlike
8.	**strong**	h.	indifferent
9.	**vacant**	i.	weak
10.	**above**	j.	ancient
11.	**early**	k.	particular
12.	**like**	l.	difficult
13.	**modern**	m.	occupied
14.	**sum**	n.	below

IV. Find the corresponding adjectives from the text and translate them into Armenian:

essence, technology, west, vary, number, intellect, difference, add, consider, vacancy, accuracy, position, advance, theory, practice, depend, materialism, calculate

V. Choose the suitable word:

1. The invention of zero is considered to be one of man's great intellectual … *(achievements / failures)*
2. The Romans used … principle in their calculation. *(exclusive / additive)*
3. The ancients employed … for computations. *(the calculator / the abacus)*
4. The zero of modern civilization … in India. *(originated / disappeared)*

5. The Mayas were in possession of the zero for more than … longer than the Spaniards. *(a millennium / a decade)*
6. The Mayas had a very … calendar system. *(simple / complex)*
7. The Babylonians used a symbol to represent … a space in their positional value numbers. *(occupied / vacant).*

VI. Match the following words with their definitions:

1.	**resistance**	a.	thousand years
2.	**civilization**	b.	lying near, close or contiguous
3.	**millennium**	c.	the art or power of resisting
4.	**adjacent**	d.	characteristic of present or recent times
5.	**abacus**	e.	advantage, profit
6.	**advance**	f.	having existed in the remote past
7.	**benefit**	g.	a member of a race formerly inhabiting Central America
8.	**Maya**	h.	a calculating device consisting of beads strung on rods in a frame
9.	**vacant**	i.	civilized peoples, states and facilities collectively
10.	**ancient**	j.	bring forward, promote
11.	**modern**	k.	having no contents, empty, devoid of something

VII. Are the following statements true or false? Correct the false statements:

1. The introduction of the zero to the mathematics of the Western World was an essential contribution to modern technological development.

2. The classical Greeks assigned a different letter of their alphabet to represent each number from 1 to 10 and each of the multiples of 10.
3. The Romans used symbols which represented a more limited number of integers such as 1, 5, 10, 50, 100, 500, 1000 and employed the additive principle to a greater degree.
4. The ancients didn't use the abacus for computations.
5. The zero of modern civilization originated in Greece.
6. The zero wasn't spread throughout the Moslem world.
7. Europe was aware of the symbol zero by 800 A.D.
8. The zero was never adopted in Europe because of the strong tradition of Roman numerals.
9. Neither the Babylonians nor the Mayas of Central America ever used the idea of zero.
10. It is ironic that our culture which has such a materialistic emphasis should be so dependent on a symbol for nothingness.
11. The benefits modern civilization derives from the use of the zero are miserable.

VIII. Insert prepositions: ***to, in, by, of, on, about, into, from.***

We now come … the numerals that are used … Europe and the Americas today, as well as … certain parts of Asia and Africa and regions such as Australasia which were settled … Europeans. First of all, it is necessary to understand that although our European and American numerals are often spoken … as Arabic, they have never been used … the Arabs. They came … us … means … a book … arithmetic which apparently was written … India … twelve hundred years ago, and was translated … Arabic soon afterward. … chance this book was carried … merchants … Europe and there was translated … Arabic … Latin. This was hundreds … years before books were first printed … Europe, and this arithmetic book was known only … manuscript form. Since it had been translated … Arabic, the numerals were supposed to be those used … the Arabs, but this was not the case. They might be called Hindu – Arabic, but since they took their present shapes … Europe they may better be called European or Modern numerals.

IX. Speak on the key points of the text: “The Digit that Means Nothing”.

X. Find a suitable title for the following text and summarize it:

Numbers are an indispensable part of our life. The clothes we wear, the houses we live in, and the food we eat, all would be different if people had not learned how to use numbers. All day long we either use numbers ourselves, or we use things that other people have made by using numbers.

It has taken thousands of years for people to learn how to use numbers or the written figures which we call “numerals”. For a long time after men began to be civilized such simple numbers as two and three were all they needed. For larger numbers they used words in their various languages which corresponded to such expressions of our own as “lots” of people, a “heap” of apples, and a “flock” of sheep. For example, a study of thirty Australian languages showed no number above four, and in many of these languages there were number names for only one and two, the larger numbers being expressed simply as “much” or “many”.[35]

The Latin word “digiti” means fingers. Because we have five fingers on each hand, people began, after many centuries to count by fives. Later they found it more convenient to count by tens using the fingers of both hands. Because we have ten toes as well as ten fingers, some people counted fingers and toes together and used a number scale of twenty. The French in early times counted by twenty. They used the word “vingt”. In the English language the word “score” meaning twenty was used. There are many evidences that twelve was often used as a scale in counting: as 12 inches=1 foot, 12 pence=1 shilling, 12 units=1 dozen, 12 lines=1 inch.[36]

The only arithmetical operation performed by early people was counting. Little by little they found out how to add, subtract, and multiply; but this was slow work and in some countries special devices were invented to make computation easier, especially in dealing with large numbers. The Romans used a counting table, or abacus, in which units, fives, tens, and so on were represented by beads which could be moved in grooves. They called these beads calculi, which is the plural of calculus, or pebble. We see here the origin of our word “calculate”. Since the syllable “calc” means lime, and marble is a kind of limestone, we see that a calculus was a small piece of marble, probably much like those used in playing marbles. Sometimes, as in the Chinese abacus described below, the calculi slid along on rods. This kind of abacus was called a suan-pan, and used in all parts of China. The Japanese used a similar instrument known as the soroban. In Russia there was employed a type of abacus known as the s’choty, a similar one was employed in Turkey (the coulba) and also in Armenia (the choreb).[37]

XI. Discuss the following text:

THE FIBONACCI SEQUENCE

Perhaps the greatest mathematician of the Middle Ages was Leonardo of Pisa, who wrote under the name of Fibonacci – a contraction of “filius Bonacci”, that is, Bonacci’s son.

It is ironic that despite his many achievements Fibonacci is remembered today mainly because the 19th century number theorist Eduard Lucas attached his name to a sequence that appears in a trivial problem in the Fibonacci’s work “The liber Abaci” (1202). The word “abaci” in the title does not refer to the abacus, but rather it means counting in general. Fibonacci posed the following problem dealing with the number of offspring generated by a pair of rabbits conjured up in the imagination:

A man put one pair of rabbits in a certain place entirely *surrounded by a wall. How many pairs of rabbits can be produced from that pair in a year, if the nature of these rabbits is such that every month each pair bears a new pair which from the second month on becomes productive?*

Assuming that none of the rabbits dies, then a pair is born during the first month, so that there are two pairs present. During the second month, the original pair has produced another pair. One month later, both the original pair and the firstborn pair have produced new pairs, so that three adult and two young pairs are present, and so on. The point to bear in mind is that each month the young pairs grow up and become adult pairs, making the new "adult" entry the previous one plus the previous "young" entry. Each of the pairs that was adult last month produces one young pair, so that the new "young" entry is equal to the previous "adult" entry.

When continued indefinitely, the sequence encountered in the rabbit problem

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377…

is called the *Fibonacci sequence* and its terms *Fibonacci numbers*.

➢ **Memorize the definition:**

The Fibonacci sequence $F_0, F_1, F_2 \ldots$ is defined as follows:

$$F_0 = 0,\ F_1 = 1 \text{ and } F_n = F_{n-1} + F_{n-2} \text{ for all integers } n \geq 2.$$

The Fibonacci sequence begins: 0, 1, 1, 2, 3, 5, 8, 13,…. There is an intimate connection between the Fibonacci sequence and the ***golden ratio*** $\alpha = \frac{1}{2}\left(1+\sqrt{5}\right)$ (which one finds when studying recurrence relations in general). Notice that α is a root of $x^2 - x - 1$ [as is $\beta = \frac{1}{2}\left(1-\sqrt{5}\right)$]. The ancient Greeks thought a rectangular figure most pleasing if its edges a and b were in the proportion.

$$a : b = b : a + b$$

It follows that $b^2 = a(a+b)$, so that $b^2 - ab - a^2 = 0$, and the quadratic formula gives $b = \frac{1}{2}\left(a \pm \sqrt{a^2 + 4a^2}\right) = a\frac{1}{2}\left(1 \pm \sqrt{5}\right)$.

Therefore,

$$b/a = \alpha \text{ or } b/a = \beta .$$

- **Prove the following theorem:**

If F_n denoted the n th term of the Fibonacci sequence, then for all $n \geq 0$.

$$F_n = \frac{1}{\sqrt{5}}\left(\alpha^n - \beta^n\right),$$

where $\alpha = \frac{1}{2}\left(1 + \sqrt{5}\right)$ and $\beta = \frac{1}{2}\left(1 - \sqrt{5}\right)$.

Lesson 10

MATHEMATICS AND ART

Mathematics is the art of giving the same name to different things.
HENRI POINCARÈ[38]

Mathematics is the sister as well as the servant of the arts and is touched with the same genius. In an age when specialization means isolation, a layman may be surprised to hear that mathematics and art are intimately related. Yet they are closely identified from ancient times and have a long historical relationship.

The ancient Egyptians and ancient Greeks knew about the golden ratio and regarded as an aesthetically pleasing ratio.

The Golden ratio roughly equal to 1.618, was first formally introduced in text by Greek mathematician Pythagoras and later by Euclid in the 5^{th} century B.C. Aside from interesting mathematical properties, there exist geometric shapes derived from the golden ratio, such as the golden rectangle, the golden triangle and Kepler's triangle. Various authors can discern the presence of the golden ratio in Egyptian pyramids, Sumerian, Greek, Chinese pieces of art. The prevalence of this special number in art and architecture even before its formal discovery by Pythagoras is perhaps evidence of an instinctive and primal human cognitive preference for the golden ratio.

The Renaissance saw a rebirth of classical Greek and Roman culture and ideas among them, the study of mathematics as a relevant subject needed to understand nature and arts.

The two major reasons drove Renaissance artists towards the pursuit of mathematics. First, painters needed to figure out how to depict three-dimentional scenes on a two-dimentional canvas. Second, philosophers and artists alike were convinced that mathematics was the true essence of the physical world and that the entire universe, including the arts, could be explained in geometric terms. In light of these factors, Renaissance artists became some of the best applied mathematicians of their times.

The revolutions in art and mathematics only deepen the relations between them. In both areas the creative process involves observation and experiment, judgment and rejection, intuition and feeling, careful calculation and analysis. Patterns in either field may illustrate, explain, or inspire work in the other.

It is a common observation that the emotional drive for creation and satisfaction from success are the same whether one constructs an object of art or a mathematical theory. The new mathematics and the new art are capable of an intimacy that we have not seen since Renaissance.[39]

🕮 TOPICAL VOCABULARY

servant	['sWvqnt]	n.	ծառա, սպասավոր
touch	[tAtS]	v.	վերաբերել, առնչվել
layman	['leimqn]	n.	ոչ մասնագետ
spatial	['speiSql]	a.	տարածական
cognitive	['kOgnitiv]	a.	իմացական
relevant	['relivqnt]	a.	տեղին, հարմար, պատշաճ
pursuit	[pq'sjHt]	n.	հետապնդում, ձգտում
depict	[di'pikt]	v.	1. պատկերել, նկարել 2. նկարագրել
judgment	['GAGmqnt]	n.	դատողություն, կարծիք
artisan	["a:ti'zxn]= ["a:ti'zan]	n.	արհեստավոր

artist	[a:tist]	n.	նկարիչ, վարպետ
prevalence	['privqlqns]	n.	գերիշխում, տարածում
discern	[di'sq:n]	v.	1. նշմարել, նկատել 2. տարբերել, զանազանել
primal	['praimql]	a.	հիմնական, գլխավոր

I. What is the Armenian for?

specialization, isolation, identify, trend, rough, shape, human, cognitive preference, figure out, emotional drive, satisfaction from success, deepen the relations, to be touched with the same genius, to begin with

II. What is the English for?

մաթեմատիկան և արվեստը սերտորեն կապված են, դասական ճարտարապետություն, քանոն և կարկին, փնտրել իդեալական (կատարյալ) համաչափություն, ստեղծագործական պրոցես, հռոմեական մշակույթ

III. a) Arrange the words in pairs similar in meaning:

1.	**artist**	a.	tendency
2.	**intimate**	b.	chase, search
3.	**artisan**	c.	proof
4.	**shape**	d.	cause
5.	**reason**	e.	form
6.	**pursuit**	f.	close
7.	**evidence**	g.	craftsman
8.	**trend**	h.	painter

b) Arrange the words in pairs opposite in meaning:

1.	**servant**	a.	failure
2.	**human**	b.	mild
3.	**construct**	c.	master
4.	**primal**	d.	inhuman
5.	**layman**	e.	destroy
6.	**rough**	f.	secondary
7.	**success**	g.	expert

IV. Match the following words with their definitions:

1.	**judgment**	a.	the act of isolating, the state of being isolated
2.	**rejection**	b.	holding as an opinion; consideration
3.	**isolation**	c.	computation
4.	**calculation**	d.	the act of denying
5.	**architecture**	e.	exceptional mental and creative power
6.	**art**	f.	the relation of one thing or part to another in respect to size, degree or quantity
7.	**solid**	g.	the designing of buildings
8.	**genius**	h.	existing in three-dimensional space; firm, hard, strong
9.	**proportion**	i.	works designed to give intellectual pleasure, as music, sculpture and esp. pictorial representation

V. Match the beginnings with their appropriate endings:

1.	Mathematics is the sister …	a.	… the best applied mathematicians of their times.
2.	It is not surprising that geometry …	b.	… the relations between them.
3.	Many trends and traditions in mathematics and art …	c.	… as well as the servant of the arts.
4.	The relations in art and mathematics only deepen …	d.	… is evident in classic architecture.
5.	Renaissance artists became …	e.	… are mixed.

VI. Form nouns from the given adjectives and translate them into Armenian:

visual, spatial, evident, deep, symmetrical, theoretical, developing, academic, successful, probable, distinct, convinced, artistic

VII. Are the statements true or false? Correct the false statements:

1. Mathematics and art are closely related.
2. Ancient people were unaware of the golden ratio.
3. Neither Pythagoras nor Euclid ever mentioned the golden ratio in their treatises.
4. Renaissance artists were of the opinion that mathematics was the true essence of the physical world.
5. Modern art is stark isolated from mathematics.
6. In both areas the creative process presupposes emotional drive inspiration and satisfaction from success.

VIII. Insert definite or indefinite articles:

… language of mathematics consists mostly of signs and symbols and, in … sense, is … unspoken language. There can be no more universal or more simple language, it is … same throughout … civilized world, though … people of each country translate it into their own particular spoken language. For instance, … symbol 5 means … same to … person in England, Spain, Italy or any other country; but in each country it may be called by … different spoken word. Some of … best known symbols of mathematics are …. Arabic numerals, … signs of addition, subtraction, multiplication, division, equality and … letters of … alphabets: Greek, Latin, Gothic.

IX. a) Speak on the key points of the text "Mathematics and art".

b) Discuss the topic: Mathematics and Renaissance artists.

X. Comment on the text:

FRACTALS

Fractal art is a form of algorithmic art created by calculating fractal objects and representing the results as still images, animations and media which has developed from the mid – 1980-s onwards. A fractal is a rough fragmented geometric shape that can be split into parts, each of which is, at least approximately, a reduced-size copy of the whole, a property called self-similarity. It was derived from the Latin ***fractus*** meaning "broken" or "fractured". A mathematical fractal is based on an equation that undergoes iteration, a form of feedback based on recursion.

The notion of fractals began to take shape in the 17th century when the great mathematician and philosopher Gottfried Leibniz considered recursive self-similarity, although he made the mistake thinking that only the straight line was self-similar in this sense. It was not until 1872 that a function appeared whose graph would today be considered a fractal, when Karl Weierstrass gave an example of a function with the non-intuitive property of being everywhere continuous, but nowhere differentiable. In 1904, Helge von Koch, dissatisfied with Weierstrass's abstract and analytic definition, gave a more geometric definition of a similar function, which is now called the Koch curve.

Iterated functions in the complex plane were investigated in the late 19th and early 20th centuries by Henri Poincaré, Felix Klein, Pierre Fatou and Gaston Julia. Without the aid of modern computer graphics, however they lacked the means to visualize the beauty of many of the objects that they had discovered.

In the 1960s, Benoit Mandelbrot started investigating self-similarity in his paper "Statistical Self-Similarity and Fractional Dimension". Finally, in 1975 Mandelbrot coined the word "fractal" to denote an object whose Hausdorff-Besicovitch dimension is greater than its topological dimension. He illustrated this mathematical definition with striking computer-constructed visualizations. These images captured the popular imagination, many of them were based on recursion, leading to the popular meaning of the term "fractal". The Julia set and Mandelbrot sets are considered icons of the fractal art.[40]

XI. Speak on the topics:

1. Mathematics and art are closely related in fractals.
2. Fractals in nature.

XII. Solve the following problems:

➢ Use mathematical induction to derive the following formula for all $n \geq 1$:

$$1(1!) + 2(2!) + 3(3!) + \cdots + n(n!) = (n+1)! - 1 .$$

- (a) Verify that for all $n \geq 1$.

$$2 \cdot 6 \cdot 10 \cdot 14 \cdot \cdots \cdot (4n-2) = \frac{(2n)!}{n!}.$$

(b) Use part (a) to obtain the inequality $2^n(n!) \leq (2n)!$ for all $n \geq 1$.

- Establish the Bernoulli inequality: If $1+a>0$ then

$$(1+a)^n \geq 1+na$$

for all $n \geq 1$.

Lesson 11

PROBABILITY

It is remarkable that a science which began with the consideration of games of chance should have become the most important object of human knowledge.

LAPLACE, PIERRE SIMON[41]

In statistics probabilities are defined as relative frequencies. If we say that the probability that it will rain on a certain day is 0.70, this means that under similar conditions we can expect it to rain 70 per cent of the time. Similarly, if we say, that the probability of a patient's recovery is 0.60, this means that 60 per cent of all people who suffer from the same illness can be expected to pull through.

If we flipped a "fair" coin we would expect heads to come up 50 per cent of the time, i.e., with a relative frequency of 0.50. This does not mean that if we flipped a coin 10 times we would necessarily get 5 heads and 5 tails, but that if we flipped the coin a large number of times we would in the long run get close to 50 per cent heads and 50 per cent tails. It is this relative frequency in the long run which we define as the probability of a given event.

Having defined the probability of an event A as the proportion of the time that we can expect A to occur in the long run, it is clear that $0 \leq P(A) \leq 1$ where P(A) stands for the probability of A. Evidently, a probability cannot exceed 1, an event cannot occur more than 100 per cent of the time, and it cannot be less than 0, an event cannot occur less than 0 per cent of the time.

Let us explain first what we mean when we say that two events are mutually exclusive if the occurrence of either prevents the occurrence of the other. If we toss a single coin heads and tails are mutually exclusive events; we will get one or the other, but never both. On the other hand, if A stands for the fact that we have apple pie for dessert while B stands for ice-cream, these two events are not mutually exclusive since we could have apple pie with ice-cream.

Theorem 1. If A and B are mutually exclusive events, the probability that either one or the other will occur is P (A or B) = P(A)+P(B). This follows immediately from the frequency definition of probability. If A happens 30 per cent of the time, B happens 40 per cent of the time, and the two events are mutually exclusive, one or the other will occur 70 per cent of the time, i.e., with a probability of 0.70.

The formula for P(A or B) can be generalized to apply also to the case where there are several mutually exclusive events.

Two events are said to be independent if the occurrence of one is in no way affected by the occurrence or nonoccurrence of the other. For example, if we toss two coins, the fact that one of them shows heads or tails has no effect on the face which is turned up in the other. Similarly, if A stands for Mr. Brown's passing an examination and B stands for his wearing green socks, these two events are clearly independent.

Theorem 2. If two events A and B are independent, the probability that they will both occur is P(A and B)=P(A)•P(B). To show that this rule follows from the frequency definition, let us consider the following illustration: if we toss two fair coins, a penny and a dime, the probability of heads for each coin is 0.50. If we toss these two coins together a great number of times, the dime can be expected to come up heads 50 per cent of the time regardless of whether the penny comes up heads or tails. Hence, the dime should come up heads in 50 per cent of the cases where the penny comes up heads. Since the probability that the penny comes up is 0.50, they will both come up heads 50 per cent, i.e., 25 per cent of the time. We thus find that the probability that they will both come up heads is 1/2•1/2=1/4.

To consider two events which are not independent, let us write P(A) for the probability that a student will pass an examination in mathematics and P(B) for the probability that he will pass an examination in physics. It is easy to see that these two events are not independent since it is well established that there exists a strong relationship between a person's ability in the given subjects. If P(A)=0.80 and P(B)=0.70 it would be quite reasonable to expect that P(A and B) is close to 0.70 since most students who pass one of these tests will also pass the other. In other words, since a student who does poorly in mathematics is likely to fail in physics, and a student who does well in mathematics is likely to get a high grade in physics.

The probability that A will happen provided that B has taken place is called the conditional probability of A relative to B and it will be written symbolically as P_B (A). With reference to our previous illustration, $P_B(A)$ is the probability that a student will pass in mathematics provided that he has passed in physics and $P_A(B)$ is a probability, that he will pass in physics, provided that he has passed in mathematics.[42]

🕮 TOPICAL VOCABULARY

per cent	["pq'sent]	n.	տոկոս
pull through	['pul 'TrH]	v.	առողջանալ, ապաքինվել, ողջ մնալ
flip a coin	['flip q' koin]	v.	մետաղադրամը վերև նետել (արծվախաղի ժամանակ)
fair coin	*մետաղադրամ, որում արծիվ թե գիր ընկնելու հավանականությունները նույնն են*		

heads or tails	['hedz L'tellz]		արծիվ, թե գիր
in the long run			*վերջ ի վերջո, երկար ժամանակ հետո*
frequency	['frikwqnsi]	n.	հաճախականություն
proportion	["prq'pLSn]	n.	մաս, մասնաբաժին
event	[i'vent]	n.	պատահույթ
toss a coin			*մետաղադրամը նետել*
mutually	['mjuCjuqli]	adv.	փոխադարձաբար
mutually exclusive			*փոխադարձաբար անհամա-պեղելի, հապում չունեցող*
illustration	["ilqs'treiSqn]	n.	օրինակ
dime	[daim]	n.	տասը ցենտի արժո-ղությամբ մետաղադրամ
regardless (of)	[ri'ga:dlis]	prep.	անկախ
reasonable	['rJzqnqbl]	a.	ողջամիտ, տրամա-բանական
do well/poorly			*լավ/վատ սովորել*
provided that	[prq'vaidid Dxt]		այն պայմանով, որ
with reference to			*վերաբերյալ, ինչ վերաբերում է*

I. What's the Armenian for?

statistics, probability, define, frequency, relative frequency, frequency definition, occurrence, establish a relationship, heads and tails, mutually exclusive, pull through, flip a coin, proportion, on the other hand, toss a coin, dessert, it follows from, generalize, illustration, turn up, event, pass an examination, relationship, subject, do well in mathematics, high grade, provided that, previous

II. What is the English for?

մետաղադրամով վիճակ գցել, նույնանման պայմաններ, հավանականություն, փոխադարձաբար անհամատեղելի, վերջիվերջո, կարելի է ընդհանրացնել, անկախ որևէ բանից, սերտ հարաբերակցություն, այլ կերպ ասած, բարձր գնահատական ստանալ

III. a) Arrange the following words in pairs similar in meaning:

1.	**define**	a.	repetition
2.	**similarly**	b.	mark
3.	**recovery**	c.	to influence
4.	**frequency**	d.	case
5.	**flip a coin**	e.	at once
6.	**an event**	f.	arise
7.	**exclusive**	g.	likewise
8.	**immediately**	h.	improvement
9.	**stand for**	i.	unique
10.	**come up**	j.	toss a coin
11.	**to affect**	k.	determine
12.	**grade**	l.	denote

b) Arrange the following words in pairs opposite in meaning:

1.	**exact**	a.	occurrence
2.	**similar**	b.	common
3.	**recovery**	c.	tails
4.	**pull through**	d.	dependent
5.	**heads**	e.	worsen
6.	**exclusive**	f.	do poorly
7.	**independent**	g.	a few
8.	**nonoccurrence**	h.	different
9.	**do well**	i.	imprecise
10.	**a great number of**	j.	lose

IV. Match the following words with their definitions:

1.	**probability**	a.	put into motion by a snap of the fingers
2.	**relative**	b.	a thing that happens or takes place, especially one of importance
3.	**proportion**	c.	be, allow, or cause to be visible
4.	**dessert**	d.	the measure of the likelihood that an event will occur
5.	**event**	e.	having existed for a long time and therefore recognized and generally accepted
6.	**show**	f.	the practice or science of collecting and analysing numerical data in large quantities
7.	**established**	g.	harmonious relation of parts to each other or to the whole
8.	**statistics**	h.	considered in relation or in proportion to something else
9.	**toss a coin**	i.	the sweet course eaten at the end of a meal

V. Give the degrees of comparison of the following adjectives:

exact, honest, relative, probable, exclusive, independent, reasonable, little, strong, high

VI. Define the type of the clause in the sentences given below:

1. If we flipped a “fair” coin we would expect heads to come up 50 per cent of the time.
2. If we toss two coins together a great number of times, the dime can be expected to come up heads 50 per cent of the time regardless of whether the penny comes up heads or tails.

3. The probability that A will happen provided that B has taken place is called the conditional probability.
4. When doing calculations it is necessary that all elementary events have a number assigned to them.
5. When flipping a coin the two possible outcomes are “heads” and “tails”.
6. As n increased to infinity the ratio would approach the probability of A.

VII. Make a suitable choice:

People appear … (to learn/having learnt) in different ways. Some people expect … (making/to make) mistakes in their studies and are capable of … (benefiting/to benefit) from their mistakes. They don’t mind … (correcting/being corrected) by their teacher and indeed often ask … (to have been/to be) corrected. Others, however, dislike … (making/to be making) mistakes. They try to avoid … (to be done/doing) anything which they might do badly. They would rather … (perfect/be perfected) something in small steps and be sure they have got it right … (that/than) attempt to do a task based on a subject they don’t feel they have finished … (explore/exploring) yet.

VIII. Are the given statements true or false? Contradict the false statements:

1. Two events are said to be independent if the occurrence of one is in no way affected by the occurrence or non-occurrence of the other.
2. Probabilities are defined as relative frequencies.
3. If we say that the probability of a patient’s recovery is 0.60, this means that 60 per cent of all people will not suffer from the same illness.

4. A student who does poorly in mathematics won't fail in physics.
5. The student who does well in mathematics is likely to get a high grade in physics.
6. Two events are mutually exclusive if the occurrence of either doesn't prevent the occurrence of the other.
7. Mr. Brown's passing an examination doesn't at all depend on his wearing green socks.

IX. Complete the following sentences using the words given below: *measure, probability measures, price, events, probability (2), mutually exclusive, occurring.*

… was defined as any measure on a collection of events – not necessarily based on the frequency of … It can be explained with a simple example: if you want to … the value of a painting you can do that in several ways – by measuring the area the painting occupies, taking into consideration the … the auctioneer gives or just by your own assessment. All of them are acceptable and are transformed into … … and depend on the particular problem. There are formulated three axioms of probability: firstly, the probability of an event … is a non-negative real number, secondly, when all the possible outcomes are known, the … of one of these outcomes is 1, and finally, the probability of … … events can be summed.

X. Render the text into Armenian:

The first discussion of the idea of mathematical probability is found in Girolamo Cardano's "Liber de Ludo Aleae" written in 1564. He was interested in the topic of decision-making and explored the ethics of gambling. Cardano commented that knowing that the chance of a fair dice coming up with a six is one in six is of no use to the gambler since probability does not predict the future. But you will be interested in it if you are trying to establish whether a gambler is fair or not and it will help you in making good decisions.

The early development of probability was driven by considering gambling problems with the exception of Pascal's wager (essentially that you've got nothing to lose by betting that God exists). These ideas about probability were collected by Jacob Bernoulli (Daniel Bernoulli's uncle) who introduced the law of large numbers, proving that if you repeat the same experiment (e.g. rolling a dice) a large number of times, the observed mean (e.g. the average of the scores you have rolled) will converge to the expected mean. For a fair dice each of the six scores is equally likely, so the expected mean is (1+2+3+4+5+6)/6= 3.5.

On the basis of Jacob Bernoulli's work, probability theory was developed by Laplace in the eighteenth century and by Fisher, Neuman and Pearson in the twentieth century. In 1933 Andrey Kolmogorov identified probability with measure theory and, in connection with statistics, probability theory became an essential tool of the scientists.

XI. Discuss the following problems on probability:

Problem 1. In a certain assembly plant, three machines, B_1, B_2 and B_3, make 30%, 45% and 25%, respectively, of the products. It is known from past experience that 2%, 3% and 2% of the products made by each machine, respectively, are defective. Now, suppose that *a* finished product is randomly selected. What is the probability that it is defective?

Problem 2. You have three bags that each contain 100 marbles. The first bag has 75 white and 25 black marbles. The second bag has 60 white and 40 black marbles. The third bag has 45 white and 55 black marbles. You choose one of the bags at random and then pick a marble from the chosen bag, also at random. What is the probability that the chosen marble is white?

XII. Speak on the key points of the text "Probability".

XIII. Read and discuss the following sayings on probability:

1. Life is a school of probability.

   (Walter Bagehot)

2. It is a truth very certain that when it is not in our power to determine what is true we ought to follow what is most probable.

   (Rene Descartes)

3. It is remarkable that a science which began with the consideration of a game of chance should have become the most important object of human knowledge.

4. The most important questions of life are, indeed, for the most part, really only problems of probability.

   (Laplace Pierre Simon)

Lesson 12

THE THEORY OF GAMES

> *Game theory is a sort of umbrella or "unified field" theory for the rational side of social science where "social" is interpreted broadly to include human as well as non-human players (computers, animals, plants).*[43]
>
> AUMANN, 1987.

Game theory is the formal study of conflict and cooperation. Its theoretic concepts are applied when the actions of several agents, who may be individuals, groups, firms or others, are interdependent. The concepts of game theory provide a language to formulate, understand and analyze different strategic situations. In mathematics game theory models such strategic situations or games in which the individual's success in making choices depends on the choices of others. This theory is widely used in social sciences, especially in economics, political science and social psychology, as well as in computer science, statistics, biology and ecology.

The earliest example of a formal game-theoretic analysis is the study of a duopoly by Antoine Cournot in 1838. In 1921 it was furthered by the mathematician Emile Borel. Some years later in 1928 when a curious paper on a rational strategy for matching pennies was presented by a young mathematician John von Neumann to the State Mathematical Society at Gottingen, Germany, a new branch of science was born which was soon to be called "The Theory of Games". Though it first seemed a little frivolous to the eminent scientists, after the publication of the monumental book "Theory of Games and Economic Behavior", which was the result of years of collaboration between mathematician John von Neumann and economist Oscar

Morgenstern, it was established as a field in its own right and was a powerful tool for the analysis of man's social relations. This work where some vast developments of the initial theory were presented, immediately attracted a great deal of attention and in the 16 years since its publication innumerable new applications of the theory have been discovered. It provided much of the basic terminology and problem setup that are still in use today. In 1950 John Nash demonstrated that finite games always have an equilibrium point at which all players choose actions which are best for them as compared with their opponents' choices. This central concept of non-cooperative game theory has been the focal point of analysis since then.

The theory was extensively developed by many scholars. It has been widely recognized as an important tool in many fields. Eight game-theorists have won Nobel Prizes in Economic Sciences and John Maynard Smith was awarded the Crafoord Prize for his application of game theory in biology.

Nowadays, the theory has been applied to such diverse subjects as anthropology and the synthesis of communication systems. The so-called "Theory of Linear Programming", for example, originated from problems in economics and has developed into a theory dealing with a maximization or minimization problem.

The mathematical foundations of game theory make it a prime tool for modeling and designing automated decision-making processes in interactive environments. The automation of strategic choices enhances the need for these choices to be made efficiently. The "prescriptive" application of game theory with the goal of improved strategic decision-making provides for structuring and analyzing problems of strategic options and considers the players' preferences and reactions providing the decision-maker with a clearer and broader view of the situation.[44]

🕮 TOPICAL VOCABULARY

conflict	['k0nflikt]	n.	հակամարտություն
unify	['junifai]	v.	միավորել
unified field			*միացյալ (համափեղ) բնագավառ*
interdependent	["intqdi'pendqnt]	a.	փոխկապակցված
formulate	['fLmjuleit]	v.	ձևակերպել
strategic situation			*ստրատեգիական իրավիճակ*
model	['moudql]	v.	մոդելավորել
duopoly	['djuq'p0li]	n.	դուոպոլիա (*տնտես.* երկու մրցակից ընկերությունների մենաշնորհ)
further	['fq:Dq]	v.	զարգացնել
curious paper			*հետաքրքիր աշխատություն*
to match pennies			*խնայել գումար*
frivolous	['frivqlqs]	a.	անլուրջ, թեթևամիտ
collaboration	[kqlxbq'reiSn]	n.	համագործակցություն
vast development			*լայն, լայնատարած զարգացում*
problem setup			*խնդրի դրվածք*
opponent	[q'pounqnt]	n.	մրցակից, հակառակորդ
focal point			*կիզակետ*

diverse	[di'vq:s]	a.	տարբեր, բազմազան
enhance	[in'ha:ns] [ɛn'ha:ns]	v.	1. խթանել, մեծացնել 2. բարձրացնել (գինը, որակը և այլն)
prime tool			*հիմնական գործիք*
prescriptive	[pri'skriptiv]	a.	կարգադրողական, հրահանգչական
preference	['prefqrqns]	n.	նախասիրություն
social science	['souSl' saiqns]		հասարակագիտություն
synthesis	['sinTqsis]	n.	սինթեզում, միաձուլում

I. What's the Armenian for?

unified field, to be interpreted broadly, theory, formal study, conflict and cooperation, theoretic concepts, an individual, a strategic situation, a duopoly, to further, to match pennies, frivolous, a monumental book, economic behavior, a problem setup, an equilibrium point, a non-cooperative game, a focal point, diverse, a prime tool, interactive environments

II. What is the English for?

տեսական գաղափարներ, լայնորեն կիրառվել, զարգացնել, համագործակցություն, տարբեր, լայնատարած, միավորել, կիզակետ, խթանել, խնդրի լուծում, գումար խնայել, անթիվ, հակառակորդ, որոշում կայացնել, թեթևամիտ, գիտնական, համատեղ բնագավառ, հրահանգ, հասարակագիտություն, քաղաքագիտություն, անալիզի մեթոդներով, ձևակերպել

III. a) Arrange the following words in pairs similar in meaning:

1.	**sort**	a.	advance
2.	**unified**	b.	define
3.	**broadly**	c.	cooperation
4.	**individual**	d.	senseless
5.	**scholar**	e.	main
6.	**curious**	f.	joined
7.	**further (v)**	g.	type, kind
8.	**formulate**	h.	widely
9.	**to match**	i.	theorist, scientist
10.	**frivolous**	j.	structure
11.	**prime**	k.	person
12.	**collaboration**	l.	important
13.	**setup**	m.	to relate
14.	**focal**	n.	interesting

b) Arrange the following words in pairs opposite in meaning:

1.	**human**	a.	agreement
2.	**broad**	b.	non-cooperative
3.	**rational**	c.	limited
4.	**conflict**	d.	similar
5.	**initial**	e.	disorganization
6.	**formal**	f.	imbalance
7.	**cooperative**	g.	tiny
8.	**innumerable**	h.	manual
9.	**vast**	i.	non-human
10.	**diverse**	j.	companion
11.	**automated**	k.	passive
12.	**equilibrium**	l.	informal
13.	**structuring**	m.	narrow
14.	**interactive**	n.	final
15.	**opponent**	o.	irrational

IV. Match the following words with their definitions:

1.	**duopoly**	a.	the way in which something, especially an organization or equipment, is organized planned or arranged
2.	**frivolous**	b.	a specialist in a particular branch of study, especially the humanities: a distinguished academic
3.	**vast**	c.	relating to the center or most important part
4.	**setup**	d.	intensify, increase or further, improve the quality, value, etc.
5.	**equilibrium**	e.	a plan of action or policy designed to achieve a major or overall aim
6.	**scholar**	f.	a state in which opposing forces or influences are balanced
7.	**enhance**	g.	monopoly of two competitive companies
8.	**preference**	h.	not having any serious purpose or value
9.	**focal**	i.	of very great extent or quantity
10.	**strategy**	j.	a greater liking for one alternative over another or others

V. Give derivatives corresponding to the following words:

formal, cooperation, equilibrium, strategy, extensively, publication, innumerable, choice, communication, efficiently, automation, origination, solve, design, process, provide, decision

VI. Insert prepositions: ***of, in, on, to, by, under.***

Nash's advisor and former Carnegie Tech professor, R.J. Duffin, wrote a letter … recommendation consisting … a single sentence: "This man is a genius". Nash was accepted … Harvard University; but the chairman … the mathematics department … Princeton, Solomon Lefschetz, offered … him the John S. Kennedy fellowship, which was enough … convince Nash that Harvard valued him less. Thus he went … Princeton where he worked … his equilibrium theory. He earned a doctorate … 1950 … a 28-page dissertation … non-cooperative games. The thesis, … which was written … the supervision … Albert W. Tucker, contained the definition and properties … what would later be called the "Nash equilibrium".

VII. Are the given statements true or false? Contradict the false statements:

1. In 1950 John Nash published his monumental book "Theory of Games and Economic Behavior".
2. In strategic situations or games the individual's success in making choices doesn't depend on the choices of the others.
3. John Nash was unable to demonstrate that finite games always have an equilibrium point at which all players choose actions.
4. The theory of games was developed by many scholars and has been recognized as an important tool in many fields.
5. The "prescriptive" application of game theory considers the player's preferences but doesn't provide him with a clearer and broader view of the situation.

VIII. Match the beginnings with their appropriate endings:

1.	The concepts of game theory provide a language …	a.	… was a powerful tool for the analysis of man’s social relations.
2.	Game theory is widely used in …	b.	… with the goal improved strategic decision-making provides for structuring and analyzing problems of strategic options.
3.	Theory of Games and Economic behavior…	c.	… at which all players choose actions, which are best for them as compared with their opponent`s choices
4.	Games with finite strategies always have an equilibrium point …	d.	… to formulate, understand and analyze different strategic situation.
5.	The prescriptive application of game theory …	e.	… social sciences, especially in economics, political science and social psychology.

IX. Render the text into Armenian:

GAME THEORY AS A METHOD

Game theory is a major method used in mathematical economics and business for modeling competing behaviors of interacting agents. Applications include a wide array of economic phenomena and approaches, such as auctions, bargaining, fair division, duopolies, oligopolies, social network formation, agent-based computational economics, general equilibrium, mechanism design, and voting systems, and across such broad areas as behavioral economics, information economics, industrial organization, and political economy.

This paper focuses on particular sets of strategies known as equilibria in games. These “solution concepts” are usually based on what is required by norms of rationality. In non-cooperative games, the most famous of these is the Nash equilibrium. A set of strategies is a Nash equilibrium if each represents a best response to the other strategies. So, if all the players are playing the strategies in a Nash equilibrium, they have no unilateral incentive to deviate, since their strategy is the best they can do given what others are doing.

The payoffs of the game are generally taken to represent the utility of individual players. Often in modeling situations the payoffs represent money, which presumably corresponds to an individual's utility. This assumption, however, can be faulty.

A prototypical paper on game theory in economics begins by presenting a game that is an abstraction of some particular economic situation. One or more solution concepts are chosen, and the author demonstrates which strategy sets in the presented game are equilibria of the appropriate type. Naturally one might wonder to what use this information should be put. Economists and business professors suggest two primary uses (noted above): descriptive and prescriptive.

X. Read the following passage on Monte Carlo Algorithms and make statements beginning with:

As far as I know ...,
As already stated ...,
Assuming that ...,
Without the assumption that ...,
It should be noted that ...,
It is difficult to ...,
It is interesting to ... .

MONTE CARLO ALGORITHMS

There exist problems for which no efficient algorithm is known to obtain a correct solution every time. A *Monte Carlo* algorithm occasionally makes a mistake but it finds a correct solution with high probability whatever the instance considered. This is a stronger claim than saying that it works correctly on a majority of instances, only failing now and again in some special cases: there must be no instance on which the probability of error is high. However, no warning is usually given when the algorithm makes a mistake.

Let p be a real number such that $0 < p < 1$. We say that a Monte Carlo algorithm is p – correct if it returns a correct answer with probability at least p , whatever the instance considered. In some cases we shall allow p to depend on the instance size but never on the instance itself. The most interesting feature of Monte Carlo algorithms is that it is often possible to reduce the error probability arbitrarily at the cost of a slight increase in computing time. We call this *amplifying the stochastic advantage.*

XI. Solve the following problem:

A and *B* are playing a fair game of *balla*. They agree to continue until one has won six rounds. The game actually stops when *A* has won five and *B* three.

How should the stakes be divided?

XII. Speak on the key points of the text: "The Theory of Games".

Lesson 13

ALGORITHMS

... what is physical is subject to the laws of mathematics and what is spiritual to the laws of God, and the laws of mathematics are but the expression of the thoughts of God.

THOMAS HILL[45]

Algorithmics is the systematic study of the design and analysis of algorithms. The rapid rise of computer science which has the study of algorithms as its focal point has made the word “algorithm” essential and actual.

There are some other words that almost, but not quite capture the concept that is needed; procedure, process, routine, method, recipe. Like these things an algorithm is a set of rules or directions (instructions) for getting a specific output from a specific input. The execution of an algorithm must not normally involve any subjective decisions, nor must it call for the use of intuition or creativity. Hence a cooking recipe can be considered to be an algorithm if it describes precisely how to make a certain dish, giving exact quantities to use and detailed instructions for how long to cook it. On the other hand, if it includes such obscure notions as “add salt to taste” or “cook until tender” then we would no longer call it an algorithm.

The distinguishing feature of an algorithm is that all vagueness must be eliminated, the rules must describe operations that are so simple and well-defined that they can be executed by a machine. Furthermore, an algorithm must always terminate after a finite number of steps.

A computer programme is the statement of an algorithm in some well-defined language, although the algorithm itself is a mental concept that exists independently of any representation. Anyone who has prepared a computer programme will appreciate the fact an algorithm must be very precisely defined, with attention to detail that is unusual in comparison with other things people do.

Programmes for numerical problems were written as early as 1800 B. C. when Babylonian mathematicians gave rules for solving many types of equations. The rules were as step-by-step procedures applied systematically to particular numerical examples. The word "algorithm" itself originated in the Middle East, although at a much later time. Curiously enough it comes from the Latin version of the last name of the Persian scholar Abu Jafar Muhammad ibn Musa al Khowaresmi (Algorithmi) whose textbook on arithmetic (c. 825 A.D.) employed for the first time Hindu positional decimal notation and gave birth to algebra as an independent branch of mathematics. It was translated into Latin in the 12th century and had a great influence for many centuries on the development of computing procedures. The name of the textbook's author became associated with computations in general and used as a term "algorithm".

Originally algorithms were concerned solely with numerical calculations; Euclid's algorithm for finding the greatest common divisor of two numbers – is the best illustration. There are many properties of Euclid's powerful algorithm which has become a basic tool in modern algebra and number theory. Nowadays the concept of an algorithm is one of the most fundamental notions not only in mathematics but in science and engineering. Experience with computers has shown that the data manipulated by programmes can represent virtually anything. In all branches of mathematics the task to prove the solvability or unsolvability of any problem requires a precise algorithm. In computer science the emphasis has now shifted to the study of various structures by which information can be represented and to the branching or decision-making aspects of algorithms, which allow them to fall on one or another sequence of operations depending on the state of affairs at the time. It is precisely these features of algorithms that sometimes make algorithmic models more suitable than traditional mathematical models for the representation and organization of knowledge.

Although numerical algorithms certainly have many interesting features, there are non-numerical ones and in fact algorithms in Cybernetics deal primarily with manipulation of symbols that need not represent numbers. Cybernetics studies the complex data structures that are necessary for the effective realization of algorithms on a computer.[46]

TOPICAL VOCABULARY

scarce	['skFqs]	a.	1. սակավ, քիչ ժամա-նակով 2. հազվագյուտ
scarcely	['skFqsli]	adv.	1. հազիվ, 2. հազիվ թե, դժվար թե
focal	['fouk(q)l]	a.	կիզակետային
procedure	[prq'sJGq]	n.	գործի ընթացք, ընթացակարգ
routine	[rH'tJn]	n.	սահմանված կարգ, որոշակի ռեժիմ
vague	[veig]	a.	անպարզ, աղոտ, անորոշ
curious	['kjuqriqs]	a.	հետաքրքրական, հետաքրքիր զարմանալի
employ	[im'pl0i]	v.	գործածել, օգտագործել, կիրառել
scholar	['sk0lq]	n.	գիտնական, գիտուն

execute	['eksikjHt]	v.	կատարել, իրագործել
associate	[q'souSieit]	v.	1. միացնել, միավորել 2. միանալ
be concerned with	[bi kqn'sWnd wiD]		գործ ունենալ
divisor	[di'vaizq]	n.	բաժանարար
manipulate	[mq'nipjuleit]	v.	հմտորեն վարվել որևէ բանի հետ, հմտորեն բանեցնել
virtually	['vWtjuqli]	adv.	փաստորեն, փաստացի կերպով, իրապես, ըստ էության
require	[ri'kwaiq]	v.	1. պահանջել հրամայել 2. կարիք ունենալ, հարկավոր, անհրաժեշտ լինել
shift	[Sift]	v.	1. տեղափոխ(վ)ել փոխադր(վ)ել 2. փոխել փոփոխել
sequence	['sikwqns]	n.	հաջորդականություն
solvability	["sOlvq'biliti]	n.	լուծելիություն
affair	[q'fFq]	n.	գործ
c. (circa)	['sWkq]	prep.	մոտավորապես

I. What is the Armenian for?

scarcely necessary, focal point, the distinguishing feature, mental concept, to exist independently, precisely defined algorithm, systematically applied procedures, to originate in the Middle East, Latin version of the word, the data manipulated by a program, to eliminate vagueness

II. What is the English for?

ճշգրիտ ալգորիթմ, սուբյեկտիվ որոշումներ, հրահանգ տալ, մեծ ազդեցություն ունենալ, հիմնական գործիք, փոխել շեշտադրումը, գործողությունների հաջորդականություն, բացի այդ, կարող են կատարվել մեքենայի կողմից, կարևոր հասկացություն, իրերի վիճակ, դասական մաթեմատիկայի մոդել

III. a) Arrange the words in pairs similar in meaning:

1.	**scarce**	a.	particular
2.	**specific**	b.	uncertain
3.	**capture**	c.	appropriate
4.	**vague**	d.	single
5.	**scholar**	e.	picture
6.	**computation**	f.	writer
7.	**illustration**	g.	quality
8.	**associate**	h.	scientist
9.	**shift**	i.	calculation
10.	**feature**	j.	idea
11.	**emphasis**	k.	rare
12.	**author**	l.	sieze
13.	**notion**	m.	move
14.	**sole**	n.	relate
15.	**suitable**	o.	stress

b) Arrange the words in pairs opposite in meaning:

1.	**precise**	a.	ignorant
2.	**finite**	b.	deform
3.	**appreciate**	c.	familiar
4.	**terminate**	d.	disprove
5.	**rise**	e.	unusual
6.	**solvability**	f.	forbid
7.	**numerical**	g.	start
8.	**allow**	h.	unsolvability
9.	**usual**	i.	fall
10.	**prove**	j.	non-numerical
11.	**unknown**	k.	depriciate
12.	**form**	l.	infinite
13.	**educated**	m.	indefinite

IV. Find the corresponding adjectives from the text and translate them into Armenian:

necessity, focus, mentality, number, particularity, position, independence, base, vary, precision, algorithm, suit, tradition, mathematics

V. Fill in the blanks with words and expressions from the text:

1. An algorithm is a set of rules or directions for getting a specific ... from a specific ...
2. The execution of an algorithm must not normally involve any ... …
3. A cooking recipe can be considered an ...
4. If it includes ... notions it would no longer be called an algorithm.

5. The textbook by the Persian scholar Al-Khowaresmi was translated into ... in the ... ...
6. A computer programmer will appreciate the fact that an algorithm must be very ... ... with attention to ...
7. Originally algorithm were concerned solely with ... ...
8. ... algorithm has become a basic tool in modern ... and ... theory.
9. In fact algorithms in Cybernetics deal primarily with manipulation of ... that need not represent numbers.

VI. Match the following words with their definitions:

1.	**algorithmics**	a.	a course or mode of action
2.	**computation**	b.	a customary procedure or course of action
3.	**routine**	c.	a learned man
4.	**associate**	d.	the study of algorithms
5.	**recipe**	e.	perform; carry out
6.	**scholar**	f.	calculation
7.	**version**	g.	link together in some conceptual relationship
8.	**execute**	h.	directions; as for preparing a specific dish
9.	**mental**	i.	particular form or variant of something
10.	**eliminate**	j.	performed by the mind or intellect
11.	**procedure**	k.	get rid of, remove; expel

VII. Are these statements true or false? Correct the false statements:

1. Algorithmics can be defined as the study of algorithms.
2. The rapid rise of computer science has made the word “algorithm” essential and actual.
3. The distinguishing feature of an algorithm is its vagueness and complexity.
4. The word “algorithm” originated in the Far East.
5. Algorithm is derived from the name of the Persian scholar Mohammed ibn-Musa al-Khowaresmi.
6. Khowaresmi’s textbook gave birth to algebra as an independent branch of mathematics.
7. The name of the text book’s author became associated with art.
8. The best illustration of numerical algorithms is the Pythagorian theorem.
9. At present the notion of an algorithm is outdated.
10. Algorithms are rarely used for solving mathematical problems.
11. Experience with computers has shown that the data manipulated by programmes represent nothing.
12. An algorithm can be modified depending on the state of affairs at the time.
13. Some specific features make algorithmic models inappropriate in comparison with traditional mathematical models.

VIII. Insert prepositions: *from, with, of, in, for, on, to.*

The word cybernetics originated … the Greek “kybernetike”, the Latin, “gubernator” and the English “governor” all meaning, … one sense or another, “control”. More recently Norbert Wiener has used the word to name his book, which deals … the activity … a group … scientists engaged … the solution … a wartime problem and some … the mathematical concepts involved. Nowadays the word has become associated … the solution … problems dealing … activities … computers. As such, the discipline must rely … the exact sciences as well as sciences such as biology, psychology, biochemistry and biophysics, neurophysiology and anatomy.

Cybernetics is a comparatively young science and yet it is increasingly applied … various branches … industry and research. Invading a wide range … fields … human activity, cybernetics endeavours to find the answer … two major questions: the best way … controlling this or that process, and the best way … utilizing a machine (if possible) … controlling this process.

IX. Speak on the key points of the text: “Algorithms”.

X. Translate the text and discuss the mathematical puzzle:

The tower of Hanoi or the tower of Brahma is a mathematical game or puzzle. According to the legend after creating the world God set on Earth three rods made of diamond and 64 rings of gold. These rings are all different in size. At the creation they were threaded on the first rod in order of size, the largest at the bottom and the smallest at the top. God also created a monastery close by the rods. The monks’ task in life is to transfer all the rings onto the second rod. The only operation permitted is to move a single ring from one rod to another in such a way that no ring is ever placed on top of another smaller one. When the monks have finished their task, according to the legend, the world will come to an end. This is probably the most reassuring prophecy ever made concerning the end of the world, for if the monks manage to move one ring per second, working night and day without ever resting nor ever making a mistake, their work will still not be finished 500 000 million years after they began. This is more than 25 times the estimated age of the Universe!

The puzzle was invented by the French mathematician Edouard

Lucas in 1883. It was based on the legend, in accordance with it when the last move of the puzzle is completed the world will end. It is not clear whether Lucas invented this legend or was inspired by it.

There are many variations on this legend. The tower may be said to be in different parts of the world and may be associated with any religion. In some versions, other elements are introduced, such as the fact that the tower was created at the beginning of the world, or that the priests or monks make only one move per day. If the legend were true, and if the priests were able to move disks at a rate of one per second, using the smallest number of moves, it would take them 2^{64}-1 seconds or roughly 585 billion years.

It would take 18,446,744,073,709,551,615 turns to finish.

The game seems impossible to many novices, yet is solvable with a simple algorithm.

Solution:

Solution to the puzzle refers to the analysis of recursive algorithms. The puzzle can be played with any number of disks. The number of moves required to solve a Tower of Hanoi puzzle is $2^n - 1$ where *n*-is the number of disks.

The objective of the puzzle is to move the entire stack to another rod, obeying the following rules:

- Only one disk may be moved at a time
- Each move consists of taking the upper disk from one of the rods and sliding it onto another rod, on top of the other disks that may already be present on that rod.
- No disk may be placed on top of a smaller disk.

A key to solving this puzzle is to recognize that it can be solved by breaking the problem down into a collection of smaller problems and further breaking those problems down into even smaller problems until a solution is reached. The following procedure demonstrates this approach.

- Label the pegs A, B, C-these labels may move at different steps
- Let *n* be the total number of discs
- Number the discs from 1 (smallest, topmost) to *n* (largest, bottommost)

To move n discs from peg A to peg C

- Move n-1 discs from A to B. This leaves disc #n alone on peg A
- Move disc #n from A to C
- Move n-1 discs from B to C so they sit on disc #n

The above is a ***recursive algorithm***: to carry out steps 1 and 3, apply the same algorithm again for n-1. The entire procedure is a finite number of steps, since at some point the algorithm will be required for n=1. This step, moving a single disc from peg A to peg B, is trivial. This approach can be given a rigorous mathematical formalism with the theory of ***dynamic programming***.[47]

Lesson 14

JOHN VON NEUMANN

If people do not believe
that mathematics is simple,
it is only because they do not
realize how complicated life is.
JOHN VON NEUMANN[48]

John von Neumann was born on December 28, 1903 as János Lajos in Budapest, Hungary to wealthy Jewish parents. His father Max Neumann, was a well-to-do banker. As a child von Neumann showed he had an incredible memory. At the age of six John was able to exchange jokes with his father in classical Greek, memorize telephone directories, and display prodigious mental calculation abilities- divide eight-digit numbers in his head. The Neumann family sometimes entertained guests with demonstrations of Johnny's ability to memorize phone books. A guest would select a page and column of the phone book at random. Young Johnny read the column over a few times, then handed the book back to the guest. He could answer any question put to him or recite names, addresses, and numbers in order.

John's father hired private tutors to give him advanced instruction in those areas in which he had displayed an aptitude. Although the family were Jewish, Max Neumann did not observe the strict practices of that religion and the household seemed to mix Jewish and Christian traditions. In 1911 John von Neumann entered the Hungarian-speaking Lutheran high school. His mathematics teacher quickly recognized von Neumann's genius and special tuition was put on for him. Recognized as a mathematical prodigy, at the age of 15 he began to study under Gábor Szegő. On their first meeting, Szegő was so impressed with the boy's mathematical talent that he was brought to tears.

In 1921 von Neumann completed his education at the Lutheran Gymnasium. His first mathematics paper was published in 1922.

Max Neumann did not want his son to take up a subject that would not bring him wealth. Max Neumann asked Theodore von Kármán to speak to his son and persuade him to follow a career in business. Perhaps von Kármán was the wrong person to ask to undertake such a task but in the end all agreed on the compromise subject of chemistry for von Neumann's university studies.

John von Neumann received his Ph.D. in mathematics from Pázmány Péter University in Budapest at the age of 22. He simultaneously earned his diploma in chemical engineering from the ETH Zurich in Switzerland at the behest of his father, who wanted his son to invest his time in a more financially viable endeavor than mathematics. Between 1926 and 1930, he taught as a Privatdozent at the University of Berlin, the youngest in its history. By age 25, he had published ten major papers, and by 30, nearly 36.

Von Neumann was invited to Princeton University, New Jersey in 1930, and, subsequently, was one of the first four people selected for the faculty of the Institute for Advanced Study (two of the others being Albert Einstein and Kurt Gödel), where he remained a mathematics professor from its formation in 1933 until his death.

In 1937, von Neumann became a naturalized citizen of the U.S. In 1938, von Neumann was awarded the Bôcher Memorial Prize for his work in analysis.

Postwar von Neumann concentrated on the development of the Institute for Advanced Studies (IAS) computer and its copies around the world. His work with the Los Alamos group continued and he continued to develop the synergism between computers capabilities and the needs for computational solutions to nuclear problems related to the hydrogen bomb. Several "supercomputers" were built by National Laboratories as copies of his machine.

The Institute of Electrical and Electronics Engineers (IEEE) continues to honor John von Neumann through the presentation of an annual award in his name. The IEEE John von Neumann Medal was established by the Board of Directors in 1990 and may be presented annually for outstanding achievements in computer-related science and technology.

In 1955, von Neumann was diagnosed with what was either bone or pancreatic cancer. He invited a Roman Catholic priest, to visit him for consultation (a move which shocked some of von Neumann's friends). The priest then administered to him the last Sacraments. He died under military security lest he reveal military secrets while heavily medicated.

His contemporary wrote "… his mind, the amulet on which he had always been able to rely, was becoming less dependable". His friend Edward Teller said, "I think that von Neumann suffered more when his mind would no longer function, than I have ever seen any human being suffer".

It would be almost impossible to give even an idea of the range of honours which were given to von Neumann. He was Colloquium Lecturer of the American Mathematical Society in 1937 and received its Bôcher Prize. He held the Gibbs Lectureship of the American Mathematical Society in 1947 and was President of the Society in 1951-53.

He was elected to many academies including the Academies of Lima, Rome, USA, Milan, Amsterdam. Von Neumann received two Presidential Awards, the Medal for Merit in 1947 and the Medal for Freedom in 1956. Also in 1956 he received the Albert Einstein Commemorative Award and the Enrico Fermi Award.

Von Neumann wrote 150 published papers in his life; 60 in pure mathematics, 20 in physics, and 60 in applied mathematics. His last work, written while in the hospital and later published in book form as *The Computer* and *The Brain*, gives an indication of the direction of his interests at the time of his death.

Hungarian American mathematician John von Neumann made major contributions to a vast range of fields, including set theory, functional analysis, quantum mechanics, ergodic theory, geometry, economics and game theory, computer science, numerical analysis, hydrodynamics (of explosions), and statistics, as well as many other mathematical fields.[49]

🕮 TOPICAL VOCABULARY

incredible	[in'kredqbl]	a.	անհավատալի, անհավանական
exchange	[iks'CeinG]	v.	փոխանակել
		n.	փոխանակում
phone directory	*հեռախոսի բաժանորդների ցուցակ*		
prodigious	[prq'diGqs]	a.	1. վիթխարի, հսկայական 2. տաղանդավոր
mental	[mentl]	a.	մտավոր, մտքում կատարվող, մտային
at random			*պատահականորեն*
recite	[re'sait]	v.	1. պատմել 2. վերարտադրել 3. թվարկել (փաստերը)
advanced	[qd'va:nst]	a.	1. առաջ քաշված 2. առաջավոր
aptitude	['xptitju:d]	n.	1. ընդունակություն 2. հակում
household	['haushould]	n.	ընտանիք, տան անդամներ
to take up			*սկսել, ձեռնարկել*
persuade	[pq'sweid]	v.	համոզել, հորդորել

undertake *(undertook-undertaken)*	["Andq'teik]	v.	1. ձեռնարկել 2. հանձն առնել
simultaneous	["simql'teinjqs]	a.	միաժամանակյա, միաժամանակ

at the behest of somebody *մեկի խնդրանքով, մեկի պնդմամբ*

viable	['vaiqbl]	a.	1. աշխույժ 2. հարմար
endeavour	[in'devq]	n.	ջանք, ճիգ
subsequently	['sAbsikwqntli]	adv.	հետագայում, հետո, ավելի ուշ

naturalized citizen *հպատակագրված, փվյալ երկրի քաղաքացիության իրավունք ստացած*

synergism	['sinqGizm]	n.	համատեղ գործողություն

pancreatic cancer *ենթաստամոքսագեղձի քաղցկեղ*

to administer somebody to the last Sacraments *փալ մեկին վերջին հաղորդություն*

reveal	[ri'vi:l]	v.	1. բացահայտել 2. ցույց տալ, ցուցաբերել
amulet	['xmjulqt]	n.	թալիսման, համայիլ
range of honours			*պարգևների շարք*
ergodic theory			*էրգոդիկ փեսություն*

I. What’s the Armenian for?

to take up a subject, naturalized citizen, an incredible memory, diploma in chemical engineering, ability to memorize phone books, to persuade somebody to follow a career in business, to display an aptitude, to invest time in a more financially viable endeavor, to display prodigious mental calculation abilities, to become less dependable, the range of honors given to von Neumann

II. What’s the English for?

ինչ-որ մեկի խնդրանքով, տղայի մաթեմատիկական տաղանդով տպավորված, իսկույն ճանաչեց Նեյմանի հանճարը, կատակներ փոխանակել դասական հունարենով, պատահական, կենտրոնանալ, ջրածնային ռումբին վերաբերող միջուկային խնդիրներ, ախտորոշվեց ոսկրի քաղցկեղ, ընտրվել շատ ակադեմիաների անդամ

III. a) Arrange the words in pairs similar in meaning:

1.	**prodigious**	a.	promoted, progressive
2.	**strict**	b.	disclose, show, expose
3.	**reveal**	c.	exact, accurate
4.	**memory**	d.	remarkable, astounding, vast
5.	**earn**	e.	remembrance, recollection
6.	**advanced**	f.	obtain, acquire, gain

b) Arrange the words in pairs opposite in meaning:

1.	**persuade**	a.	supply, plenty, abundance, satisfy
2.	**contemporary**	b.	stupidity, solidity, inanity
3.	**lack**	c.	discourage, restrain, dissuade
4.	**genius**	d.	insecure, dangerous, unsafe
5.	**secure**	e.	antecedent, prior, asynchronous
6.	**undertake**	f.	write, read, hear, quote, misrepresent
7.	**recite**	g.	abandon, decline, neglect, omit

IV. Match the following words with their definitions:

1.	**private**	a.	(loud noise caused by a) sudden and violent bursting
2.	**range**	b.	1. kept from the knowledge of others 2. mystery
3.	**vast**	c.	1. consulting or being consulted 2. meeting for consulting
4.	**explosion**	d.	1. coming or happening every year 2.lasting for only one year or season
5.	**consultation**	e.	immense, extensive
6.	**annual**	f.	(contrasted with minor) greater or more important
7.	**at random**	g.	1. (opposite of public) of, for the use of, concerning, one person or group of persons, not people in general 2. secret; kept secret
8.	**major**	h.	without order, happening by chance
9.	**secret**	i.	row, line or series of things

V. Find the corresponding nouns from the text:

academic, capable, to analyze, to memorize, busy, to recognize, to achieve, secure, able, to present, to demonstrate

VI. Choose the suitable word:

1. The Neumann family sometimes entertained guests with demonstrations of Johnny's ability to … phone books**.** *(memorize / memorizing)*
2. His contemporary wrote "His mind, the amulet on which he had always been able to rely, was becoming … dependable". *(more / less)*

3. On their first meeting, Szegő was so impressed with the boy's mathematical talent that he was brought …. *(to tears / to laughter)*
4. A guest would select a page and column of the … book at random. *(phone / phone's)*
5. Several "supercomputers" were built … National Laboratories as copies of Neumann's machine. *(with / by)*
6. Neumann's first … paper was published in 1922. *(mathematics / mathematical)*

VII. Match the beginnings with their appropriate endings:

1.	As a child von Neumann showed he had …	a.	… the range of honours which were given to von Neumann.
2.	It would be almost impossible to give even an idea of …	b.	… the Albert Einstein Commemorative Award and the Enrico Fermi Award.
3.	At the age of six …	c.	… von Neumann's genius and special tuition was put on for him.
4.	In 1956 he received …	d.	… an incredible memory.
5.	His mathematics teacher quickly recognized …	e.	… John was able to exchange jokes with his father in classical Greek

VIII. Put in definite or indefinite articles where necessary:

Von Neumann invariably wore … conservative grey flannel business suit, once riding down … Grand Canyon astride … mule in … three-piece pin-stripe, and he enjoyed throwing … large parties at his home in Princeton, occasionally twice … week. His white clapboard house at 26 Westcott Road was one of … largest in Princeton. Despite being … notoriously bad driver, he nonetheless enjoyed driving (frequently while reading … book) – occasioning numerous arrests as well as … accidents. When Cuthbert Hurd hired him as … consultant to IBM, Hurd often quietly paid … fines for his traffic tickets.

IX. Fill in the blanks with prepositions if necessary:

Analyzing the qualities … mind that made possible von Neumann's extraordinary contributions, Eugene Wigner emphasized the accuracy … his logic, his brilliance, and his exceptional memory. Observing that von Neumann was ever-ready to help and was genuinely interested … every problem that presented a challenge, Professor Wigner said … that he himself had learned more mathematics … von Neumann than anyone else and much more … the "essence … creative thinking … mathematics" than a lifetime's study … without von Neumann could have taught him. Professor Wigner also quoted … Atomic Energy Commission Chairman Lewis L. Strauss's comment … von Neumann: "If he analyzed a problem, it was not necessary to discuss … it any further. It was clear what had to be done".

X. Are these statements true or false? Contradict the false statements:

1. John von Neumann received his Ph.D. in mathematics from Pázmány Péter University in Budapest at the age of 45.
2. The Neumann family sometimes entertained guests with demonstrations of Johnny's ability to make up funny stories.
3. He was Colloquium Lecturer of the American Mathematical Society in 1937 and received its Bôcher Prize.

4. Von Neumann wrote 10 published papers in his life; 2 in pure mathematics, 2 in physics, and 6 in applied mathematics.
5. He was elected to many academies including the Academies of the USA, Lima, Rome, Milan, Amsterdam. Von Neumann received two Presidential Awards, the Medal for Merit in 1947 and the Medal for Freedom in 1956.

XI. Complete the following sentences using the words given below:

contribution, President, scientists, since, honor, United States, development, stamps.

On February 15, 1956, Neumann was presented with the Presidential Medal of Freedom by … Dwight Eisenhower.

On May 4, 2005 the … Postal Service issued the American Scientists commemorative postage stamp series, a set of four 37-cent self-adhesive … in several configurations. The … depicted were John von Neumann, Barbara McClintock, Josiah Willard Gibbs, and Richard Feynman.

The John von Neumann Award of the Rajk László College for Advanced Studies was named in his …, and has been given every year … 1995 to professors who have made an outstanding … to the exact social sciences and through their work have strongly influenced the professional … and thinking of the members of the college.

XII. Render the text into Armenian:

The following problem can be solved either the easy way or the hard way.

Two trains 200 miles apart are moving toward each other; each one is going at a speed of 50 miles per hour. A fly starting on the front of one of them flies back and forth between them at a rate of 75 miles per hour. It does this until the trains collide and crush the fly to death. What is the total distance the fly has flown?

The fly actually hits each train an infinite number of times before it gets crushed, and one could solve the problem the hard way with pencil and paper by summing an infinite series of distances. The easy way is as follows:

Since the trains are 200 miles apart and each train is going 50 miles an hour, it takes 2 hours for the trains to collide. Therefore the fly was flying for two hours. Since the fly was flying at a rate of 75 miles per hour, the fly must have flown 150 miles. That's all there is to it.

When this problem was posed to John von Neumann, he immediately replied, "150 miles".

"It is very strange," said the poser, "but nearly everyone tries to sum the infinite series".

"What do you mean, strange?" asked Von Neumann. "That's how I did it!"[50]

XIII. Speak on the key points of the text "John Von Neumann".

Lesson 15

THE GAME OF CHESS

Chess is one of the few mind games that is quite popular among the generation of today. A recreational and competitive game, it basically involves participation by two players. Though the origin of the game is pretty old, it continues to indulge people till date. It is played by millions of people worldwide, in clubs, online, by correspondence, in tournaments and also casually. In fact, chess has managed to become one of the games played at the international level and form a part of the Olympic Games.

Chess is said to have originated in India, where it was played as far back as the 6th century AD. The following legend of chess was recorded in Ancient India. It is still under doubt whether it's true or a myth. According to the legend, the chess was invented by King Shirham's vizier Sissa Ben Dahir and presented to the king by the latter. Sissa Ben was a talented mathematician.The King was very pleased and asked Sissa Ben whether he could do anything to thank him for the present. His vizier's desire seemed quite easy to fulfil. He asked his king to give him a grain of wheat to put on the first square of the chessboard, and two grains to put on the second square, and four grains to put on the third square and so on doubling the number for each square that came next. He said he wanted the grains to cover all 64 squares of the board. The king thought his vizier did not ask too much for what he had done and said that he would certainly give him what he asked for. And he told his men to bring a large bag full of wheat. But when the counting began, the bag was emptied before the twentieth square was covered with wheat. More bags were brought before the king but the number of grains needed increased so rapidly that the king soon saw he would not be able to keep his word.

The thing is that a bushel of wheat contains about 5.000.000 grains. The king had, certainly, no idea of it and did not expect that he would have to give 4000 billion bushels to Sissa Ben. Since the world production of wheat averages about 2.000.000.000 bushels a year, the amount of wheat asked by the vizier equaled the world's wheat production for the period of some two thousand years. Thus king Shirham found that he either had to remain constantly in debt to his vizier or cut his head off. He thought it best to choose the latter alternative.

The chess was played in ancient India in a crude form called "chaturanga". The term "chaturanga" meant four divisions of the military, namely-infantry, cavalry, elephants and chariots. All the four divisions found representation in the game, in the form of pawn, knight, bishop and rook, respectively. As the game went to Persia, it came to be known as "shatranj" and witnessed many new rules being made. The name of the "king" piece changed from the Sanskrit "rajah" to the Persian "shah". From "shah" all European names for the game are derived. Shahmat, which means "the king dead" became the expression "checkmate".

Somewhere around the 9th century, chess made inroads into Western Europe and Russia. By the year 1000, everyone in Europe knew about the game.

Chess started to develop a corpus of theory somewhere around the end of 15^{th} century. The oldest book on chess till date, is "Repeticion de Amores y Arte de Ajedrez" ("Repetition of Love and the Art of Playing Chess"), by Spanish churchman Luis Ramirez de Lucena. It was published in Salamaca, in the year 1497. In 18^{th} century the center of European chess life moved from Southern European countries to France. The 19^{th} century saw chess organizations, chess clubs, chess books, chess journals and, inter-city chess matches came into being.

Transformation of chess into sports.

It was in the 1851 that chess transformed completely into sport, with the first modern chess tournament being held in London. The first official World chess Championship is regarded as the one organized in 1886, in which Wilhelm Steinitz beat Johannes Zukertort. After Steinitz came Emanuel Lasker (German), a mathematician who maintained the title of master for 27 years, the longest tenure of all World Champions. It was Jose Raul Capablanca, a prodigy from Cuba, who broke the German dominance by winning the title. After winning for the next seven years, he finally lost to Russian-French Alexander Alekhine. World Chess Federation (FIDE) was founded in Paris, in 1924. Three years later, the world saw the formation of the Women's World chess Championship, whose first winner was Czech-English master Vera Menchik. The former World Chess Champion is Viswanathan Anand, who won the championship tournament in September 2007 and defended his title until 2013. The current World Chess Champion is a Norwegian chess grandmaster Magnus Carlsen.[51]

📖 TOPICAL VOCABULARY

game	[geim]	n.	խաղ
popular	['pOpjulq]	a.	հանրաճանաչ
generation	["Genq'reiSn]	n.	սերունդ
recreational	['rekrieiSqnl]	a.	1. թարմացնող, ուժերը վերականգնող 2. հետաքրքիր, զվարճալի
competitive	[kqm'petitiv]	a.	մրցակցական
indulge	[in'dAlG]	v.	տարվել, անձնատուր լինել

correspondence	["kOris'pOndqns]	n.	թղթակցություն, նամակագրություն
tournament	['tuqnqmqnt]	n.	մրցախաղ, մրցում, տուրնիր
casual	['kxZjuql]	a.	պատահական
originate	[q'riGineit]	v.	ծագել, սկիզբ առնել, առաջանալ
crude	[krHd]	a.	1. խակ, թերահաս 2. չմշակված, անմշակ
infantry	['infqntri]	n.	հետևազոր, հետևակ
cavalry	['kxvqlri]	n.	հեծելազոր
chariot	['Cxriqt]	n.	կառք
pawn	['pLn]	n.	զինվոր
knight	[nait]	n.	շախմատի ձի
bishop	['biSqp]	n.	շախմատի փիղ
rook	[ruk]	n.	շախմատի նավակ
witness	['witnis]	v.	վկա, ականատես լինել
version	['vWSn]	n.	տարբերակ
derive	[di'raiv]	v.	ծագել, սկիզբ առնել
inroad	['inroud]	n.	ներխուժում
descendant	[di'sendqnt]	n.	հետնորդ, ժառանգ
corpus	['kLpqs]	n.	ժողովածու, հավաքածու (տարեգրությունների, օրենքների)
prodigy	['prOdiGi]	n.	շնորհալի, տաղանդավոր մարդ
tenure	['tenjuq]	n.	տիրության ժամկետ

I. What is the Armenian for?

a recreational competitive game, to indulge people, by correspondence, to cross international boundaries, to make inroads, a corpus of theory, to increase rapidly, to remain constantly in debt, to cut one's head off, to choose the latter alternative

II. What is the English for?

երկու խաղացողի մասնակցությամբ, ամբողջ աշխարհում, առաջանալ Հնդկաստանում, գոհացնել թագավորին, կատարել խոստումը, դատարկել պարկը, ցորենի քանակը, պարսկական «շահ» բառը դարձավ հանրաճանաչ, շախմատի տեսության ժողովածու, իսպանացի եկեղեցական

III. a) Arrange the words in pairs similar in meaning:

1.	**casual**	a.	competition
2.	**tournament**	b.	accidental
3.	**witness**	c.	nearly
4.	**rule**	d.	quantity
5.	**master**	e.	perform
6.	**invent**	f.	option
7.	**thankful**	g.	include
8.	**produce**	h.	leader
9.	**contain**	i.	spectator
10.	**alternative**	j.	law
11.	**fulfil**	k.	discover
12.	**amount**	l.	create
13.	**about**	m.	grateful

b) Arrange the words in pairs opposite in meaning:

1.	**cover**	a.	slow
2.	**modern**	b.	complicated
3.	**the latter**	c.	decrease
4.	**increase**	d.	the former
5.	**popular**	e.	uncover
6.	**empty**	f.	unknown
7.	**constant**	g.	improbable
8.	**rapid**	h.	elaborate
9.	**probable**	i.	variable
10.	**easy**	j.	old
11.	**crude**	k.	fill

IV. Form new words using suffixes and prefixes:

invent, cover, know, probable, popular, rapid, please, constant, publish, certain, legend, expect, contain, origin

V. Choose the suitable word:

1. Chess is one of the few ... games that is quite popular nowadays. (*mind / body*).
2. Though the origin of the game is pretty old, it continues ... people till date. (*to annoy / to indulge*).
3. The chess was played in ancient India in a ... form called “chaturanga”. (*elaborate / crude*).
4. According to an old legend the game of chess was ... by Sissa Ben Dahir. (*invent / find*).
5. The king was very ... with the present. (*disappointed / pleased*).
6. The vizier’s desire was quite ... to fulfil. (*difficult / easy*).
7. The king soon saw he would not be able to keep his ... (*word / documents*).

8. He had to choose ... alternative. (*the former / the latter*).
9. The author of the oldest book on chess till date is a ... churchman. (*German / Spanish*).

VI. Match the following words with their definitions:

1.	**chess**	a.	particular form or variant of something
2.	**casual**	b.	print and offer for sale, as a book etc.
3.	**competition**	c.	a body of writings, as laws
4.	**invent**	d.	give pleasure or satisfaction
5.	**vizier**	e.	something which one person is under an obligation to pay or render to another
6.	**alternative**	f.	ruler's senior aide or minister
7.	**debt**	g.	favoured or admired by the people, commonly known
8.	**please**	h.	create, produce or construct by original thought or ingenuity
9.	**popular**	i.	stating or offering either of two things expressing alternation
10.	**bushel**	j.	one descended from an ancestor
11.	**version**	k.	a game for two played on a board of 64 checkered squares
12.	**publish**	l.	happening by chance
13.	**descendant**	m.	unit of dry measure
14.	**corpus**	n.	a match between contestants

VII. Are these statements true or false? Correct the false statements:

1. Chess is an outdated game which is unpopular nowadays.
2. It is only preserved as a part of the Olympic Games.
3. The game of chess originated in France.
4. The game was invented and presented to the king by his vizier.
5. The king was displeased with the game and punished the vizier.
6. Pawn, knight, bishop and rook represent four ancient military divisions.
7. From "shah" all Euroepen names for the game are derived.
8. In the 19th century nobody ever heard about chess.
9. The first modern chess tournament was held in London in 1851.
10. Emanuel Lasker maintained the title of a master for 2 years.
11. World Chess Federation (FIDE) was founded in Paris, in 1924.

VIII. Speak on the key points of the text: "The Game of Chess".

IX. Read and discuss the text (state the methods of reasoning mathematicians and chessplayers use):

Chess and Mathematics

Mathematics is the queen
of natural knowledge.
C.F. GAUSS [52]

Chess is a fascinating and really very difficult field of research for mathematicians. On the other hand many more or less great chess players, along the history, since many centuries, were strongly interested in mathematics and particularly in mathematical problems connected with the chess game. Mathematical chess problem is a mathematical problem which is formulated using chessboard or chess pieces. These problems belong to recreational mathematics. The most known problems of this kind are Eight queens puzzle or Knight's Tour problems, which have connection to graph theory and combinatorics. Many famous mathematicians studied mathematical chess problems, for example, Euler, Legendre and Gauss. Besides finding a solution to a particular problem, mathematicians are usually interested in counting the total number of possible solutions, finding solutions with certain properties, as well as generalization of the problems to NxN or rectangular boards. Not all mathematical study of chess is purely recreational. The so-called rook polynomials have found applications in matrix theory and number theory. There's even a Rook Reciprocity Theorem!

How are chess theory and mathematics interrelated? According to Klein (1962), the major method of obtaining knowledge is reasoning. The most commonly used methods of reasoning in mathematics are reasoning by analogy, induction and deduction. Reasoning by analogy means what is true for the similar case should be true of the one in question.

The essence of induction is that one observes repeated occurrences of the same phenomenon and concludes that the phenomenon will always occur. This method is widely used in experimentation. In deductive reasoning we start with certain statement called premises and assert a conclusion which is a necessary or inescapable consequence of the premises. Let us consider an example, if we accept as basic facts that honest people return found money and that John is honest, we may conclude unquestionably that John will return money that he finds. It should be mentioned, that the conclusion drawn by analogy or induction has only probability of being correct, whereas the conclusion drawn by deduction necessarily holds. All mathematical proofs must be deductive. Let us consider the game of chess and try to find any special method of reasoning that chess players could use. Chess is a process of thought conditioned and limited by the rules of the game. The judgments of thoughts are visibly expressed upon the chessboard in movements of various forces. In the first place, to be a good chessplayer one needs visual imagery. Before you make a contemplated move, you have to visualize how the board will look after you make it, and then how it will be changed by your opponent's response. A chessplayer needs a good memory, patience and imagery, which are the links of associative reasoning. Inductive reasoning is not much used in chess, but it pays dividends in business and professional life.[53]

X.

- **Memorize the principle of mathematical induction:**

Induction reasoning is used in mathematics to help guess what might be true. Mathematical induction is often used as a method of definition as well as a method of proof.

The assumptions made in carrying out the induction step are known as the induction hypotheses.

If with each positive integer *n* there is associated a statement P_n, then all the statements P_n are true, provided the following two conditions are satisfied:

(1) P_1 is true.

(2) Whenever *k* is a positive integer such that P_k is true, then P_{k+1} is also true.

➢ **Discuss the theorem: "A Horse of a Different Colour".**

Theorem. All horses are the same colour.

Proof We shall prove that any set of horses contains only horses of a single colour. In particular, this will be true of the set of all horses. Let H be an arbitrary set of horses. Let us prove by mathematical induction on the number of horses in H that they are all the same colour.

- ***Basis*:** The case $n=0$ is trivially true, if there are no horses in *H*, then surely they are all the same colour! (If you do not like this argument, check the next simplest case: if $n=1$, then there is only one horse in *H*, and again it is vacuously clear that "they" are "all" the same colour).
- ***Induction step*:** Consider any number *n* of horses in *H*. Call these horses $h_1, h_2, \cdots h_n$. Assume the induction hypothesis that any set of $n-1$ horses contains only horses of a single colour (but of course the horses in one set could a priori be a different colour from the horses in another). Let H_1 be the set obtained by removing horse h_1 from H, and let H_2 be defined similarly: see the figure below:

$$\begin{array}{llllll} H_1: & & h_2 & h_3 & h_4 & h_5 \\ H_2: & h_1 & & h_3 & h_4 & h_5 \end{array}$$

Figure: Horses of the same colour (n=5)

There are $n-1$ horses in each of these two new sets. Therefore, the induction hypothesis applies to them. In particular, all the horses in H_1 are of a single colour, say c_1 , and all the horses in H_2 are also of a single (possibly different) colour, say c_2 . But is it really possible for colour c_1 to be different from colour c_2? Surely not, since horse h_n belongs to both sets and therefore both c_1 and c_2 must be the colour of that horse! Since all the horses in H belong to either H_1 or H_2 (or both), we conclude that they are all the same colour $c = c_1 = c_2$. This completes the induction step and the proof by mathematical induction.

Before you continue, figure out the fallacy in the above "proof". If you think the problem is that our induction hypothesis ("any set of $n-1$ horses must contain only horses of a single colour") was absurd, think again!

***Solution*:** The problem is that "h_n belongs to both sets" is not true for $n = 2$ since h_2 does not belong to H_2! Our reasoning was impeccable for the basis cases $n = 0$ and $n = 1$. Moreover, it is true that our theorem follows for sets of n horses assuming that it is true for $n-1$, but only when $n \geq 3$. We can go from 2 to 3, from 3 to 4, and so on, but not from 1 to 2. Since the basis cases contain only 0 and 1, and since we are not allowed to go from 1 to 2, the induction step cannot get started. This small missing link in the proof is enough to invalidate it completely. We encountered a similar situation when we proved that $n^3 < 2^n$: the induction step did not apply for $n < 5$, and thus the fact that the statement is true for $n = 0$ and $n = 1$ was irrelevant. The important difference was that $n^3 < 2^n$ is true for $n = 10$, and therefore also for all larger values of n.

Lesson 16

GEORG CANTOR

No one shall expel us from the
Paradise that Cantor has created.
DAVID HILBERT[54]

Georg Ferdinand Ludwig Philipp Cantor was born in St. Petersburg, Russia, on March 3.1845. Georg's background was very diverse. His father was a Danish Jewish merchant while his mother was a Russian musician Maria Anna Bohm. The family stayed in Russia for eleven years until the father's ailing health forced them to move to the more acceptable environment of Frankfurt, Germany, the country Georg would call home for the rest of his life. Georg, the oldest of six children, was an outstanding violinist, having inherited his parents' considerable musical and artistic talents. At the same time, he excelled in mathematics. His father saw this gift and tried to push his son into the more profitable but less challenging field of engineering. Georg was not at all happy about this idea but he lacked the assertiveness to stand up to his father and relented. However, after several years of training he became so fed up with the idea that he begged his father to become a mathematician.

In 1862, Georg Cantor entered the University of Zurich. After receiving a substantial inheritance upon his father's death in 1863, Cantor shifted his studies to the University of Berlin. There he studied under some of the greatest mathematicians of the day including Kronecker and Weierstrass. In 1867, Berlin University granted him the PhD for a thesis on number theory. After teaching briefly in a Berlin girls' school, Cantor took up a position at the University of Halle. All Cantor's professional career, some 44 years, was spent at Halle University, a small school without particular reputation.

In 1874, Cantor married and eventually had six children. It was that same year of 1874 that Cantor published his first paper on the theory of sets. While studying a problem in analysis, he had dug deeply into its "foundations", especially infinite sets. What he found flabbergasted him so much that he wrote to a friend: "I see it but I don't believe it". In a series of papers from 1874 to 1897, he was able to prove among other things that the set of integers had an equal number of members as the set of even numbers, squares, cubes; that the number of points in a line segment is equal to the number of points in an infinite line, a plane and 3 dimensional space; and that the quantity of transcendental numbers, numbers such as $\boldsymbol{\pi}$ and $\boldsymbol{e}$ that can never be the solution to any algebraic equation with integer coefficients, were much larger than the number of integers.

Interestingly, the Jesuits also used his theory to "prove" the existence of God and the Holy Trinity. However, Cantor, who was also an excellent theologian, quickly distanced himself away from such "proofs". Before in mathematics, infinity had been a taboo subject. Previously, Gauss had stated that infinity should only be used as "a way of speaking" and not as a mathematical value. Most mathematicians followed his advice and stayed away. However, Cantor would not leave it alone. He considered infinite sets not as merely going on forever but as completed entities, that are having an actual though infinite number of members. He called these actual infinite numbers transfinite numbers. By considering the infinite sets with a transfinite number of members, Cantor was able to come up with his amazing discoveries. For his work, he was promoted to full professorship in 1879.

However, his new ideas also gained him numerous enemies. Many mathematicians just would not accept his groundbreaking ideas that shattered their safe worlds of mathematics. One great mathematician, Henri Poincare expressed his disapproval, stating that Cantor's set theory would be considered by future generations as "a disease from which one has recovered". However, he was kinder than another critic, Leopold Kronecker. Cantor's former professor, Leopold Kronecker became the focus of Cantor's troubles, a sort of personal devil. Kronecker, who headed mathematics at Berlin until his death in 1891, became increasingly uncomfortable with the prospect of having Cantor as a colleague, perceiving him as a "corrupter of youth" for teaching his ideas to a younger generation of mathematicians. Among other things, he delayed or suppressed completely Cantor's and his followers' publications, raged both written and verbal personal attacks against him, belittled his ideas in front of his students and blocked Cantor's life ambition of gaining a position at the prestigious University of Berlin. Not all the mathematicians were antagonistic to Cantor's ideas. Some greats such as Mittag-Leffler, Karl Weierstrass, and long-time friend Richard Dedekind supported his ideas and attacked Kronecker's actions. However, it was not enough. Like with his father before, Cantor simply could not handle it. Stuck in a third-rate institution, stripped of well-deserved recognition for his work and under constant attack by Kronecker, he suffered the first of many nervous breakdowns in 1884. The rest of his life was spent in and out of mental institutions and his work nearly ceased completely.

Much too late for him to really enjoy it, his theory finally began to gain recognition by the turn of the century. In 1904, he was awarded a medal by the Royal Society of London and was made a member of both the London Mathematical Society and the Society of Sciences in Gottingen. He died in a mental institution on January 6.1918.

Today, Cantor's work is widely accepted by the mathematical community. His theory on infinite sets reset the foundation of nearly every mathematical field and brought mathematics to its modern form. In addition, his work has helped to explain Zeno's paradoxes that plagued mathematics for 2500 years. However, his theory also has led to many new questions, especially about set theory, that should keep mathematicians busy for centuries.[55]

🕮 TOPICAL VOCABULARY

diverse	[dai'vWs]	a.	զանազան, բազմազան
ailing	['eiliN]	a.	վատառողջ, հիվանդ
environment	[in'vaiqrqnmqnt]	n.	շրջապատ, միջավայր
inherit	[in'herit]	v.	ժառանգել
challenge	['CxlinG]	n. v.	մրցահրավեր, մարտահրավեր, հրավիրել (մրցման, մարտի)
assertiveness	[q'sWtivnis]	n.	հաստատակա-մություն
relent	[ri'lent]	v.	գթաշարժվել, փափկել, մեղմանալ
substantial	[sqb'stxnSql]	a.	էական, զգալի
flabbergast	['flxbqga:st]	v.	ապշեցնել
shatter	['Sxtq]	v.	ջարդվել, կոտրվել, փշրվել
prospect	['prOspekt]	n.	հեռանկար
perceive	[pq'sJv]	v.	գիտակցել, հասկանալ, ընկալել
corrupt	[kq'rApt]	v.	փչացնել, այլասերել
breakdown	['breikdaun]	n.	քայքայում, կործանում
taboo	[tq'bH]	n.	տաբու, արգելում

strip	[strip]	v.	խլել. կողոպտել
plague	[pleig]	v.	տանջել, ձանձրացնել
cease	[sJz]	v.	դադարել, դադարեցնել
suppress	[sq'pres]	v.	արգելել, թույլ չտալ
promote	[prq'mout]	v.	բարձրացնել, առաջ քաշել (պաշտոնում)
belittle	[bi'litl]	v.	նվաստացնել, նսեմացնել
block	[bl0k]	v.	արգելք լինել, խոչընդոտել

I. What is the Armenian for?

ailing health, outstanding violinist, profitable field of engineering, to lack the assertiveness, to become fed up with the idea, to be promoted to full professorship, to feel uncomfortable with the prospect, to shatter their safe worlds of mathematics, to reset the foundation, to be antagonistic to Cantor's ideas, to delay or suppress Cantor's publications

II. What is the English for?

գթաշարժվել, մեծ ժառանգություն, արգելված առարկա, հրաշալի աստվածաբան, չափազանց զարմացնել, հավանություն չտալ, ձեռք բերել բազմաթիվ թշնամիներ, խոչընդոտել ծրագրերի իրականացմանը, սաստիկ զայրացած հարձակվել, ճանաչում ստանալ, լայնորեն ընդունվել մաթեմատիկոսների հանրության կողմից, տառապել նյարդային համակարգի քայքայումից, դադարեցնել աշխատանքը

III. a) Arrange the words in pairs similar in meaning:

1.	**flabbergast**	a.	hinder
2.	**background**	b.	basis
3.	**merchant**	c.	substance
4.	**environment**	d.	terminate
5.	**gift**	e.	medium
6.	**contemporary**	f.	talent
7.	**beg**	g.	trader
8.	**grant**	h.	education
9.	**foundation**	i.	astonish
10.	**entity**	j.	up-to-date
11.	**cease**	k.	ask
12.	**block**	l.	award

b) Arrange the words in pairs opposite in meaning:

1.	**outstanding**	a.	demote
2.	**gifted**	b.	insignificant
3.	**disease**	c.	encourage
4.	**profitable**	d.	similar
5.	**gain**	e.	dull
6.	**recover**	f.	unprofitable
7.	**substantial**	g.	lose
8.	**enemy**	h.	ordinary
9.	**suppress**	i.	health
10.	**promote**	j.	ally
11.	**diverse**	k.	worsen

IV. Find the corresponding nouns from the text and translate them into Armenian:

inherit, assert, exist, disapprove, add, theoretical, spatial, gifted, recognise, follow, publish, solve, prove, advise, troublesome, valuable, critical, dead, generate

V. Match the words with their definitions:

1.	**gift**	a.	collapse as from illness
2.	**advice**	b.	break in pieces
3.	**theology**	c.	unlimited quantity or number
4.	**critic**	d.	something that has real existence, a thing
5.	**ambition**	e.	a ban or prohibition
6.	**antagonism**	f.	condemn, feel or express disfavor
7.	**award**	g.	mutual resistance or opposition of two forces in action
8.	**disapprove**	h.	bestow as a prize or reward
9.	**taboo**	i.	eager desire for distinction, power or fame
10.	**infinity**	j.	one who appraises the merit of others' works esp. artistic or literary
11.	**entity**	k.	the systematic study of God and His divinity
12.	**shatter**	l.	suggestion
13.	**breakdown**	m.	something given; a donation; a present

VI. Choose the suitable word:

1. Cantor's background was very ... (*poor / diverse*)
2. Georg ... the assertiveness to stand up to his father's idea. (*lacked / displayed*)
3. Cantor received a substantial ... upon his father's death. (*support / inheritance*)
4. Infinity was a ... subject in mathematics. (*compulsory / taboo*)
5. He called these actual infinite numbers ... numbers. (*odd / transfinite*)
6. By considering infinite sets with a transfinite number of numbers Cantor came up with ... discoveries. (*contradictory / amazing*)

7. Cantor's new ideas gained him numerous ... . (*allies / enemies*)
8. Professor Kronecker became the focus of Cantor's ... (*troubles / success*)
9. He ... Cantor's and his followers' publications. (*praised / suppressed*)
10. Cantor's work is widely ... by the mathematical community. (*accepted / rejected*)

VII. Are these statements true or false? Correct the false statements:

1. Georg Cantor was born in Germany, in 1812.
2. His father was a merchant, while his mother was a musician.
3. Georg was an outstanding violinist and at the same time he excelled in mathematics.
4. Cantor studied in the University of Berlin under prominent mathematicians Kronecher and Weiestrass
5. St. Petersburg University granted him Ph.D for a thesis on number theory.
6. G. Cantor never married and had no children.
7. In 1874 Cantor published his first scientific treatise on the theory of sets.
8. He was demoted for his work on transfinite numbers.
9. Cantor's new ideas gained him numerous rivals.
10. None of the mathematicians supported his ideas.
11. Georg Cantor suffered nervous breakdowns and died in a mental institution.
12. Cantor's theory on infinite sets penetrated nearly every mathematical field and brought mathematics to its modern form.

VIII. a) Fill in the words given in brackets:

(desire, significant, amateur, restriction, to be performed, counterparts, were not solved, were very popular, passed, specified).

b) Comment on the text:

Greek mathematics is … for the questions it raised and did not answer. Among such questions are three famous construction problems known to every … in mathematics. They are referred to as "squaring the circle", "doubling the cube" and "trisecting the angle". To square the circle means to construct a square, the area of which is equal to the area of a given circle. To double a cube means to construct the side of a cube whose volume shall be double that of a given cube. To trisect an angle means to divide any angle into three equal parts. These constructions are … only with an unmarked ruler and a compass. No other instruments are to be used.

The reason for this … sheds light on the classic attitude towards mathematics. A ruler and a compass are the physical … suggesting the concepts of a straight line and a circle. This restriction, self-imposed and arbitrary, was motivated by the … to keep geometry simple and harmonious. The three construction problems … in Greece. The first historical reference to them states that the philosopher Anaxagoras … his time in prison trying to square the circle. Despite the repeated efforts of the best Greek mathematicians the problems … Nor were they to be solved for the next two thousand years. It was finally proved that the constructions cannot be performed under the conditions …

IX. Speak on Georg Cantor's life, his thorny way to recognition and his revolutionary theory.

X. Prove:

- Consider the function defined by

$$T(n)=\begin{cases}\dfrac{3n+1}{2} & \text{for } n \text{ odd}\\[2ex] \dfrac{n}{2} & \text{for } n \text{ even}\end{cases}$$

The $3n+1$ conjecture is the claim that starting from any integer $n>1$, the sequence of iterates $T(n), T(T(n)), T(T(T(n))), \cdots$, eventually reaches the integer 1 and subsequently runs through the values 1 and 2. This has been verified for all $n \le 10^{16}$. Confirm the conjecture in the cases $n = 21$ and $n = 23$.

- Suppose that the numbers a_n are defined inductively by $a_1 = 1$, $a_2 = 2$, $a_3 = 3$ and $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for all $n \ge 4$. Use the Second Principle of Finite Induction to show that $a_n < 2^n$ for every positive integer *n*.

- If the numbers a_n are defined by $a_1 = 11$, $a_2 = 21$ and $a_n = 3a_{n-1} - 2a_{n-2}$ for $n \ge 3$, prove that $a_n = 5 \cdot 2^n + 1$, $n \ge 1$.

Lesson 17

SET THEORY

If you stop to think of all the new things that have been developed since you can remember, you will agree with the often repeated comment that you live in a changing world. Nothing can live and be useful without becoming adjusted to the conditions around it. This is as true of mathematics and other areas of knowledge as it is of plants and people. Better understanding of the meaning of these relationships and operations can sometimes be gained by studying them from a different viewpoint or expressing them in different terms and symbols.

One of the more recent approaches to the meanings of algebra is based on the idea of sets of objects or elements. The concept of a set is outwardly simple and has an intuitive counterpart in everyday experience. Although the mathematician in formally developing mathematical systems would prefer to consider the term set to be undefined, it is easy to suggest clearly both the meaning and the use of the term. A set is any well-defined collection of distinct objects, and it is synonymous with the terms aggregate or class. Although this concept is quite general, we must be careful to observe two things: a set consists of distinct elements of objects, so that every element of a set is a separate object, and no two elements of a set are identical. A set is usually represented by a capital letter. Any object that belongs to a set is called an element of the set. All elements of a set are to be enclosed in braces { } so that there should be no misunderstanding of what is included in the set.

We will always consider an elements as a member (or object) of some set; elements will not be permitted to exist without a parent set. It is evident that stating the property possessed by the elements of a set is the most meaningful way to specify a set. We are free to consider sets consisting of a finite number of elements, sets containing infinitely many elements or even sets consisting of only one element, say the set $\{a\}$. Indeed, one element sets are very important, for the set $\{a\}$ is not the same thing as the element a. Furthermore, the elements of sets may be any definite objects whatever and they may themselves be sets (in which case we will call the set "a collection of sets" rather than a set of sets). We will also admit the "empty set" or the set which consists of no elements into our family sets. Although it may appear strange to admit the existence of the "empty or null set", this concept is not as artificial as it might first appear to be.

Operations with Sets Two elements are equal, $x=y$, if x and y are the same element. Thus the word "equals" in mathematics means that the elements x and y are two names for the same object. This leads to definition for the equality of two sets: *The sets A, B are equal if they contain the same elements*.

If we are given a set, it seems natural to think of subdividing the set and to consider parts of it. Parts of a set are called subsets, but we must be more specific about this definition: The set R is a subset of the set T if every element of R is an element of T.

The Union of the sets A and B is the set of all elements which belong either to A or to B or to both A and B.

The product (or intersection) of the sets A and B is the set of elements which belong to both A and B.

The difference $B - A$ is the subset of B consisting of all elements of B that are not in A.

We will require that an element always be a member of some set, and all sets under discussion be subsets of some given (or universal) set.

The Number of Elements in a Set Sometimes we can observe that two sets do not have the same number of elements without counting the elements in either set: if we arrive at a cinema which is open and see a line of people waiting outside, we ordinarily conclude that the set of seats in the cinema contains fewer elements than the set of individuals desiring to see the film at the time, and in drawing this conclusion we do not know the number of elements in either set.

So we determine whether two sets have the same number of elements by attempting to “pair off” elements in the two sets. If the pairing process of correspondence is complete – there are no elements “left over” in either set – we say they have the same number of elements. This pairing principle is the basis for an important concept in mathematics known as one-to-one correspondence.

The sets S and T are placed in one-to-one correspondence if there is a pairing of the elements in the sets such that each element in S corresponds to one and only one element in T and each element in T corresponds to one and only one element in S.

When two sets can be placed into a one-to-one correspondence, they are referred to as equivalent. Two sets can be equivalent, of course, although their elements differ vastly; a set of fifteen elephants is equivalent to a set of fifteen football stadiums or to any other set containing fifteen elements.[56]

TOPICAL VOCABULARY

comment	['k0ment]	n.	մեկնաբանություն
true	[trH]	a.	ճշգրիտ, ստույգ, ճշմարիտ
adjust	[q'GAst]	v.	հարմարեցնել
plant	[pla:nt]	n.	բույս, սածիլ
fundamental	["fAndq'mentl]	a.	հիմնական, էական
likely	['laikli]	a.	հավանական
gain	['gein]	v.	1. ձեռք բերել, ստանալ 2. շահել, տանել (խաղը, գործը)

approach	[q'prouC]	n.	մոտեցում
intuitive	[in'tjHitiv]	a.	ինտուիտիվ, ներըմբռնողական, կռահողական
counterpart	['kauntqpa:t]	n.	երկնմանակ, կրկնորդ
experience	[iks'piqriqns]	n.	փորձառություն, կյանքի փորձ
distinct	[dis'tiNkt]	a.	1. տարբեր, առանձնա-հատուկ 2. պարզ, հստակ, որոշակի
aggregate	['xgrigit]	n.	ամբողջություն
synonymous	[si'n0nimqs]	a.	հոմանիշ
enclose	[ink'louz]	v.	ներփակել, մեջը դնել
particular	[pq'tikjulq]	a.	1. հատուկ, առանձնահա-տուկ 2. սպեցիֆիկ, յուրահատուկ
specify	[s'pesifai]	v.	ճիշտ որոշել, սահմանել
admit	[qd'mit]	v.	ընդունել, ճիշտ համարել
artificial	["a:ti'fiSql]	a.	արհեստական
equivalent	['ikwivqlqnt]	a.	համարժեք, համազոր
correspondence	["k0ris'p0ndqns]	n.	համապատասխանու-թյուն

I. What's the Armenian for?

often repeated comment, a changing world, to become adjusted to the conditions, fundamental processes, basic truth, recent approach, to be likely to change, to draw the conclusion, infinite number of terms

II. What's the English for?

գիտելիքի ոլորտ, կրկնորդ, համարել «բազմություն» տերմինը չսահմանված, քննարկվող բազմություններ, ներկայացվել մեծատառով, ձևավոր փակագծերի մեջ առնել, ներառել բազմության մեջ, պատկանել հավաքածուին

III. Arrange the words in pairs similar in meaning:

1.	**develop**	a.	accept
2.	**remember**	b.	utilize
3.	**repeat**	c.	coincide
4.	**change**	d.	reiterate
5.	**employ**	e.	false
6.	**misunderstand**	f.	recall
7.	**limit**	g.	correct
8.	**recent**	h.	different
9.	**distinct**	i.	current
10.	**satisfy**	j.	misinterpret
11.	**area**	k.	bound
12.	**true**	l.	field
13.	**artificial**	m.	advance
14.	**admit**	n.	please
15.	**correspond**	o.	alter

IV. Make opposites of these words by adding the following prefixes (*un, dis, mis, in, im*):

agree, developed, changing, use, different, defined, enclose, satisfy, likely, limited, easy, distinct, personal, similar, finite, understand

V. Find the corresponding nouns from the text and translate them into Armenian:

to comment, useful, to condition, algebraic, to relate, to operate, to term, symbolic, to mean, elementary, conceptual, experienced, mathematical, parental, to collect, numerical, to discuss, linear, to conclude, basic, to correspond

VI. Match the following words with their definitions:

1.	**truth**	a.	gross amount
2.	**aggregate**	b.	duplicate
3.	**knowledge**	c.	a part of any aggregate or whole
4.	**define**	d.	contain as a part, take in
5.	**member**	e.	remark or observation
6.	**comment**	f.	make to fit or conform; adapt
7.	**mathematics**	g.	mentally conceived image
8.	**concept**	h.	the science of structure, order and relation that has evolved from elemental practices of counting, measuring and describing the shapes of objects
9.	**adjust**	i.	state the meaning of, describe, explain
10.	**include**	j.	awareness of facts, truths or principles
11.	**counterpart**	k.	conformity of assertion to fact or reality

VII. Fill in the blanks with words and expressions from the text:

1. Nothing can live and be useful without … … to the conditions around.
2. It is generally agreed that we live in a ... …
3. This is true of … and other areas of …
4. The … of sets plays an essential part in …
5. The concept of a set is outwardly simple and has an intuitive … in everyday …
6. All elements of a set are to be … in braces.
7. The pairing principle is the basis for an important concept in mathematics known as … …
8. If two sets can be placed into one-to-one correspondence they are considered to be …

VIII. Insert articles where necessary:

Algebra in … Egypt appeared almost as soon as in ... Babylonia; but ... Egyptian algebra lacked ... sophistication in method shown by ... Babylonian algebra, as well as its variety in types of equations solved. For linear equations ... Egyptians used ... method of solution consisting of ... initial estimate followed by ... final correction, ... method now known as ... "rule of false position". ... algebra of Egypt, like that of ... Babylonia, was rhetorical.

... numeration systems of ... Egyptians, relatively primitive in comparison with that of ... Babylonians, help to explain ... lack of sophistication in ... Egyptian algebra. ... European mathematicians of ... sixteenth century had to extend ... Hindu-Arabic notion of number before they could progress significantly beyond ... Babylonian results in solving equations.

IX. Speak on the key points of the text: "Set Theory".

X. Comment on the text:

The more we study mathematics the more we see that the ideas and conceptions involved become more divorced and remote from experience, and the role played by the mind of the mathematician becomes larger and larger. The gradual introduction of new concepts which more and more depart from forms of experience finds its parallel in geometry and many of the specific geometrical terms are mental creations.

As mathematicians nowadays working in any given branch discover new concepts which are less and less drawn from experience and more and more from human mind the development of concepts is progressive and later concepts are built on earlier notions. These facts have unpleasant consequences. Because the more advanced ideas are purely mental creations rather than abstractions from physical experience and because they are defined in terms of prior concepts it is more difficult to understand them and illustrate their meanings even for a specialist in some other province of mathematics. Nevertheless, the current introduction of new concepts in any field enables mathematics to grow rapidly. Indeed, the growth of modern mathematics is, in part, due to the introduction of new concepts and new systems of axioms.[57]

XI. What is implied in the following quotations?

1. "Mathematics is the music of reason." (*James Joseph Sylvester*)
2. "The most painful thing about mathematics is how far away you are from being able to use it after you have learned it." (*James Newman*)
3. "Mathematics is a game played according to certain simple rules with meaningless marks on paper." (*David Hilbert*)

4. “The study of mathematics like the Nile, begins in minuteness but ends in magnificence.” (*Charles Caleb Colton*)
5. “It is generally recognized that women are better than men at languages, personal relations and multi-tasking, but less good at map-reading and spatial awareness. It is therefore not unreasonable to suppose that women might be less good at mathematics and physics.” (*Stephen Hawking*)
6. “It may be true, that men, who are mere mathematicians, have certain specific shortcomings, but that is not the fault of mathematics for it is equally true of every other exclusive occupation.” *(Carl Friedrich Gauss)*
7. “If I were again beginning my studies, I would follow the advice of Plato and start with mathematics.” (*Galileo Galilei*)
8. “All science requires mathematics. The knowledge of mathematical things is almost innate in us. This is the easiest of sciences, a fact which is obvious in that no one’s brain rejects it; for laymen and people who are utterly illiterate know how to count and reckon.” (*Roger Bacon*)
9. “Mathematics has beauty and romance. It’s not a boring place to be, the mathematical world, it’s worth spending time there.” (*Marcus de Suntoy*)
10. “In mathematics the art of proposing a question must be held of higher value than solving it.” (*Georg Cantor*)
11. “If there is a God he’s a great mathematician”. (*Paul Dirac*)
12. “Small minds discuss persons, average minds discuss events, great minds discuss ideas, really great minds discuss mathematics. (*Anon*)

XII. Prove:

- Establish the formulas below by mathematical induction:

(a) $1+2+3+\cdots+n=\dfrac{n(n+1)}{2}$ for all $n\geq 1$.

(b) $1+3+5+\cdots+(2n-1)=n^2$ for all $n\geq 1$.

(c) $1\cdot 2+2\cdot 3+3\cdot 4+\cdots+n(n+1)=\dfrac{n(n+1)(n+2)}{3}$ for all $n\geq 1$.

(d) $1^2+3^2+5^2+\cdots+(2n-1)^2=\dfrac{n(2n-1)(2n+1)}{3}$ for all $n\geq 1$.

(e) $1^3+2^3+3^3+\cdots+n^3=\left[\dfrac{n(n+1)}{2}\right]^2$ for all $n\geq 1$.

- If $r\neq 1$, show that for any positive integer n,

$$a+ar+ar^2+\cdots+ar^n=\frac{a\left(r^{n+1}-1\right)}{r-1}$$

- Use the Second Principle of Finite Induction to establish that for all $n\geq 1$.

$$a^n-1=(a-1)\left(a^{n-1}+a^{n-2}+a^{n-3}+\cdots+a+1\right)$$

[Hint: $a^{n+1}-1=\left(a+1\right)\left(a^n-1\right)-a\left(a^{n-1}-1\right)$.]

Lesson 18

THE PRINCE OF AMATEURS

His most notorious theorem baffled the
greatest minds for more than three centuries.
But after 10 years of work, one mathematician cracked it.
SIMON SINGH, KENNETH A.RIBET[58]

Pierre de Fermat was born on August 20, 1601, in Beaumont-de-Lomagne, a small town in southwest France. He pursued a career in local government and the judiciary. To insure impartiality, judges were discouraged from socializing, and so each evening Fermat would retreat to his study and concentrate on his hobby, mathematics. Although an amateur, Fermat was highly accomplished and was largely responsible for the development of the probability theory and the foundation of calculus. Isaac Newton, the father of modern calculus, stated that he had based his work on “Monsieur Fermat’s method of drawing tangents”.

Above all, Fermat was a master of number theory – the study of integers and their relationships. He would often write to other mathematicians about his work on a particular problem and ask if they had the ingenuity to match his solution. These challenges, and the fact that he would never reveal his own calculations, caused others a great deal of frustration. Rene Descartes, perhaps most noted for inventing coordinate geometry, called Fermat a braggart, and the English mathematician John Wallis once referred to him as “that damned Frenchman”.

Fermat penned his most famous challenge, his so-called last theorem, while studying the ancient Greek mathematical text *Arithmetica*, by Diophantus of Alexandria. The book discussed positive integer solutions to the equation $a^2+b^2=c^2$, Pythagoras’s formula describing the relation between the sides of a right triangle. This equation has many sets of integer solutions, such as a=3, b=4, c=5, which are known as Pythagorean triples. Fermat took the formula one step further and concluded that there are no nontrivial solutions for a whole family of similar equations, $a^n + b^n = c^n$, where n represents any integer greater than 2.

It seems remarkable that although there are infinitely many Pythagorean triples, there are no Fermat triples. Even so, Fermat believed he could support his claim with a rigorous proof. All his other theorems were proved, one by one, until only Fermat's last remained.

Numerous mathematicians battled the last theorem and failed. In 1742 Leonhard Euler, the greatest number theorist of the 18th century, became so frustrated by his inability to prove the last theorem that he asked a friend to search Fermat's house in case some vital scrap of paper was left behind. In the 19th century Sophie Germain – who, because of prejudice against women mathematicians, pursued her studies under the name of Monsieur Leblanc – made the first significant breakthrough. Germain proved a general theorem that went a long way toward solving Fermat's equation for values of *n* that are prime numbers greater than 2 and for which 2n+1 is also prime. But a complete proof for these exponents, or any others, remained out of her reach.

At the start of the 20th century Paul Wolfskehl, a German industrialist, bequeathed 100.000 marks to whoever could meet Fermat's challenge. Ironically, just as the Wolfskehl Prize was encouraging enthusiastic amateurs to attempt a proof professional mathematicians were losing hope. The problem held a special place in the hearts of number theorists, and they regarded Fermat's last theorem in the same way that chemists regarded alchemy. It was a foolish romantic dream from a past age.[59]

baffle	[bxfl]	v.	մոլորեցնել, փակուղու մեջ գցել
amateur	['xmqtW]	n.	սիրող, ոչ պրոֆեսիոնալ
pursue	[pq'sjH]	v.	1. շարունակել 2. զբաղվել (գիտությամբ, արվեստով և այլն)
judiciary	[GH'diSiqri]	n.	դատավորներ, դատավորական կազմ
impartiality	[im"pa:Si'xliti]	n.	անաչառություն, անկողմնակալություն, արդարացիություն
impartial	[im'pa:S(q)l]	a.	անաչառ
socialize	['souSqlaiz]	v.	հաղորդակցվել, շփվել
retreat	[ri'trJt]	v.	մեկուսանալ, առանձնանալ
accomplished	[q'kOmpliSt]	a.	կայացած

theory of probability *հավանականությունների տեսություն*

calculus **(*pl. calculi*)**	['kxlkjulqs] *['kxlkjulai]*	n.	1. հաշիվ 2. մաթեմատիկական անալիզ
ingenuity	["inGi'njuiti]	n.	հնարագիտություն, ճարտարամտություն
match	[mxC]	v.	համապատասխանել, համեմատել
reveal	[ri'vi:l]	v.	ցույց տալ, բացահայտել

frustration	[frAs'treiSn]	n.	խափանում (պլան-ների), խորտակում (հույսերի)
be frustrated	[bi frAs'treitid]	v.	հուսահատության մեջ ընկնել
braggart	['brxgqt]	n.	պարծենկոտ մարդ
support one's claim			*հիմնավորել պնդումը*
trivial	[triviql]	a.	1.առօրյա, սովորա-կան 2. չնչին, աննշան
rigorous	['rigqrqs]	a.	1. խիստ 2. ճշգրիտ
one by one			*մեկը մյուսի եփևից*
battle	['bxtl]	v.	հակադրվել, չընդունել
vital	['vaitql]	a.	էական, կարևոր
scrap of paper			*թղթի փոքր կտոր*
prejudice	['preGudis]	n.	կանխակալ կարծիք
breakthrough	['breikTrou]	n.	1. բեկում 2. հաղթա-նակ, ձեռքբերում *(գի-տական ոլորտում)*
prime number			*պարզ թիվ*
out of reach			*հասանելիության սահմաններից դուրս*
bequeath	[bi'kwJD]	v.	կտակել, ժառանգել
meet the challenge			*ընդունել մարտահրավերը*
regard	[ri'ga:d]	v.	համարել, ընդունել
hold a place			*տեղ զբաղեցնել*

I. What is the Armenian for?

to pursue a career, local government, to insure impartiality, to be discouraged from, to be responsible for, foundation of calculus, the method of drawing tangents, above all, a particular problem, to match the solution, to cause frustration, to pen, to take the formula one step further, sets of solutions, a triple, some vital scrap of paper, because of prejudice, to make the first significant breakthrough, to go a long way toward, integer

II. What is the English for?

անկողմնակալ լինել, կայացած, ոչ մասնագետ, հուսահատվել, մարտահրավեր, անալիզի հիմունքներ, հնարամտություն, համապատասխանել, ամբողջ թվով լուծումներ, ապացուցել, նշանակալից նվաճում/հաղթանակ, կտակել, կանխակալ կարծիքի պատճառով

III. a) Arrange the following words in pairs similar in meaning:

1.	**accomplished**	a.	sketch, design
2.	**baffle**	b.	gifty
3.	**amateur**	c.	cleverness
4.	**foundation**	d.	question
5.	**draw**	e.	disappointment
6.	**ingenuity**	f.	commonplace
7.	**challenge**	g.	consider
8.	**breakthrough**	h.	get together
9.	**integer**	i.	display
10.	**trivial**	j.	continue
11.	**socialize**	k.	confuse
12.	**regard**	l.	whole number
13.	**frustration**	m.	layman
14.	**reveal**	n.	achievement
15.	**pursue**	o.	base

b) Arrange the following words in pairs opposite in meaning:

1.	**retreat**	a.	trivial
2.	**discourage**	b.	prejudiced
3.	**reveal**	c.	blessed
4.	**significant**	d.	ability
5.	**impartial**	e.	at the end of
6.	**nontrivial**	f.	advance
7.	**damned**	g.	insignificant
8.	**greater than**	h.	encourage
9.	**support**	i.	less than
10.	**inability**	j.	contradict
11.	**at the start of**	k.	conceal

IV. a) Explain the meaning of the modal verb 'would' in the sentences given below:

1. Fermat would retreat to his study and concentrate on his hobby, mathematics. 2. He would often write to other mathematicians about his work on a particular problem. 3. The fact that he would never reveal his own calculations caused others a great deal of frustration.

b) Now bring your own examples.

V. Arrange the following words according to the parts of speech they belong to; adjective or adverb:

local, highly, largely, small, modern, particular, whole, own, famous, positive, infinitely, similar, nontrivial, great, rigorous, truly, last, significant, complete, ironically, enthusiastic, special

VI. Match the following words with their definitions:

1.	**braggart**	a.	to deprive of courage or confidence
2.	**pursue**	b.	the feeling of being upset or annoyed as a result of being unable to change or achieve something
3.	**match**	c.	consisting of or involving three items or people
4.	**socialize**	d.	get in touch with
5.	**responsible**	e.	leave (property) to a person by a will
6.	**ingenuity**	f.	having an obligation to do something
7.	**encourage**	g.	a person who boasts about their achievements or possessions
8.	**accomplished**	h.	the quality of being clever, original, and inventive
9.	**frustration**	i.	continue or proceed along
10.	**discourage**	j.	well educated and having good social skills
11.	**bequeath**	k.	a struggle to succeed or survive
12.	**battle**	l.	give support, confidence, or hope to (someone)
13.	**triple**	m.	correspond or cause to correspond in some essential respect; make or be harmonious

VII. Make the following sentences passive:

1. He pursued a career in local government and judiciary.
2. Fermat penned his most famous challenge, his so-called last theorem while studying the ancient Greek mathematical text *Arithmetica*.
3. The book discussed positive whole-number solutions to the equation $a + b = c$.

4. Numerous mathematicians battled the last theorem.
5. Sophie Germain pursued her studies under the name of Monsieur Leblanc.
6. Number theorists regarded Fermat's last theorem in the same way that chemists regarded alchemy.

VIII. Are the given statements true or false? Contradict the false statements:

1. Although an amateur Fermat pursued a career in local government and the judiciary.
2. Fermat was a master of number theory – the study of integers and their relationships.
3. His challenges didn't cause any frustration at all.
4. Although there are infinitely many Pythagorean triples, there are no Fermat triples.
5. Numerous mathematicians baffled Fermat's last theorem and were able to prove it.

IX. Match the beginnings with their appropriate endings:

1.	Isaac Newton had based his work …	a.	… called Fermat a braggart.
2.	English mathematician John Wallis …	b.	… on "Monsieur Fermat's method of drawing tangents".
3.	Paul Wolfskehl, a German industrialist …	c.	… once referred to him as "that damned Frenchman".

X. Paraphrase the following passage:

THE CHILDHOOD DREAM

Children, of course, love romantic dreams. And in 1963, at the age of 10, English mathematician Andrew Wiles became enamored with Fermat's last theorem. He read about it in his local library in Cambridge, England, and promised himself that he would find a proof. His schoolteachers discouraged him from wasting time on the impossible. His college lecturers also tried to dissuade him. Eventually his graduate supervisor at the University of Cambridge steered him toward more mainstream mathematics, namely into the fruitful research area surrounding objects called elliptic curves. Elliptic curves are not ellipses. They are named such because they are described by cubic equations like those used for calculating the perimeter of an ellipse. The ancient Greeks originally studied elliptic curves, and they appear in *Arithmetica*.

Little did Wiles know that this training would lead him back to Fermat's last theorem and in his *Annals of Mathematics* published in 1995 Wiles claimed that he had solved this puzzle though the mathematicians have continued to squabble.

XI. Speak on the key points of the text: "The Prince of Amateurs".

XII. Render the following passage into English using the following words and word combinations:

to graduate with honours, to hold professorship, candidate of sciences, to award, to make an effort, Ph.D., membership, scientific paper, social activity, to bring up generations, to pay attention to, natural sciences, high-qualified specialists, monumental

Հայ անվանի գիտնականը՝ մայր բուհի ռեկտոր

Սերգեյ Ալեքսանդրի Համբարձումյանը ծնվել է 1922թ. Ալեքսանդրապոլում (Գյումրի) իրավաբանի ընտանիքում: 1942թ. նա գերազանցությամբ ավարտել է Երևանի Պոլիտեխնիկական ինստիտուտը և անցել դասախոսական աշխատանքի նույն ինստիտուտում: 1946թ-ին նա արդեն գիտությունների թեկնածու էր, իսկ 1952թ-ին նրան շնորհվում է տեխնիկական գիտությունների դոկտորի գիտական աստիճան:

Ս. Համբարձումյանը մեծ ջանքեր է գործադրում Հայաստանում մեխանիկայի և մաթեմատիկայի զարգացման համար, որի ապացույցն է Մաթեմատիկայի և Մեխանիկայի ինստիտուտների ստեղծումը: 1977-1991թթ. նա Հայաստանի մայր բուհի՝ Երևանի պետական համալսարանի ռեկտորն էր, միաժամանակ լինելով աշխարհի բազմաթիվ համալսարանների դոկտոր ու պատվավոր անդամ: Նրա 200-ից ավելի գիտական աշխատությունները, որոնք թարգմանվել ու հրատարակվել են աշխարհի տարբեր լեզուներով, կարևոր առաջընթաց են նշանավորել ժամանակակից գիտության մեջ՝ նպաստելով տեխնիկական գիտությունների տարբեր ճյուղերի զարգացմանը:

Ս. Համբարձումյանի գիտական գործունեությունը ուղեկցվել է նաև գիտական ու մանկավարժական բարձրորակ կադրերի մի քանի սերունդների աճեցմամբ, հասարական ու քաղաքացիական ակտիվ գործունեությամբ: Նա հատուկ ուշադրություն է դարձրել ոչ միայն բնական գիտությունների, այլև հայագիտության զարգացմանը: Նրա նախաձեռնությամբ են հրատարակվել դարերի ընթացքում ստեղծված հայ պատմագրության ու գրականության կոթողային գանձերը:

Անվանի գիտնականը կնքել է իր մահկանացուն 2018 թվականին՝ 97 տարեկանում:

XII. Speak on the key points of the text: “The Prince of Amateurs”.

Lesson 19

NEWTON AND LEIBNIZ

No one must think Newton's great creation can be overthrown by Relativity or any other theory. His clear and wide ideas will forever retain their significance.

A. EINSTEIN[60]

The "invention" of the calculus is sometimes ascribed to two men, Newton and Leibniz. In reality, the calculus is the product of a long evolution that neither began nor finished by Newton and Leibniz, but in which both played a decisive part. The rudimentary notion of a limiting process emerged in the 1680s as Isaac Newton and Gottfried Leibniz struggled with the creation of the Calculus. Though each person's work was initially unknown to the other and their creative insights were quite different both realized the need to formulate a notion of function and the idea of quantities being "close" to one another. Two central problems held their attention. First, the problem of tangents; determining the tangent lines to a given curve, the fundamental problem of the differential calculus. Second, the problem of quadrature: determining the area under a given curve, the fundamental problem of the integral calculus. The vital observation made by Newton and, independently, by Leibniz was that areas under curves could be calculated by reversing the differentiation process. This exciting technique, one that solved previously difficult area problems with ease, sparked enormous interest among the mathematicians of the era and led to a coherent theory that became known as the differential and integral calculus.

Isaac Newton (1642-1727) was born in Woolsthorpe, in Lincolnshire, England, on Christmas Day; his father, a farmer, had died three months earlier. At the age of 11 he was sent to boarding school in Grantham. Fortunately, a perceptive teacher noticed his mathematical talent and, in 1661, Newton entered Trinity College at Cambridge University, where he studied with Isaac Barrow. When the bubonic plague struck in 1665-1666, leaving dead nearly 70.000 persons in London, the university closed and Newton spent two years back in Woolsthorpe. It was during this period that he formulated his basic ideas concerning optics, gravitation, and his method of "fluxions", later called "calculus". He returned to Cambridge in 1667 and was appointed Professor in 1669. His theories of universal gravitation and planetary motion were published to world acclaim in 1687 under the title "Philosophiae Naturalis Principia Mathematica" ("The Mathematical Principles of Natural Philosophy"). However, he neglected to publish his method of inverse tangents for finding areas, and this led to a controversy over priority with Leibniz.

In 1703 he was elected President of the royal Society and held the office till his death in 1727.

At his funeral, Newton was eulogized as "the greatest genius that existed". His place of burial in Westminster Abbey is a popular tourist site.

Gottfried Wilhelm Leibniz (1646-1716) was born in Leipzig, Germany. He was six years old when his father, a professor of philosophy, died and left his son the key to his library and a life of books and learning. Leibniz entered the University of Leipzig at the age of 15, graduated at the age of 17, and received a Doctor of Law degree from the University of Altdorf four years later. He wrote on legal matters, but was more interested in philosophy. He also developed original theories about language and the nature of the universe. In 1672, he went to Paris as a diplomat for four years. While living in Paris he began to study mathematics with the Dutch mathematician Christian Huygens. His travels to London to visit the Royal Academy further stimulated his interest in mathematics. His background in philosophy led him to very original, though not always rigorous, results.

Unaware of Newton's unpublished work, Leibniz published papers in the 1680s that presented a method of finding areas that is known today as the Fundamental Theorem of Calculus. Unfortunately, some followers of Newton accused Leibniz of plagiarism, resulting in a dispute that lasted until Leibniz's death. Their approaches to calculus were quite different and it is now evident that their discoveries were made independently. Leibniz is now renowned for his work in philosophy, but his mathematical fame rests on his creation of the calculus. Leibniz introduced the term "function" to indicate a quantity that is depended on a variable, and invented "infinitesimally small" numbers as a way of handling the concept of a limit. He coined the term "calculus" for the new method of calculation, invented $\frac{d_x}{d_y}$ and elongated S notations for the integral that are used today.

Newton and Leibniz developed and organized the subject, making it one of the most powerful chapters of mathematics. The invention of the calculus initiated a period of exceedingly rapid development both in mathematics and in its applications to physics, astronomy, engineering, etc. In the latter half of the seventeenth century and the entire eighteenth century, much effort was devoted to the important task of developing further the ideas of Newton and Leibniz and their manifold applications in physical science.[61]

🕮 TOPICAL VOCABULARY

rudimentary	["rudi'mentqri]	a.	1. տարրական 2. թերաճ, չզարգացած

emerge	[i'mWG]	v.	երևան գալ, հայտնվել, առաջանալ
refer	[ri'fW]	v.	1. հղել 2. վերաբերել որևէ բանի
initial	[i'niSql]	a.	սկզբնական
insight	['insait]	n.	խորաթափանցու-թյուն (մտքի)
coherent	[kou'hiqrqnt]	a.	1. համաձայնեցված, հաջորդական, հետևողական, 2. կապակցված
acclaim	[q'kleim]	v.	ողջունել, ազդարարել
stimulate	['stimjuleit]	v.	1. խթանել 2. քաջալերել
rigorous	['rigqrqs]	a.	1. խիստ 2. ճշգրիտ
accuse	[q'kjHz]	v.	մեղադրել
plagiarism	['pleiGiqrizm]	n.	գրագողություն
dispute	[dis'pjHt]	n.	բանավեճ, վիճաբանություն
renowned	[ri'naund]	a.	հռչակավոր, ականավոր, փառաբանված
perceptive	[pq'septiv]	a.	ընկալող, ըմբռնող
fluxion	[flAkSqn]	n.	ածանցյալ
appoint	[q'pOint]	v.	նշանակել
inverse	['invWs]	a.	հակադարձ, հակառակ, շրջված

controversy	['k0ntrqvWsi]	n.	վիճաբանություն, բանավեճ
priority	[prai'0riti]	n.	առաջնություն
funeral	['fjHnqrql]	n.	թաղում
eulogize	['jHlqGaiz]	v.	գովասանական ճառ արտասանել
ease	[Jz]	n.	հեշտություն
vital	[vaitl]	a.	1. կենսական 2. էական, կարևոր, անհրաժեշտ
spark	[spa:k]	v.	1. կայծ արձակել, կայծ տալ 2. բռնկվել
coin	['k0in]	v.	ստեղծել նոր բառեր
office	['0fis]	n.	պաշտոն, դիրք
manifold	['mxnifould]	a.	1. բազմազան 2. բազմաթիվ
neglect	[ni'glekt]	v.	կարևորություն չտալ

I. What is the Armenian for?

the rudimentary notion, creative insights, an exciting technique, a coherent theory, universal gravitation, held their attention, to be renowned, with ease, to make discoveries independently, to reverse the differentiation process

II. What is the English for?

վճռորոշ դեր, կարևոր դիտարկում, մեծ հետաքրքրություն առաջացնել, մեղադրել գրագողության մեջ, ուշադիր ուսուցիչ, մոլորակների շարժում, ճշգրիտ արդյունքներ, հակասություն առաջնայնության վերաբերյալ, ստեղծել “calculus” տերմինը, զբաղեցնել պաշտոնը մինչև կյանքի վերջ, բազմաթիվ կիրառություններ

III. a) Arrange the words in pairs similar in meaning:

1.	**acclaim**	a.	progression
2.	**stimulate**	b.	decide
3.	**rigorous**	c.	distinguished
4.	**accuse**	d.	reverse
5.	**dispute**	e.	encourage
6.	**renowned**	f.	controversy
7.	**inverse**	g.	exact
8.	**sequence**	h.	blame
9.	**determinate**	i.	approve

b) Arrange the words in pairs opposite in meaning:

1.	**decisive**	a.	shorten
2.	**ease**	b.	advanced
3.	**priority**	c.	constant
4.	**emerge**	d.	meaningless
5.	**coherent**	e.	disappear
6.	**variable**	f.	inferiority
7.	**rudimentary**	g.	difficulty
8.	**alongate**	h.	indecisive

IV. Form nouns from the given verbs and translate them into Armenian:

differentiate, depend, know, perceive, notice, formulate, acclaim, emerge, appoint, publish, bury, die, stimulate, originate, introduce, coin, eulogize, approach, indicate

V. Fill in the blanks with words and expressions from the text:

1. The rudimentary notion of a limiting process emerged in …
2. In reality, the calculus is the product …
3. Both Newton and Leibniz played … in the development of the Calculus.
4. According to Leibniz and Newton areas under curves could be calculated by …
5. Newton studied at boarding school where … noticed his mathematical gift.
6. In 1661, Newton entered … at Cambridge University.
7. In 1669 he was appointed …
8. Newton was elected … and held the office …
9. At his funeral Newton was eulogized …
10. Leibniz entered … at the age of 15.
11. He received … from the University of Altdorf.
12. Leibniz began to study mathematics with …
13. Some followers of Newton accused Leibniz of …
14. His visit to Royal Academy in London … his interest in mathematics.
15. Leibniz is renowned for his work in philosophy, but his mathematical fame rests …

VI. Match the words with their definitions:

1.	**rudimentary**	a.	a line no part of which is straight
2.	**sequence**	b.	argue, debate
3.	**limit**	c.	the ceremony of burying a dead person
4.	**perceptive**	d.	attraction
5.	**eulogize**	e.	nominate or assign; designate
6.	**renowned**	f.	select for office by vote
7.	**plagiarism**	g.	epidemic disease
8.	**plague**	h.	the offering of another artistic or literary work as one's own
9.	**elect**	i.	famous
10.	**appoint**	j.	praise highly
11.	**gravitation**	k.	quick to perceive
12.	**funeral**	l.	boundary
13.	**dispute**	m.	succession
14.	**curve**	n.	undeveloped

VII. Are the statements true or false? Correct the false statements:

1. Mathematical analysis deals with limiting concepts.
2. Leibniz and Newton played insignificant role in Mathematics.
3. The technique invented by Newton and Leibniz aroused great interest among the mathematicians of the era.
4. They created the theory of games.

5. Newton studied at Trinity College at Cambridge University and in 1669 was appointed Professor of the same University.
6. Newton had to return to Woolsthorpe because of the plague. That was very fruitful period in his life.
7. Newton was the first to publish his treatise on the calculus and there was not any controversy between two scientists.
8. Leibniz and Newton were born in France.
9. Leibniz studied neither law, nor philosophy.
10. Christian Hygens, a mathematician from Holland, taught Leibniz mathematics.
11. Leibniz was never accused in plagiarism by some followers of Newton.
12. Leibniz created some terms which are widely used in mathematics.
13. Leibniz is famous both as a philosopher and as a mathematician.

VIII. Insert prepositions: *of, on, to, among.*

Although Apollonius was an astronomer … merit, and although he wrote … a variety … mathematical subjects, his chief claim … fame rests on his extraordinary and monumental “Conic Sections”, a work that earned him the title, … his contemporaries, … the “Great Geometer”. Apollonius’ “Conic Section” is a thorough investigation … these curves and it completely superseded all earlier works ... the subject. It was Apollonius who supplied the terms “ellipse”, ”parabola” and “hyperbola”.

IX. Discuss the following mathematical statements and correct them if they are wrong:

1. Every square is a rhombus.
2. Every trapezoid is a parallelogram.
3. No parallelogram is a trapezoid.
4. The opposite sides of a parallelogram are congruent to each other.

5. A rectangle that is inscribed in a circle is a square.
6. No trapezoid has a pair of congruent sides.
7. Two sides of a scalene triangle are unequal.
8. A straight line extends indefinitely only in one direction.
9. No trapezoid has two right angles.
10. All three sides of an isosceles triangle are equal.
11. The bases of a parallelepiped are circles.
12. No rhombus is a trapezoid.

X. Speak on the key points of the text: "Newton and Leibniz".

XI. Memorize the following definitions:

➢ ***Definition 1.* Instantaneous Rate of Change**

The **Instantaneous rate of change** of f with respect to x at x_0 is the derivative

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

provided the limit exists.

➢ ***Definition 2.* The Definite Integral as a Limit of Riemann Sums**

Let f be a function defined on a closed interval $[a,b]$. For any partition P of $[a,b]$, let the numbers c_k be chosen arbitrarily in the subintervals $\left[x_{k-1}, x_k\right]$.

If there exists a number I such that

$$\lim_{\|P\| \to 0} \sum_{k=1}^{n} f(c_k)\Delta x_k = I$$

no matter how P and the c_k 's are chosen, then f is **integrable** on $[a,b]$ and I is the definite integral of f over $[a,b]$.

➢ ***Definition 3.*** **Area under a Curve (as a Definite Integral)**

If $y = f(x)$ is nonnegative and integrable over a closed interval $[a,b]$, then the area under the curve $y = f(x)$ from *a* to *b* is the integral of f from *a* to *b:*

$$A = \int_a^b f(x)dx\,.$$

Lesson 20

THE THEORY OF ERRORS

Errors like straws upon the surface flow;
he who would search for pearls must dive below.
JOHN BRYDEN[62]

From the scientific point of view Thomas Simpson, in his *Miscellaneous Tracts* (1757), was the first to examine critically the implications of taking the mean of a set of astronomical observations of the same event. Thus this theory, now an integral part of the subject of the significance of errors, owes its origin to astronomical needs. Naturally, the French experimentalists were by now equally concerned with the same problem. In 1770 Lagrange published his memoir on the method of taking the best value from a series of observations.

All these results are obtained by expansion of multinominal expressions and other purely algebraic processes; but at the same time a new conception was introduced by Simpson and Lagrange which proved later to be exceedingly fertile in analysis- the idea of an error curve. For reasons to be explained, "errors" or divergences from the "true" value necessarily consist of a discontinuous set of data but apart from the calculus of finite differences the whole field of mathematics concerned itself with "continuous" phenomena. Thus, in the face of mathematical limitations, the facts regarding the nature of error were altered to suit, and both Simpson and Lagrange introduced the notion of continuous variation in error. The analogy did not proceed very far; but nevertheless, the concept of errors in a continuum x with a probability function F (x) had now found its place.

The critical work of the French Encyclopédistes did not proceed far, conducted as it was by individuals who were for the most part non-mathematicians and who failed therefore to distinguish between mathematically and socially important considerations. Even a distinguished mathematician like D’Alembert, who directed his criticism at the fundamental definitions in probability theory, succeeded only in arriving at the most preposterous conclusions. The Marquise de Condorcet dealt with such questions as the probability of election of a candidate by a given number of voters, and the probability of a tribunal arriving at a true verdict in a trial. In view of his faith in the necessary progress of the human race towards happiness and perfection, it is one of the ironies of history that he himself was condemned by the revolutionary tribunal.

It is during this period that the problem of “inverse probability”, first considered by James Bernoulli, again shows itself, in two posthumous memoirs by Bayes which appeared in the Philosophical Transaction for 1764-1765.[63]

TOPICAL VOCABULARY

miscellaneous	["misq'leinjqs]	a.	զանազան, բազմազան
tract	[tr x kt]	n.	բրոշյուր, տրակտատ
implication	["impli'keiSn]	n.	եզրակացություն
observation	["Obzq: 'veiSn]	n.	դիտում, զննում
integral	['intigrql]	n.	ինտեգրալ
memoir	['memwa:(r)]	n.	հուշեր, հուշագրություններ *pl.* տեղեկագիր, գիտական աշխատությունների ժողովածու
expansion	[iks'p x nSn]	n.	վերլուծություն

multinominal	['mAlti'nOminql]	a.	բազմանդամ
conception	[kqn'sepSn]	n.	1. հասկացություն 2. մտահղացում
exceedingly	[ik'si:diNli]	adv.	չափազանց
fertile	['fq:tail]	a.	*փխբ.* արգասավոր, առատ
curve	[kq:v]	n.	կոր
divergence	[dai'vq:Gqns]	n.	տարբերություն, խոտորում, շեղում
value	['vxlju:]	n.	արժեք
alter	['O:ltq]	v.	փոխ(վ)ել, վերափոխել
notion	[nouSn]	n.	1.հասկացություն 2. գաղափար
variation	[veqri'eiSn]	n.	1. փոփոխություն 2. շեղում, վարիացիա
analogy	[q'nxlqGi]	n.	համանմանություն, նմանություն
proceed	[prq'si:d]	v.	առաջ գալ, առա-ջանալ
distinguish	[dis'tiNgwiS]	v.	տարբեր(վ)ել, բնորոշել
distinguished	[dis'tiNgwiSt]	a.	ականավոր, հռչա-կավոր
continuum ***(pl. continua)***	[kqn'tinjuqm] *[kqn'tinjuq]*		կոնտինուում
probability	["prObq'biliti]	n.	հավանականություն
definition	["defi'niSn]	n.	սահմանում

preposterous	[pri'pOstqrqs]	a.	1. անմիտ 2. սխալ, աղավաղված 3. ոչ ռացիոնալ, ոչ բանական
conclusion	[kqn'klu:Zn]	n.	եզրակացություն, եզրափակում, ամփոփում
election	[i'lekSn]	n.	ընտրություն
voter	['voutq]	n.	ընտրող, քվեարկու- թյան մասնակից
tribunal	[tri'bju:nql]	n.	տրիբունալ, դատա- րան
verdict	['vq:dikt]	n.	դատավճիռ, դատողություն
trial	['traiql]	n.	դատական քննու- թյուն, դատ
condemn	[kqn'dem]	v.	դատապարտել, դա- տավճիռ կայացնել
inverse	['in'vq:s]	a.	հակադարձ
posthumous	['pOstjumqs]	a.	ետմահու
transaction	[trxn'zxkSn]	n.	գործարք
consideration	[kqn"sidq'reiSn]	n. *pl.*	կարծիք, նկատառում

I. What is the Armenian for?

to be exceedingly fertile, were altered to suit, philosophical transaction, divergence, continuous variation, implication, to be condemned by, miscellaneous, owes its origin to astronomical needs, continuum, multinominal expressions, to be concerned with, inverse probability, discontinuous set of data, scientific point of view, preposterous conclusions, one of the ironies of history, posthumous memoirs, distinguished mathematician, significance of errors, in the face of mathematical limitations

II. a) Arrange the words in pairs similar in meaning:

1.	**error**	a.	consider, notice, view
2.	**origin**	b.	improvement, advancement
3.	**regard**	c.	idea, conception, opinion
4.	**implicate**	d.	genuine, exact
5.	**progress**	e.	irrational, absurd
6.	**true**	f.	mistake, fault
7.	**preposterous**	g.	productive, fruitful
8.	**irony**	h.	beginning, source
9.	**fertile**	i.	sarcasm, satire
10.	**notion**	j.	involve, compromise

b) Arrange the words in pairs opposite in meaning:

1.	**proceed**	a.	disbelief, infidelity
2.	**analogy**	b.	improbability, unlikelihood
3.	**faith**	c.	hazard, disregard
4.	**progress (n.)**	d.	recede, stop
5.	**introduce**	e.	termination, end
6.	**origin**	f.	justify, approve
7.	**trial**	g.	conclude
8.	**fail**	h.	delay, failure, regress
9.	**condemn**	i.	dissimilarity
10.	**probability**	j.	succeed

III. Match the following words with their definitions:

1.	**field**	a.	choosing or selection (of candidates for an office, etc.)
2.	**calculus**	b.	give judgment against
3.	**miscellaneous**	c.	be published, come into view
4.	**election**	d.	area or department of study or activity
5.	**appear**	e.	getting farther apart from a point or from each other as they progress
6.	**curve**	f.	of mixed sorts, having various qualities and characteristics
7.	**divergence**	g.	inverted; reversed in position, direction or relations
8.	**condemn**	h.	1. a compact connected metric space 2. nondenumerable set of real numbers, denoted *c*
9.	**memoirs**	i.	branch of mathematics that deals with variable quantities, used to solve many mathematical problems
10.	**inverse**	j.	an object similar to a line but that need not be straight
11.	**continuum**	k.	person's written account of his own life or experience

IV. Give derivatives corresponding to the following words:

perfect, appear, theory, observe, diverge, vary, define, express, differ, limit

V. Write the plural forms of:

data, phenomena, calculus, analysis, continuum, matrix, lemma, axis

VI. Find the corresponding nouns from the text:

implicate, significant, expand, vary, analogous, introduce, note, express, differ, happy, conclude

VII. Choose the correct preposition:

1. The concept of errors … a continuum x … a probability function F(x) had now found its place. *(in / of), (with / without)*
2. D'Alembert, who directed his criticism … the fundamental definitions … probability theory, succeeded only … arriving … the most preposterous conclusions. *(at / of), (in / on), (at / in), (to / at)*
3. The French experimentalists were … now equally concerned … the same problem. *(till / by), (with / of)*
4. … view of his faith … the necessary progress of the human race … happiness and perfection, it is one of the ironies … history that he himself was condemned … the revolutionary tribunal. *(in / on), (at / in), (towards / for), (of / for), (at / by)*
5. … the face … mathematical limitations, the facts regarding the nature … error were altered to suit, and both Simpson and Lagrange introduced the notion of continuous variation … error. *(in / on), (on / of), (by / of), (at / in)*

VIII. Choose the suitable word:

1. In 1770 Lagrange published his … on the method of taking the best value from a series of observations. *(memoir/ memory)*
2. A new … was introduced by Simpson and Lagrange which proved later to be exceedingly fertile in analysis - the idea of an error curve. *(conception /conceive)*
3. Thomas Simpson was the first to examine critically the implications of taking the mean of a set of … observations of the same event. *(astronomy/ astronomical)*

4. The critical work of the French Encyclopédistes did not … far. *(proceed/process)*
5. The Marquise de Condorcet dealt with such questions as the … of election of a candidate by a given number of voters. *(probable/probability)*
6. A … mathematician like D'Alembert, who directed his criticism at the fundamental definitions in probability theory, succeeded only in arriving at the most preposterous conclusions. *(distinguished/distinguishable)*

IX. Match the beginnings with their appropriate endings:

1.	The French experimentalists were by now …	a.	… altered to suit, and both Simpson and Lagrange introduced the notion of continuous variation in error.
2.	It is one of the ironies of history that the Marquis de Condorcet himself was …	b.	… the whole field of mathematics concerned itself with "continuous" phenomena.
3.	In the face of mathematical limitations, the facts regarding the nature of error were …	c.	… equally concerned with the same problem.
4.	Errors or divergences from the "true" value necessarily consist of a discontinuous set of data but apart from the calculus of finite differences …	d.	… appeared in the "Philosophical Transactions" for 1764-1765.
5.	The problem of "inverse probability", in two posthumous memoirs by Bayes, …	e.	… condemned by the revolutionary tribunal.

X. Insert definite or indefinite articles if necessary:

Mathematics as …, viewed as … whole, is … collection of branches. … largest branch is that which builds on …ordinary whole numbers, fractions, and irrational numbers, or what, collectively is called … real number system. Arithmetic, algebra, … study of … functions, … calculus, differential equations, and various other subjects which follow … calculus in logical order are all developments of … real number system. This part of mathematics is termed … mathematics of number. … second branch is … geometry consisting of several geometries.

XI. Comment on the following quotes of outstanding people:

1. Errors using inadequate data are much less than those using no data at all.[64]

   (*Charles Babbage*)

2. Mathematicians do not study objects, but relations between objects. Thus, they are free to replace some objects by others so long as the relations remain unchanged. Content to them is irrelevant: they are interested in form only.[65]

   (*Henri Poincaré*)

3. Mathematicians are like Frenchmen: whatever you say to them they translate into their own language and forthwith it is something entirely different.[66]

   (*Johann Wolfgang von Goethe*)

4. To a mathematician, real life is a special case.

   (*Johann Wolfgang von Goethe*)

5. Everyone knows what a curve is, until he has studied enough mathematics to become confused through the countless number of possible exceptions.[67]

   (*Felix Klein*)

6. You know we all became mathematicians for the same reason: we were lazy.[68]

(*Max Rosenlicht*)

7. The life of a mathematician is dominated by an insatiable curiosity, a desire bordering on passion to solve the problems he is studying.[69]

(*Jean Dieudonne*)

XII. Render the text into English:

Արտաշես Շահինյանի դերը համալսարանական կրթության և գիտության զարգացման գործում

Հայ մաթեմատիկոս, ակադեմիկոս և մանկավարժ, հայկական մաթեմատիկական գիտության դպրոցի հիմնադիր Արտաշես Շահինյանը ծնվել է 1906թ. դեկտեմբերի 19-ին Ալեքսանդրապոլում (այժմ՝ Գյումրի): 1924թ. ավարտել է միջնակարգ դպրոցը, 1926թ.՝ Երևանի արդյունաբերական տեխնիկումը և ընդունվել ԵՊՀ տեխնիկական ֆակուլտետ: Չորրորդ կուրսից փոխադրվել է ֆիզիկամաթեմատիկական ֆակուլտետ, որն ավարտել է 1930թ.-ին:

1929-1934թթ. Ա. Շահինյանը աշխատել է ԵՊՀ-ում, սկզբում որպես մաթեմատիկայի ամբիոնի ասիստենտ, ապա՝ դոցենտ և ամբիոնի վարիչի պաշտոնակատար: 1934-1937թթ. նա սովորել է Լենինգրադի պետական համալսարանին առընթեր մաթեմատիկայի և մեխանիկայի գիտահետազոտական ինստիտուտի ասպիրանտուրայում: 1939թ. պաշտպանել է թեկնածուական ատենախոսություն: 1938-1944թթ. եղել է ԵՊՀ բարձրագույն երկրաչափության ամբիոնի վարիչ, 1939-1942թթ.՝ ֆիզիկամաթեմատիկական ֆակուլտետի դեկան: 1944թ. պաշտպանել է դոկտորական ատենախոսություն և մինչև 1978թ. եղել է մաթեմատիկական անալիզի և ֆունկցիաների տեսության ամբիոնի վարիչ: 1945թ. նրան շնորհվել է պրոֆեսորի կոչում:

1945թ. Շահինյանը ընտրվել է ՀԽՍՀ ԳԱ թղթակից անդամ, 1947թ.՝ ակադեմիկոս: 1945-1950թթ. նա ՀԽՍՀ ԳԱ մաթեմատիկայի և մեխանիկայի սեկտորի վարիչն էր, 1950-1959թթ.՝ ԳԱ մաթեմատիկայի և մեխանիկայի ինստիտուտի տնօրենը, 1950-1963թթ.՝ ՀԽՍՀ ԳԱ ֆիզիկամաթեմատիկական գիտությունների բաժանմունքի ակադեմիկոս-քարտուղարը:

1965թ. հիմնադրել է ԵՊՀ-ին կից ֆիզիկամաթեմատիկական դպրոցը, որը 1986թ-ից կրում է Ա. Շահինյանի անունը:

1961 թ. արժանացել է ՀԽՍՀ գիտության վաստակավոր գործչի կոչման: Եղել է ՀԽՍՀ Գերագույն խորհրդի VII և VIII գումարումների պատգամավոր: Պարգևատրվել է Աշխատանքային կարմիր դրոշի (2), ժողովուրդների բարեկամության շքանշաններով:

Ա. Շահինյանը վախճանվել է 1978թ. մայիսի 14-ին Երևանում:[70]

XIII. Speak on the key points of the text "The Theory of Errors".

Lesson 21

THOMAS SIMPSON

"Perfect numbers like perfect men are rare."
Rene Descartes[71]

The British mathematician, inventor Thomas Simpson was born in 1710, on August 20. He lived in Leicestershire, England where his father was a weaver. Simpson's father naturally expected his son to take up the same profession as his. Thomas Simpson received little formal education and taught himself mathematics. He first worked as a weaver. However, with the occurrence of a solar eclipse in 1724, Thomas Simpson turned to "mathematical interests", changing his life forever. After all, a solar eclipse "eclipsed" his world of weaving.

He left home early, for by 1724 he was reported to be in nearby Nuneaton, practicing as an astrologer. There he accepted a job as a schoolteacher. From 1725, when Simpson was fifteen years old, until around 1733, he taught mathematics in Nuneaton. His mathematical interests were aroused when a pedlar gave him a copy of the popular textbook, Cocker's Arithmetic. After mastering Cocker's Arithmetic and the concepts of algebra, Simpson became the most distinguished of a group of itinerant lecturers who taught in the London coffee houses. At this time coffee houses were sometimes called the Penny Universities because of the cheap education they provided. They charged an entrance fee of one penny and then while customers drank coffee they listened to lectures.

By 1735, Simpson was able to solve puzzles concerning infinitesimal calculus.

He was an early member of the Spitalfields Mathematical Society being one of 49 members in 1736. This Society operated as a working men's club.

In 1737, Simpson published his textbook, "A New Treatise of Fluxions." It was a high-quality textbook devoted to the calculus of fluxions, the Newtonian version of the infinitesimal calculus. Simpson is best known for his work on interpolation and numerical methods of integration. "Simpson's Rule," a method for numerical integration, or the numerical approximation of definite integrals, is:

$$\int_a^b f(x)\,dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

This is used to approximate the area under a curve. Parabolas are used to approximate each part of the curve, and, by integrating and adding the areas under each parabola, Simpson's Rule yields the approximate area. This is an improvement from the Rectangle and Trapezoidal Rules because it does not use straight lines to approximate the curve.

In 1740 he published his another book "The Nature and Laws of Chance." It focused on probability theory, following in the footsteps of De Moivre. Simpson also worked on the Theory of Errors, and tried to prove that "the arithmetic mean was better than a single observation". His justification of this appeared in his 1757 memoir – an attempt to show the advantage arising by taking the mean of a number of observations in practical astronomy.

In 1743 Simpson was appointed as the head of mathematics at the Royal Military Academy at Woolwich. Simpson took up the post and his appointment there had an impact on the mathematical topics he investigated. He began research on engineering problems and problems relating to fortifications. Two years after his appointment, Simpson was elected a fellow of the Royal Society. In 1758 he was also elected a foreign member of the Royal Swedish Academy of Sciences.

Simpson published *Mathematical Dissertations* in 1743 which discussed the attraction of the solid obtained by rotating an ellipse around one of its axes. His two volume work *The Doctrine and*

Application of Fluxions in 1750 contains work of Cotes and is considered by many to be the best work on Newton's version of the calculus published in the 18th century.

From 1754 to 1760 he also served as editor of the Ladies Diary – a journal that sought to interest the "fair sex" in "Mathematics and Philosophical Knowledge". He had published in the *Ladies Diary* from the time he came to London in 1736. He answered problems posed in this journal, but used a variety of pseudonyms such as Marmaduke Hodgson, Hurlothrumbo, Kubernetes, Patrick O'Cavannah, and Anthony Shallow. It was his obvious mathematical skills demonstrated in these solutions which first brought him to the attention of other mathematicians of the day. Other periodicals which he published in were the *Gentleman's Magazine, Miscellanea Curiosa Mathematica,* and the *Gentleman's Diary.*

Among other works, Simpson published a series of mathematics "texts", Algebra (1745), Geometry (1747), and Trigonometry (1748), in which he introduced the current abbreviations for the trigonometric functions. Simpson died on May 14, 1761. Today, the very brilliant and well-rounded mathematician is remembered for his contribution to numerical integration: "Simpson's Rule" which "has become a popular and useful special case of the Newton-Cotes formula for approximating an integral".[72]

🕮 TOPICAL VOCABULARY

inventor	[in'ventq]	n.	գյուտարար
weaver	['wi: vq]	n.	ջուլհակ
occurrence	[q'karqns]	n.	դեպք, պատահար
solar	['soulq(r)]	a.	արևի, արեգակնային
eclipse	[i'klips]	n.	խավարում

pedlar	['pedlq(r)]	n.	չարչի, շրջիկ մանրավաճառ
master	['ma:stq]	n.	1. վարպետ 2. մագիստրոս
		v.	1. տիրապետել 2. հաղթահարել
itinerant	[ai'tinqrqnt]	a.	շրջիկ
to charge an entrance fee			*մուտքի վճար գանձել*
customer	['kAstqmq]	n.	գնորդ, հաճախորդ
infinitesimal	['infini'tesimql]	a.	անվերջ փոքր
		n.	անվերջ փոքր մեծություն
treatise	['tri:tiz]	n.	տրակտատ, գիտական շարադրություն
fluxion	['flAkSn]	n.	ածանցյալ
interpolation	[in"tq:pq'leiSn]	n.	ինտերպոլացիա
approximation	[q"prOksi'meiSn]	n.	մոտարկում
parabola	[pq'rxbqlq]	n.	պարաբոլ
yield	[ji:ld]	v.	1. բերել, տալ արդյունք 2. ենթարկվել
improvement	[im'pru:vmqnt]	n.	բարելավում, կատարելագործում
rectangle	['rektxNgl]	n.	ուղղանկյուն
to follow in one's footsteps			*մեկի օրինակին հետևել*
observation	["Obzq: 'veiSn]	n.	դիտարկում, զննում

justification	["GAstifi'keiSn]	n.	1. արդարացում 2. հաստատում (փաստերով)
to be appointed			*նշանակվել*
to take up the post			*զբաղեցնել պաշտոնը*
to have an impact on			*ազդեցություն ունենալ*
investigate	[in'vestigeit]	v.	1.քննել 2.հետազոտել, ուսումնասիրել
research	[ri'sq:C]	n.	գիտական ուսումնասի-րություն, հետազոտություն
relate (to)	[ri'leit]	v.	կապել, կապակցել վերաբերել՝ կապ, առնչություն ունենալ
fortification	["f0:tifi'keiSn]	n.	1.ամրաշինության գիտություն 2. ամրություն-ներ
rotate	[rou'teit]	v.	1. պտտ(վ)ել 2. հերթափոխ(վ)ել, հերթով միմյանց հաջորդել
variety	[vq'raiqti]	n.	1. բազմազանություն 2. մեծ քանակություն
pseudonym	['sju:dqnim]	n.	կեղծանուն
periodical	["piqri'0dikql]	n.	պարբերական
abbreviation	[q"bri:vi'eiSn]	n.	1. կրճատում 2. հապավում
trigonometric	["trigq'n0metrik]	a.	եռանկյունաչափական

I. What's the Armenian for?

with the occurrence of a solar eclipse, the most distinguished, single observation, calculus of fluxions, justification, by rotating an ellipse around one of its axes, to serve as, research on engineering problems, to provide a cheap education, observations in practical astronomy, periodical, a brilliant and well-rounded mathematician, problems relating to fortifications, work on interpolation and numerical methods of integration

II. What's the English for?

մուտքի վճար, շրջիկ դասախոսներ, սխալների տեսություն, ազդեցություն ունենալ, մեծ թվով կեղծանուններ, կենտրոնանալ հավանականության տեսության վրա, բարձրորակ դասագիրք, նշանակվել որպես, շրջիկ մանրավաճառ, մաթեմատիկական հմտություններ, երկհատոր աշխատություն, ցույց տալ առավելությունը, բարդ խնդիր (գլուխկոտրուկ) լուծել, անվերջ փոքր մեծությունների տեսության վերաբերյալ, ինտեգրման թվային մեթոդներ

III. a) Arrange the words in pairs similar in meaning:

1.	**nearby**	a.	regular, methodical
2.	**report**	b.	apparent, evident
3.	**obvious**	c.	announce, notify
4.	**naturally**	d.	supply
5.	**formal**	e.	adjacent, neighboring, close
6.	**provide**	f.	originally

b) Arrange the words in pairs opposite in meaning:

1.	**accept**	a.	recall, withdraw
2.	**cheap**	b.	ugly, unfair
3.	**fair**	c.	odious, unpopular, exclusive
4.	**appoint**	d.	dear, expensive, costly
5.	**popular**	e.	hollow, soft, elastic, weak
6.	**solid**	f.	refuse, reject, decline

IV. Match the following words with their definitions:

1.	**solar**	a.	move round a central point
2.	**customer**	b.	charge or payment or professional advice or services
3.	**treatise**	c.	examine, inquire into; make a careful study of
4.	**investigate**	d.	person who goes from house to house selling small articles
5.	**fee**	e.	of the sun
6.	**pedlar**	f.	ability to do something well
7.	**pseudonym**	g.	person who buys things especially one who gives his custom to a shop
8.	**skill**	h.	power of pulling towards
9.	**attraction**	i.	book, etc. that deals systematically with one subject
10.	**rotate**	j.	name taken, especially by an author, instead of his real name

V. Write the plural forms of:

ellipse, diary, datum, schoolteacher, formula, crisis

VI. Give the corresponding adjectives of the nouns given below:

sun, know, number, justification, Sweden, attention, trigonometry, distinguish

VII. Match the beginnings with their appropriate endings:

1.	Simpson first worked as …	a.	... to solve puzzles concerning infinitesimal calculus.
2.	Simpson is best known for his work on …	b.	… the Royal Military Academy at Woolwich.
3.	His mathematical interests were aroused when a pedlar gave him …	c.	… of the Royal Swedish Academy of Sciences.
4.	In 1758 he was also elected a foreign member …	d.	… a weaver.
5.	By 1735, Simpson was able …	e.	… interpolation and numerical methods of integration.
6.	In 1743 Simpson was appointed as the head of mathematics at …	f.	… a copy of the popular textbook, Cocker's Arithmetic.

VIII. Choose the suitable word:

1. With the occurrence of a … eclipse in 1724, Thomas Simpson turned to "mathematical interests", changing his life forever. *(solar / sun)*
2. "A New Treatise of Fluxions" was "a high-quality textbook

devoted to the … of fluxions, the Newtonian version of the infinitesimal calculus”. *(calculi / calculus)*

3. Simpson also worked on the Theory of Errors, and tried to … that “the arithmetic mean was better than a single observation”. *(prove / proof)*
4. “The Nature and Laws of Chance” focused on … theory, following in the footsteps of De Moivre. *(probable / probability)*
5. Simpson began research on … problems and problems relating to fortifications. *(engineering / engineer’s)*
6. From 1754 to 1760 he also served as editor of the … – a journal that sought to interest the “fair sex” in “Mathematics and Philosophical Knowledge.” *(Gentleman’s Diary / Ladies Diary)*
7. “Simpson’s Rule” has become a popular and ... special case of the Newton-Cotes formula for approximating an integral. *(useful / useless)*

IX. Fill in the blanks with prepositions:

English mathematician, physicist and astronomer R. Cotes was born … Leicestershire … July 10, 1682. … 1699 he became a student … Trinity college, Cambridge. … 1707 he was appointed the first Plumian professor of astronomy and natural philosophy …Cambridge. … 1711 he was made a fellow … the Royal Society. Cotes, one … the foremost … the young disciples who gathered around Sir Isaac Newton, had a major influence … the evolving Newtonian philosophy, largely … his preface …the important second edition … Newton’s “Principa”. … the preface Cotes answered Newton’s many critics with a plea … the new methods, insights, assumptions, and theological fruitfulness … Newtonianism, and attacked the absurdities … the Cartesian and Leibnizian philosophies.

Cotes died … Cambridge … June 5, 1716 … the age of 33. Although his mathematical abilities elicited … Newton the judgment, “Had Mr. Cotes lived we might have known something”, he had published little … his short life. Several … his works … optics, astronomical observations, analysis … errors, and applications … logarithms were published posthumously, as were his lectures … hydrostatics and pneumatics and his correspondence with Newton.[73]

X. Are the following statements true or false? Contradict the false statements:

1. Simpson tried to prove that “the arithmetic mean was better than a single observation.”
2. Simpson was an American mathematician and inventor.
3. Simpson taught mathematics in Leicestershire.
4. Simpson used a lot of pseudonyms in his life.
5. In 1758 he was elected a foreign member of the French Academy of Sciences.
6. Nowadays, Simpson is remembered for his contribution to numerical integration.
7. The London coffee houses were also named “Penny Universities” because of the expensive education they provided.

XI. Make up sentences of your own starting with:

At the beginning …/First… / Firstly…
To start with …
In theory …
In practice …
Due to the fact that …
In conclusion …
To sum up …

XII. Render the text into English:

Արտաշես Շահինյանը՝
հայկական մաթեմատիկական գիտության դպրոցի հիմնադիր

Հայ մաթեմատիկոս, ակադեմիկոս և մանկավարժ, հայկական մաթեմատիկական գիտության դպրոցի հիմնադիր Արտաշես Շահինյանի աշխատանքներով Հայաստանում սկիզբ են դրվել մաթեմատիկայի բնագավառի համակարգված գիտական հետազոտություններին: Նա զբաղվել է մաթեմատիկական անալիզում ֆունկցիաների մոտավորության տեսությամբ, կոմպլեքս փոփոխական ֆունկցիաների տեսությամբ: Աշխատանքները վերաբերում են կոմպլեքս տիրույթում մոտավորությունների տեսությանը, կոմպլեքս և իրական փոփոխականների ֆունկցիաների տեսության մի շարք նոր ուղղություններին: Ֆունկցիաների տեսության բնագավառում Շահինյանի ներդրումները միջազգային ճանաչում են ստացել:

Ա. Շահինյանի անունով են կոչվել ԵՊՀ-ին կից իր նախաձեռնությամբ ստեղծված ֆիզմաթ թեքումով Արտաշես Շահինյանի անվան ֆիզմաթ դպրոցը, փողոցներ Երևանում և Գյումրիում, 1999թ.-ին Գյումրիում կանգնեցվել է հուշակոթող: 1978թ.-ին ԵՊՀ-ում սահմանվել է Շահինյանի անվան կրթաթոշակ:

Նրա բրոնզաձույլ կիսանդրին ԵՊՀ կենտրոնական մասնաշենքի նախասրահում խորհրդանշում է ականավոր գիտնական մանկավարժի դերը համալսարանական գիտության և կրթության զարգացման գործում:[74]

XIII. Speak on the key points of the text “Thomas Simpson”.

Lesson 22

ADA LOVELACE

Beautiful, charming, temperamental,
an aristocratic hostess.
CHARLES BABBAGE[75]

Augusta Ada Lovelace was a British mathematician and musician who is best known as the first computer programmer. She was born on December 10, 1815 in London, being the only legitimate child of the British poet Lord George Gordon Byron and the self-proclaimed "Princess of Parallelograms" Annabella Milbanke Noel.

Ada was a few-month baby when her father went abroad and she never saw him again as he didn't return to England having died in Greece. Lady Byron was given the sole custody of her daughter and she tried to do everything possible to bring up her child a well-educated person. She considered mathematics a good subject for training the mind and believed that music would provide the girl with the right social skills. So a number of private tutors were employed to organize Ada's education. On discovering that her daughter preferred geography to arithmetic Lady Byron insisted that the child be driven too hard and worked long at her lessons, especially at mathematics. She even replaced the tutors and kept a constant pressure on her daughter. In the result Ada excelled in mathematics, also becoming an accomplished musician and linguist.

At the age of 17 Ada Lovelace was introduced to a prominent mathematician in England Mary Sommerville who was one of the few successful women in the field. She had published *The Mechanism of the Heavens* on mathematical astronomy. In 1832 they became friends and the one person Ada most longed to meet was Mary Sommerville –

a role model who was also a mathematician! She encouraged Ada in her mathematical and technological studies.

In November, 1834 at a dinner party at Mrs. Sommerville's Ada Lovelace met Charles Babbage and heard his ideas for a new calculating machine, the Analytical Engine. He conjectured that the engine could not only foresee but could also act on that foresight. Ada was so impressed by the "universality of his ideas" and by observing what Babbage had designed, she soon became an expert of the inventor's work. Lovelace saw a tremendous potential in the machine. She understood it better than most other people older and more experienced than her. She commented that such a machine might be used to compose complex music, to produce graphics and would be of practical as well as of scientific use. This paper was the summit of her career but still she felt it was unbecoming for a woman of her social class to publish anything so "unfeminine" and modestly signed with only her initials "A.A.L.". It was nearly 30 years before the identity of "A.A.L." was commonly known. She basked in the admiration that she received from her friends who knew who AAL was. Ada suggested writing a plan to Babbage for how the engine might calculate Bernoulli numbers which is now regarded as the first "computer program".

Ada Lovelace's achievements were so remarkable that mathematicians of the time thought her a magnificent addition to their number. She figures in the history of the Calculating Engines as Charles Babbage's interpretress and his "fair lady". Babbage described her as "that Enchantress who has thrown her magical spell around the most abstract of Sciences and has grasped it with a force which few masculine intellects could have exerted over it", or, in another occasion, as "The Enchantress of Numbers"

But, unfortunately, she had some serious health problems and was wrecked with pain. In 1852 when she was only 36 years of age, Ada died and was buried at her request beside her father Lord Byron who had also passed away as young as her.

It was Ada Lovelace who together with Charles Babbage laid some of the early conceptual and technical foundations for high technology by helping develop an early computer. The Analytical Engine was able to read data from a deck of punched cards, store data and perform arithmetic operations.

In 1979 the United State Department of Defense named its universal computer programming language "ADA" after Ada Lovelace. It is based on the language PASCAL which is a general-purpose language designed to be readable and easily maintained, besides it is efficient for machines and easy to use.[76]

🕮 TOPICAL VOCABULARY

legitimate	[li'Gitimit]	a.	օրինավոր, օրինածին
self-proclaimed	[self"prq'kleimd]	a.	ինքն իրեն հռչակած, ինքնակոչ
sole	[soul]	a.	միակ, ամբողջ
custody	['kAstqdi]	n.	խնամակալություն
tutor	['tjutq]	n.	տնային մասնավոր ուսուցիչ
excel	[ik'sel]	v.	գերազանցել, աչքի ընկնել
long to	[lON]	v.	1. շատ ցանկանալ,

			փափագել 2. կարոտել
conjecture	[kqn'GekCq]	v.	ենթադրել
foresight	['fLsait]	n.	կանխատեսում, կանխատեսու-թյուն
tremendous	[trə'mendəs]	a.	մեծ, հսկայական
universality	["junivq'sxliti]	n.	համընդհանրու-թյուն, բազմակողմա-նիություն
summit	['sAmit]	n.	1. գագաթ 2. գագաթնա-կետ
unbecoming	["Anbi'kAmiN]	a.	անպատշաճ, անվայելուչ
interpretress	[in'tWpritris]	n.	թարգմանչուհի, մեկնաբան
bask	['ba:sk]	v.	հաճույք ստանալ, երանության մեջ լինել
wreck	[rek]	v.	քայքայել,կոր-ծանել
deck	[dek]	n.	ցուցատախտակ
pass away	[pa:s q'wei]	v.	մահանալ
punched card			*ծակապաքարտ*
maintain	['mein'tein]	v.	պահպանել

I. What's the Armenian for?

parallelogram, a legitimate child, an aristocratic hostess, a good subject for training the mind, to long, prominent, to keep a constant pressure on, to replace, a tremendous potential, initials, to foresee, to conjecture, to put into an appropriate human context, to comment, to produce graphics, to bask in, magnificent, temperamental, at the request, easy to use, maintain, wrecked with pain, to lay the conceptual and technical foundations, to become an expert

II. What is the English for?

երեխա դաստիարակել, միտքը կոփել, խնամակալություն, տնային մասնավոր ուսուցիչ, թարգմանչուհի, գործունեության գագաթնակետը, կանխատեսում, կնոջն անվայել, մասնագետ, գերազանցել, հրաշալի, երանության մեջ լինել, պահպանել, համապատասխան, անվան սկզբնատառերը, գաղափարների համընդհանրություն, գրաֆիկներ ստեղծել, ինքնակոչ, հսկայական կարողություն

III. a) Arrange the following words in pairs similar in meaning:

1.	**best known**	a.	care
2.	**legitimate**	b.	substitute
3.	**die**	c.	distinguished
4.	**custody**	d.	to desire
5.	**be employed**	e.	predict
6.	**replace**	f.	pass away
7.	**tutor**	g.	suitable
8.	**excel**	h.	enjoy
9.	**maintain**	i.	legal
10.	**to long**	j.	create
11.	**appropriate**	k.	perfect, surpass
12.	**foresee**	l.	save
13.	**compose**	m.	teacher
14.	**bask**	n.	be engaged

b) Arrange the following words in pairs opposite in meaning:

1.	**self-proclaimed**	a.	tiny
2.	**to conjecture**	b.	theoretical
3.	**universality**	c.	bottom
4.	**tremendous**	d.	depress
5.	**practical**	e.	to prove
6.	**summit**	f.	particularity
7.	**modest**	g.	beginner
8.	**admiration**	h.	order
9.	**well-educated**	i.	undeclared
10.	**addition**	j.	special-aimed
11.	**general-purpose**	k.	illiterate
12.	**request**	l.	subtraction
13.	**expert**	m.	immodest
14.	**encourage**	n.	disgust

IV. Are the given statements true or false? Contradict the false statements:

1. Augusta Ada Lovelace was the daughter of the British poet Lord George Gordon Byron and the self-proclaimed "Princess of Parallelograms" Anabella Milbanke Noel.
2. Ada Lovelace is best known as the first computer programmer.
3. Lovelace was unable to realize Charles Babbage's ideas for a new calculating machine.
4. The identity of "A.A.L." was at once commonly known as the author of the paper who saw a tremendous potential in Babbage's Analytical Engine.
5. In 1979 the United State Department of Defense named its universal computer programming language "ADA" after Ada Lovelace which was based on the language PASCAL.

V. Match the following words with their definitions:

1.	**calculating machine**	a.	achieving maximum productivity with minimum wasted effort or expense
2.	**potential**	b.	a thing done successfully with effort, skill, or courage
3.	**initials**	c.	relating to or based on mental concepts
4.	**identity**	d.	a woman who receives or entertains guests
5.	**wreck**	e.	cards with holes made with a special tool
6.	**achievements**	f.	a close similarity or affinity
7.	**request**	g.	something one has got due to his knowledge or good skill
8.	**conceptual**	h.	an asking for something
9.	**punched cards**	i.	computing engine, a computer
10.	**hostess**	j.	destroy or severely damage something
11.	**efficient**	k.	first letters of a person's name

VI. What words were used in the text to characterize Ada Lovelace both as a woman and as a mathematician? Use some of them to describe your fellow student or to present your model of a woman-scientist.

VII. Insert prepositions: ***in, on, at, of, with, for, by, out, from***

A career is one that pays well … which you have a broad choice … full-time and part-time jobs, … which there is some sort … barrier to entry so that you won't have to compete … a lot … other applicants, … which there are good jobs … every country and you can enjoy job security … middle age and not be driven … … young people.

Most workers, however, get a lot … satisfaction … meeting new people, working … others collaboratively, being thanked … customers, teaching, having a direct positive impact … other people. In fact, the only science job that regularly offers any … these satisfactions is professor.

"Science is a wonderful thing if one does not have to earn one's living … it."(Albert Einstein)

VIII. Match the beginnings with their appropriate endings:

1.	Mathematics was considered to be …	a.	… the Mechanism of the Heavens on mathematical astronomy.
2.	Music would provide …	b.	… an appropriate human context.
3.	Mary Sommerville published …	c.	… a good subject for training the mind.
4.	Ada attempted to put mathematics and technology into …	d.	… read data from a deck of punched cards, store data and perform arithmetic operations.
5.	The Analytical Engine was able to …	e.	… with the right social skills.

IX. Render the text into Armenian:

WOMEN IN SCIENCE

Immemorial dispute who are cleverer: men or women, is not solved till now. At home women win and it is difficult to find any argument against this statement. But men have an unarguable success on their professional path. For example, from four hundred of Nobel winners only nine are women.

Women have pursued science for more than four thousand years. The very first historical mention of a female name was of En Hedu'Anna (about 2354 B.C.), who was the daughter of Sargona, the founder of Sargonidov dynasty, in Babylon. She was appointed the main nun in the Moon Temple, and those years temples of Babylon were known as scientific institutions where mathematical sciences were developed.

Another woman-scientist from the ancient times was Hypatia from Alexandria who lived in the middle of IV century. She had a keen interest in mathematics, astronomy and astrology. She had invented some astronomical tools and developed the idea of conic sections, the hyperbola, the parabola and the ellipse.

Women have made a valuable contribution to science, and though some of them faced difficulties in getting the due recognition of their work, with the passing years the society realized the value of their scientific work and today they are held in high regard.

X. Speak on the key points of the text: "Ada Lovelace".

PROPER NAMES

Abraham de Moivre	['eibrqhqm dq' m0iviq]	Աբրահամ դը Մուավր

Abraham de Moivre (1667–1754) was a French mathematician known for de Moivre’s formula, one of those that link complex numbers and trigonometry, and for his work on the normal distribution and probability theory. He was a friend of Isaac Newton, Edmond Halley, and James Stirling.

De Moivre wrote a book on probability theory, The Doctrine of Chances, said to have been prized by gamblers. De Moivre first discovered Binet’s formula, the closed-form expression for Fibonacci numbers linking the nth power of the golden ratio φ to the nth Fibonacci number. He also was the first to postulate the Central Limit Theorem, a cornerstone of probability theory.

Abram Besicovitch	['xbrqm "besi'k0viC]	Աբրամ Բեզիքովիչ

Abram Besicovitch (1891–1970) was a Russian mathematician, who worked mainly in England. He was born in Berdyansk on the Sea of Azov. He worked mainly on combinatorial methods and questions in real analysis, such as the Kakeya needle problem and the Hausdorff-Besicovitch dimension.

Abu Jafar Muhammad ibn Musa al Khowaresmi	[xbu dZifa: mu'hamqd ibn 'mjuzq xl'k0resmi]	Աբու Ջաֆար Մուհամմադ իբն Մուսա ալ Խորեսմի

Abu Jafar Muhammad ibn Musa al Khowaresmi (c. 780 BC – 850 BC) was a Muslim mathematician and astronomer whose major works introduced Hindu-Arabic numerals and the concepts of algebra into European mathematics, Latinized versions of his name and of his most famous book title live on in the terms algorithm and algebra.

Albert Einstein	['xlbqt 'ainstain]	Ալբերտ Էյնշտեյն

Albert Einstein (1879–1955) a German-born theoretical physicist. He developed the general theory of relativity, one of the two pillars of modern physics. Einstein's work is also known for its influence on the philosophy of science. Einstein is best known in popular culture for his mass–energy equivalence formula $E = mc^2$ (which has been dubbed "the world's most famous equation"). He received the 1921 Nobel Prize in Physics for his "services to theoretical physics", in particular his discovery of the law of the photoelectric effect, a pivotal step in the evolution of quantum theory.

Alexander Alekhine	["xlig'za:ndq 'xlekin]	Ալեքսանդր Ալյոխին

Alexander Alekhine (1892–1946) was a Russian chess grandmaster and the fourth World Chess Champion. He is widely considered one of the greatest chess players ever.

Amalie Emmy Noether	['xmqli emi 'nouTq]	Ամալի Էմմի Նոդեր

Amalie Emmy Noether (1882–1935) was a German mathematician known for her contributions to abstract algebra and theoretical physics. She was described by Pavel Alexandrov, Albert Einstein, Jean Dieudonné, Hermann Weyl, and Norbert Wiener as the most important woman in the history of mathematics. As one of the leading mathematicians of her time, she developed the theories of rings, fields, and algebras. In physics, Noether’s theorem explains the connection between symmetry and conservation laws.

Annabella Milbanke Noel	['xnqbelq 'mIlbeNk'n0Il]	Անաբելա Միլբենք Նոյլ

Annabella Milbanke Noel (1792-1860) was mathematics-loving wife of Lord Byron.

Antoine Cournot ['antuqn'kHnqt] Անտուան Կուրնո

Antoine Cournot (1801–1877) was a French philosopher and mathematician who also contributed to the development of economics theory. He took two doctoral degrees, one in mechanics and one in astronomy. In addition, he published a number of articles and even acquired a degree in law.

Aristotle ['ærI"stoutql] Արիստոտել

Aristotle (384–322 BC) was a Greek philosopher and scientist. At eighteen, he joined Plato’s Academy in Athens and remained there until the age of thirty-seven. His writings cover many subjects: including physics, biology, zoology, metaphysics, logic, ethics, aesthetics, poetry, theater, music, rhetoric, linguistics, politics and government constituting the first comprehensive system of Western philosophy. Shortly after Plato died, Aristotle left Athens and, at the request of Philip of Macedon, tutored Alexander the Great getting many opportunities and an abundance of supplies. He established a library in the Lyceum which aided in the production of many of his hundreds of books. Aristotle’s views on natural sciences profoundly shaped medieval scholarship. All aspects of Aristotle’s philosophy continue to be the object of active academic study today.

Arthur Cayley ['a:Tq 'keili] Արթուր Քեյլի

Arthur Cayley (1821–1895) was a British mathematician who helped founding the British school of pure mathematics. He is most famous for developing the algebra of matrices and non-Euclidean and n-dimensional geometry. Cayley wrote over a thousand papers in his lifetime.

Augustin Louis Cauchy [O'gqstqn lui kO'SJ] Ավգուստին Լուիս Կոշի

Auguistin Louis Cauchy (1789–1857) was a French mathematician who pioneered in analysis and the theory of substitution groups.

Augusta Ada Lovelace ['Lgqstq 'eIdq'lOvleIs] Ավգուստա Ադա Լովլես

Augusta Ada Lovelace (1815–1852) was a British mathematician and writer, chiefly known for her work on Charles Babbage's early mechanical general-purpose computer, the Analytical Engine. Her notes on the engine include the first algorithm to be carried out by a machine. For this reason, she is often regarded as the first computer programmer. Lovelace was the only legitimate child of the poet George Gordon, Lord Byron and his wife Anne Isabella Milbanke. Byron separated from his wife a month after Ada was born and left England forever. Four months later, eventually dying of disease in the Greek War of Independence when Ada was eight years old. As a teen, her mathematical talents led her to friendship with fellow British mathematician Charles Babbage also known as father of computers. Between 1842 and 1843, she translated an article by Italian military engineer Luigi Menabrea on the engine. These notes contain what many consider to be the first computer program – that is, an algorithm designed to be carried out by a machine. Lovelace's notes are important in the early history of computers. She also developed a vision of the capability of computers to go beyond mere calculating or number-crunching. She is often regarded as the first computer programmer..

Australasia ['Lstrql'eiSq] Օվկիանիա

Australasia – a region of Oceania, comprises Australia, New Zealand, the island of New Guinea and neighbouring islands in the Pacific ocean.

Babylon ['bxbilqn] Բաբելոն

Babylon (1894 BC – 539 BC) – the capital of Babylonian Empire – the most powerful state in the ancient world. Babylon was known for its impressive walls and buildings, its reputation as a great seat of learning and culture.

Beaumont-de-Lomagne ['bumoun dqI0'ma:nI] Բյումոնտ դը Լոմանի

Beaumont-de-Lomagne is a commune in southern France where Pierre de Fermat was born. There is a statue and museum to him in the town.

Benjamin Pierce ['bFnGqmIn'pIqs] Բենջամին Փիրս

Benjamin Pierce is an American professor of computer science at the University of Pennsylvania. His research includes work on programming languages, static type systems, distributed programming, mobile agents, and process calculi.

Benoit Mandelbrot ['benua "mxndqI'br0t] Բենուա Մանդելբրոթ

Benoit Mandelbrot (born in Poland 1924) was largely responsible for the present interest in fractal geometry. He showed how fractals can occur in many different places, both in mathematics and elsewhere in nature.

Bertrand Russell ['bertrqnd 'rAsql] Բերտրան Ռասսել

Bertrand Russell (1872 – 1970) was a British philosopher, logician, mathematician, historian, writer, social critic and political activist. He was born in Monmouthshire into one of the most prominent aristocratic families in Britain.

In the early 20th century, Russell led the British "revolt against idealism". He is considered one of the founders of analytic philosophy along with his predecessor Gottlob Frege, colleague G. E. Moore, and his protnġn Ludwig Wittgenstein. He is widely held to be one of the 20th century's premier logicians. With A.N. Whitehead he wrote Principia Mathematica, an attempt to create a logical basis for mathematics. His philosophical essay "On Denoting" has been considered a "paradigm of philosophy". His work has had a considerable influence on logic, mathematics, set theory, linguistics, artificial intelligence, cognitive science, computer science, and philosophy, especially the philosophy of language, epistemology, and metaphysics.

Bôcher prize ['bOSq "praiz] Բոշերի անվան մրցանակ

Bôcher prize – the oldest of the prizes offered by the American Mathematical Society.

The Bôcher Memorial Prize was founded by the American Mathematical Society in 1923 in memory of Maxime Bôcher with an initial endowment of $1,450 (contributed by members of that society). It is awarded every three years (formerly every five years) for a notable research memoir in analysis that has appeared during the past six years in a recognized North American journal or was authored by a member of the Society. This provision, introduced in 1971 and modified in 1993, is a liberalization of the terms of the award. The current award is $5,000.

Budapest ['bu:dqpest]=['budapFSt] Բուդապեշտ

Budapest – the capital and the largest city of Hungary, and one of the largest cities in the European Union. It is the country's principal political, cultural, commercial, industrial, and transportation centre, sometimes described as the primate city of Hungary. Budapest became a single city occupying both banks of the river Danube with the unification of Buda, Óbuda and Pest on the west bank, with Pest on the east bank on 17 November 1873.

Cambridge ['keImbrIG] Քեմբրիջ

Cambridge is a university city and the county town of Cambridgeshire, England. Cambridge is most widely known as the home of the University of Cambridge, founded in 1209 and consistently ranked one of the top five universities in the world.

Camille Jordan	[kq'mil 'GLdqn]	Քամիլ Ջորդան

Camille Jordan (1838–1922) was a French mathematician whose work on substitution groups (permutation groups) and the theory of equations first brought full understanding of the importance of the theories of the eminent mathematician Evariste Galois.

Charles Babbage	['Ca:lz 'bxbIG]	Չարլզ Բեբիջ

Charles Babbage (1791–1871) was an English mathematician, philosopher, inventor and mechanical engineer. Babbage originated the concept of a programmable computer and is considered the "father of the computer". Babbage is credited with inventing the first mechanical computer that eventually led to more complex designs. Parts of Babbage's uncompleted mechanisms are on display in the London Science Museum.

Cosimo II de Medici	[k0z'imqtH de'mediCi]	Կոզիմո II դե Մեդիչի

Cosimo II de Medici (12 May 1590 – 28 February 1621) was Grand Duke of Tuscany from 1609 until 1621. For the most of his eleven-year reign, he delegated the administration of Tuscany to his ministers. He is best remembered as the patron of Galileo Galilei, his childhood tutor.

David Hilbert	['deivid 'hilbqrt]	Դավիթ Հիլբերտ

David Hilbert (1862–1943) was a German mathematician is recognized as one of the most influential and universal mathematicians of the 19th and early 20th centuries. Hilbert discovered and developed a broad range of fundamental ideas in many areas, including invariant theory and the axiomatization of geometry. He also formulated the theory of Hilbert spaces, one of the foundations of functional analysis.

Hilbert adopted and warmly defended Georg Cantor’s set theory and transfinite numbers. A famous example of his leadership in mathematics is his 1900 presentation of a collection of problems that set the course for much of the mathematical research of the 20th century.

Hilbert and his students contributed significantly to establishing rigor and developed important tools used in modern mathematical physics. Hilbert is known as one of the founders of proof theory and mathematical logic.

Diophantus of Alexandria	["da iq'f x ntqs qv x lig'z x ndriq]	Դիոֆանտուս Ալեքսանդրիացի

Diophantus of Alexandria was an Alexandrian Greek mathematician (about AD 201-285) sometimes called “the father of algebra” and the author of the book Arithmetica.

Emanuel Lasker [i'mxnjuql 'la:skq] Էմանուել Լացկեր

Emanuel Lasker (1868–1941) was a German chess player, mathematician, and philosopher who was World Chess Champion for 27 years (from 1894 to 1921). In his prime Lasker was one of the most dominant champions, and he is still generally regarded as one of the strongest players ever.

Émile Borel ['emil'b0rql] Էմիլ Բորել

Émile Borel (1871–1956) was a French mathematician and politician. As a mathematician, he was known for his founding work in the areas of measure theory and probability. The concept of a Borel set is named in his honor. One of his books on probability introduced the amusing thought experiment that entered popular culture under the name infinite monkey theorem or the like. He also published a series of papers (1921–27) that first defined games of strategy.

Erlangen ['Frla:N] Էրլանգեն

Erlangen – a Middle Franconian city in Bavaria, Germany. Erlangen has more than 100,000 inhabitants.

As of 2015 Erlangen is dominated by the University of Erlangen-Nuremberg and the numerous branch offices of Siemens

AG, as well as a large research Institute of the Fraunhofer Society and the Max Planck Institute for the Science of Light. An event that left its mark on the city was the settlement of Huguenots after the revocation of the Edict of Nantes in 1685.

Felix Klein's Erlangen program of 1872, considering the future of research in mathematics, is so called because Klein then taught at the University of Erlangen-Nuremberg.

Euclid	['jHklid]	Էվկլիդես

Euclid (c. 300 BC) the most prominent Greek mathematician of antiquity, best known for his treatise on geometry "The Elements".

Évariste Galois	[i'verist 'gxlwa]	Էվարիստ Գալուա

Évariste Galois (1811–1832) was a French mathematician famous for his contributions to the part of higher algebra now known as group theory.

Felix Klein	['feliks 'klain]	Ֆելիքս Կլայն

Felix Klein (1849–1925) was a German mathematician and mathematics educator, known for his work in group theory, complex analysis, non-Euclidean geometry, and on the connections between geometry and group theory. His 1872 Erlangen Program, classifying geometries by their underlying symmetry groups, was a hugely influential synthesis of much of the mathematics of the day.

Francois Viete ['franswa 'viet] Ֆրանսուա Վիետ

Francois Viete (1540–1603) was a French mathematician, the father of modern algebra and the foremost mathematician of the 16^{th} century.

Gábor Szegö ['ga:b0 'sFg0:] Գաբոր Սեգա

Gábor Szegö (1895–1985) a Hungarian mathematician was one of the foremost analysts of his generation and made fundamental contributions to the theory of Toeplitz matrices and orthogonal polynomials.

He wrote over 130 papers in several languages. Each of his four books, several written in collaboration with others, has become a classic in its field. The monograph Orthogonal polynomials, published in 1939, contains much of his research and has had a profound influence in many areas of applied mathematics, including theoretical physics, stochastic processes and numerical analysis.

Galileo Galilei [gali'lF:o gali'l F:i] Գալիլեո Գալիլեյ

Galileo Galilei (1564–1642), was an Italian astronomer, physicist, engineer, philosopher, and mathematician who played a major role in the scientific revolution during the Renaissance. His achievements include improvements to the telescope and consequent astronomical observations. Galileo has been called the “father of modern observational astronomy”, the “father of modern physics”, and the “father of modern science”.

His contributions to observational astronomy include the telescopic confirmation of the phases of Venus, the discovery of the four largest satellites of Jupiter (named the Galilean moons in his honour), and the observation and analysis of sunspots. Galileo also worked in applied science and technology, inventing an improved military compass and other instruments.

Gaston Julia ['gxstqn 'Guliq] Գաստոն Ջուլիա

Gaston Julia (1893–1978) – one of the two main inventors of iteration theory and the modern theory of fractals, won the grand Prix from the French Academy of Sciences in 1918.

Geoffrey Hardy ['GFfri 'ha:di] Ջեֆրի Հարդի

Geoffrey Hardy (1877–1947) was an English mathematician known for his achievements in number theory and mathematical analysis. He is usually known by those outside the field of mathematics for his essay from 1940, A Mathematician's Apology, which is often considered one of the best insights into the mind of a working mathematician written for the layman.

Georg Cantor	[geLg kxn'tL]	Գեորգ Կանտոր

Georg Cantor (1845–1918) was a German mathematician who founded set theory and introduced the mathematically meaningful concept of transfinite numbers, indefinitely large but distinct from one another.

Girolamo Cardano	[Girq'lamou kW'dano]	Ջերոլամո Կարդանո

Girolamo Cardano (1501–1576) was an Italian physician, mathematician and astrologer, who gave the first clinical description of typhus fever and whose book Ars magua (The Great Art, or the Rules of Algebra) is one of the cornerstones in the history of algebra.

Gòttingen	['gq:tingqn]	Գոթինգեն

Göttingen – a university town in Lower Saxony, Germany. It is the capital of the district of Göttingen. The River Leine runs through the town. The origins of Göttingen lay in a village called Gutingi, first mentioned in a document in 953 AD. The city was founded northwest of this village, between 1150 and 1200 AD, and adopted its name.

Today, Göttingen is famous for its old university (Georgia Augusta, or “Georg-August-Universität”), which was founded in 1734 and became the most visited university of Europe. In 1837, seven professors protested against the absolute sovereignty of the kings of Hanover; they lost their offices, but became known as the “Göttingen Seven”. Its alumni include some well-known historical figures: the Brothers Grimm, Heinrich Ewald, Wilhelm Eduard Weber and Georg Gervinus. Also, German Chancellors Otto von Bismarck and Gerhard Schröder attended law school at the Göttingen University. Karl Barth held his first professorship here. Some of the most famous mathematicians in history, Carl Friedrich Gauss, Bernhard Riemann and David Hilbert, were professors at Göttingen.

Greece	['grJs]	Հունաստան

Greece is a country in southeastern Europe, known in Greek as Hellas or Ellada, and consisting of a mainland and an archipelago of islands. Greece is the birthplace of Western philosophy (Socrates, Plato, and Aristotle), literature (Homer and Hesiod), mathematics (Pythagoras and Euclid), history (Herodotus), drama (Sophocles, Euripedes, and Aristophanes), the Olympic Games, and democracy.

Gustaf Mittag-Leffler	[gustqf mitig'leflq]	Գուստաֆ-Միթագ Լեֆլեր

Gustaf Mittag Leffler (1846–1927) was a Swedish mathematician whose mathematical contributions are connected chiefly with the theory of functions.

Harry Markowitz ['hxri'ma:kqwits] Հերի Մարկովիչ

Harry Markowitz (born: August 24, 1927) is an American economist. He is a recipient of the 1989 John von Neumann Theory Prize and the 1990 Nobel Memorial Prize in Economic Sciences and the author of: "Portfolio Selection: Efficient Diversification of Investments".

Helge von Koch ['helgq v0n 'k0C] Հելգե ֆոն Կոխ

Helge von Koch (1870–1924) was a Swedish mathematician famous for his discovery of the von Koch snowflake curve, a continuous curve important in the study of fractal geometry.

Henri Poincaré ['henri pwanka'ra:] Հենրի Պուանկարե

Henri Poincaré (1854–1912) was a French mathematician, one of the greatest mathematicians and mathematical physicists at the end of 19th century. He made a series of profound innovations in geometry, the theory of differential equations, electromagnetism, topology, and the philosophy of mathematics.

Hungary ['hANgqri] Հունգարիա

Hungary – a landlocked country in Central Europe. It is situated in the Carpathian Basin and is bordered by Slovakia to the north, Romania to the east, Serbia to the south, Croatia to the southwest, Slovenia to the west, Austria to the northwest, and Ukraine to the northeast. The country's capital and largest city is Budapest. Hungary is a member of the European Union, NATO, the OECD, the Visegrnd Group, and the Schengen Area. The official language is Hungarian, which is the most widely spoken non-Indo-European language in Europe.

Isaak Newton ['aizqk 'njHtqn] Իսահակ Նյուտոն

Isaak Newton (1646–1727) was an English mathematician, physicist, the most original and influential theorist in the history of science. In addition to his invention of the infinitesimal calculus and a new theory of light and color Newton transformed the structure of physical science with his three laws of motion and the law of universal gravitation. His famous masterpiece is "The Principia".

James Bernolli ['Geimz bqr'nu:li] Ջեյմս Բեռնուլի

James Bernolli (1655 – 1705) was an early proponent of Leibnizian calculus and had sided with Leibniz during the Leibniz–Newton calculus controversy. He is known for his numerous contributions to calculus, and along with his brother Johann, was one of the founders of the calculus of variations. However, his most important contribution was in the field of probability, where he derived the first version of the law of large numbers in his work Ars Conjectandi.

Jean – Baptist le Rond d'Alembert	['Za:n 'bxptist lqr0: "dxlqm'bFqr]	Դալամբեր

Jean – Baptist le Rond d'Alembert (1717–1783) French mathematician, mechanician, physicist, philosopher, and music theorist. Until 1759 he was also co-editor with Denis Diderot of the Encyclopédie. D'Alembert's formula for obtaining solutions to the wave equation is named after him. The wave equation is sometimes referred to as d'Alembert's equation.

Johannes Zukertort	[j0'hanis "zukq't0:t]	Յոհաննես Ցուկերթորթ

Johannes Zukertort (1842–1888) was a leading Polish chess master. He was one of the leading world players for most of the 1870s and 1880s, and lost to Wilhelm Steinitz in the World Chess Championship 1886, generally regarded as the first World Chess Championship match.

John Maynard Smith	['G0n'meinWd'smiT]	Ջոն Մայնարդ Սմիթ

John Maynard Smith (1920–2004) was a British theoretical evolutionary biologist and geneticist. Originally an aeronautical engineer during the Second World War, he took a second degree in genetics. Smith was instrumental in the application of game theory to evolution. In 1973 he formalised a central concept in evolutionary game theory called the evolutionarily stable strategy (ESS). This area of research culminated in his 1982 book Evolution and the Theory of Games.

John von Neumann ['G0n'f0n'n0imqn] Ջոն ֆոն Նեյման

John von Neumann (1903–1957) was a Hungarian-American pure and applied mathematician, physicist, inventor, polymath, and polyglot. He made major contributions to a number of fields, including mathematics (foundations of mathematics, functional analysis, ergodic theory, geometry, topology, and numerical analysis), physics (quantum mechanics, hydrodynamics, fluid dynamics and quantum statistical mechanics), economics (game theory), computing (Von Neumann architecture, linear programming, self-replicating machines, stochastic computing), and statistics. He was a pioneer of the application of operator theory to quantum mechanics, in the development of functional analysis, a principal member of the Manhattan Project and the Institute for Advanced Study in Princeton (as one of the few originally appointed), and a key figure in the development of game theory and the concepts of cellular automata, the universal constructor, and the digital computer.

Von Neumann's mathematical analysis of the structure of self-replication preceded the discovery of the structure of DNA.

Joseph-Louis Lagrange	['Zouzef lui lq'gra:nZ] = [lq'greinG]	Ժոզեֆ Լուի Լագրանժ

Joseph-Louis Lagrange (1736–1813) was an Italian Enlightenment Era mathematician and astronomer who made significant contributions to the fields of analysis, number theory, and both classical and celestial mechanics.

In 1766, on the recommendation of Euler and d'Alembert, Lagrange succeeded Euler as the director of mathematics at the Prussian Academy of Sciences in Berlin, Prussia, where he stayed for over twenty years, producing volumes of work and winning several prizes of the French Academy of Sciences. Lagrange's treatise on analytical mechanics offered the most comprehensive treatment of classical mechanics since Newton and formed a basis for the development of mathematical physics in the nineteenth century.

Karl Weierstrass	['ka:l 'wJrStras]	Կարլ Վայերշտրաս

Karl Weierstrass (1815–1897) was a German mathematician, one of the founders of the modern theory of functions.

Kenneth Ribet	['kFniT'ribit]	Քենեթ Ռիբեթ

Kenneth Ribet (born June 28, 1948) is an American mathematician, currently a professor of mathematics at the University of California, Berkeley. His mathematical interests include algebraic number theory and algebraic geometry.

Kurt Gödel	['kq:t 'gq:d(q)l]	Կուրտ Գյոդել

Kurt Gödel (1906 – 1978) an Austrian, and later American, logician, mathematician, and philosopher. Considered with Aristotle and Gottlob Frege to be one of the most significant logicians in history, Gödel made an immense impact upon scientific and philosophical thinking in the 20th century, a time when others such as Bertrand Russell, A. N. Whitehead, and David Hilbert were pioneering the use of logic and set theory to understand the foundations of mathematics.

Laplace Pierre Simon	[la'pla:s'piq 'simqn] = [lq'pla:s]	Լապլաս Պիեռ Սիմոն

Laplace Pierre Simon (1749 –1827) was an influential French scholar whose work was important to the development of mathematics, statistics, physics, and astronomy. He summarized and extended the work of his predecessors in his five-volume Mécanique Céleste (Celestial Mechanics) (1799–1825).

This work translated the geometric study of classical mechanics to one based on calculus, opening up a broader range of problems. In statistics, the Bayesian interpretation of probability was developed mainly by Laplace. Laplace is remembered as one of the greatest scientists of all time. Sometimes referred to as the French Newton or Newton of France, he possessed a phenomenal natural mathematical faculty superior to that of any of his contemporaries.

Leicestershire ['leistqSiq] Լեյչեստերշիր

Leicestershire – a landlocked county in the English Midlands. It takes its name from the City of Leicester, traditionally its administrative center, although the City of Leicester unitary authority is today administered separately from the rest of Leicestershire.

The county has a population of just under 1 million with over half the population living in Leicester's built-up area. The county covers an area of 2,084 km2.

Leonhard Euler ['lenha:t '0ilq] Լեոնարդ Էյլեր

Leonhard Euler (1707–1783) was a pioneering Swiss mathematician and physicist. He made important discoveries in infinitesimal calculus and graph theory. He also introduced much of the modern mathematical terminology and notation, particularly for mathematical analysis, such as the notion of a mathematical function. He is also renowned for his work in mechanics, fluid dynamics, optics, astronomy, and music theory.

Leopold Kronecher ['lioupould 'kr0nqkq] Լեոպոլդ Քրոնեկեր

Leopold Kronecher (07.12.1823–29.12.1891) was a German mathematician whose primary contributions were in the theory of equations and higher algebra.

Louis Bachelier	['lui:'bxCq'liq]	Լուիս Բաշելիեր

Louis Bachelier (1870–1946) was a French mathematician at the turn of the 20th century. He is credited with being the first person to model of Brownian motion. It discussed the use of Brownian motion to evaluate stock options and is historically the first paper to use advanced mathematics in the study of finance. Bachelier is considered a pioneer in the study of financial mathematics and stochastic processes.

Luis Ramirez De Lucena	[lHis rq'mirez dq 'lusinq]	Լուիս Ռամիրես դե Լուսենա

Luis Ramirez De Lucena (c. 1465 – c. 1530) was a Spanish chess player who published the first still existing chess book. He was probably the son of the humanist writer and diplomat Juan de Lucena, from a family of Jews who converted to Roman Catholicism.

Magnus Carlsen	['magnqs 'ka:ls(q)n]	Մագնուս Կարլսոն

Magnus Carlsen defeated V. Anand in the World Chess Championship 2013, becoming the new world chess champion.

Marquis de Condorcet	['ma:kwis dq'k0nd0r'se]	Մարի Ժան Անտուան Նիկոլա դը Կոնդորսե, մարքիզ

Marquis de Condorcet (1743–1794) was a French philosopher, mathematician, and early political scientist whose Condorcet method in voting tally selects the candidate who would beat each of the other candidates in a run-off election. Unlike many of his contemporaries, he advocated a liberal economy, free and equal public instruction, constitutionalism, and equal rights for women and people of all races. His ideas and writings were said to embody the ideals of the Age of Enlightenment and rationalism, and remain influential to this day. He died a mysterious death in prison after a period of flight from French Revolutionary authorities.

Mary Sommerville	['mxri's0mqvil]	Մերի Սոմերվիլ

Mary Sommerville (1780–1872) was a Scottish science writer and polymath, at a time when women's participation in science was discouraged. She studied mathematics and astronomy, and was nominated to be jointly the first female member of the Royal Astronomical Society at the same time as Caroline Herschel.

Merton Miller	['mqtn'milq]	Մերտոն Միլլեր

Merton Miller (1923–2000) – an American economist and the co-author of the Modigliani–Miller theorem, which proposed the irrelevance of debt-equity structure. He was awarded by Nobel Memorial Prize in Economic Sciences.

Monsieur Leblanc	[mq'sjq: lq'blank]	Մըսյե Լեբլանկ

Monsieur Leblanc is Sophie Germain's pseudonym.

Niels Henrik Abel	[nilz 'henrik 'eibql]	Նիլս Հենրիկ Աբել

Niels Henrik Abel (1802–1829) was a Norwegian mathematician, a pioneer in the development of several branches of modern mathematics. In 2002 the Abel Prize was established in his memory.

Nuneaton	[nq'nJtqn]	Նունեատոն

Nuneaton is the largest town in the Borough of Nuneaton and Bedworth and in the English county of Warwickshire. The population of Nuneaton's eleven wards in 2011 was 81,877.

Oscar Morgenstern	['Oskq'mLgn'stWn]	Օսկար Մորգենշտեյն

Oscar Morgenstern (1902–1977) was a German –born economist. In collaboration with mathematician John von Neumann, he founded the mathematical field of game theory and its application to economics. The collaboration between economist Morgenstern and mathematician von Neumann led to the birth of entirely new areas of investigation in both mathematics and economics.

Padua	['pxdjuq]	Պադուա

Padua is a city and commune in northern Italy. It is the capital of the province of Padua and the economic and communications hub of the area.

Paul Wolfskehl	['pLl "w0lf'skel]	Փոլ Վոլֆսքել

Paul Wolfskehl (1856-1906) was a Jewish-German industrialist with an interest in mathematics.

Pierre de Fermat	[piq dq' fer'ma]	Պիեռ դե Ֆերմա

Pierre de Fermat (1601–1665), was a French mathematician. His work on curves led directly to the general methods of calculus introduced by Newton and Leibniz. He is also recognized as the founder of the theory of numbers.

Pierre Fatou [piq fq'tu] Պիեռ Ֆատու

Pierre Fatou (28.02.1878 – 10.08.1929) was a French mathematician and astronomer. He is known for major contributions to several branches of analysis. The Fatou lemma and the Fatou set are named after him.

Pythagoras [pai'Tagqrqs] = [pi'Txgqrqs] Պյութագորաս

Pythagoras (570 – 495 BC) was a Greek philosopher, mathematician. He was born on the island of Samos, and around 530 BC he established some kind of school. Pythagoras made influential contributions to philosophy. He is best known for the Pythagorean theorem which bears his name. to mathematics or natural philosophy. It was said that he was the first man to call himself a philosopher, or lover of wisdom, and Pythagorean ideas exercised a marked influence on Plato, and through him, all of Western philosophy.

Raul Jose Capablanca ['raul 'houze "keipq'blankq] Խոսե Ռաուլ Կապաբլանկա

Raul Jose Capablanca (1888–1942) was a Cuban chess player who was world chess champion from 1921 to 1927. Considered one of the greatest players of all time, he was renowned for his exceptional endgame skill and speed of play. He was exceptionally difficult to beat, losing only 35 first class games in his entire career.

René Descartes [rq'ne dei'ka:(r)t] Ռենե Դեկարտ

René Descartes (1596–1650) – was a French philosopher, mathematician, and scientist. Descartes's influence in mathematics is equally apparent; the Cartesian coordinate system – allowing reference to a point in space as a set of numbers, and allowing algebraic equations to be expressed as geometric shapes in a two or three-dimensional coordinate system (and conversely, shapes to be described as equations) – was named after him. He is credited as the father of analytical geometry, the bridge between algebra and geometry, used in the discovery of infinitesimal calculus and analysis. Descartes was also one of the key figures in the scientific revolution.

Richard Dedekind ['ritSqd dedq'kind] Ռիչարդ Դեդեկինդ

Richard Dedekind (1831–1916) was a German mathematician who developed a major redefinition of irrational numbers in terms of arithmetic concepts. Although not fully recognized in his lifetime, his treatment of the ideas of the infinite and of what constitutes a real number continues to influence modern mathematics.

Robert John Aumann ['rLbqt'G0n'auman] Ռոբերտ Ջոն Աուման

Robert John Aumann (born June 8, 1930) is an Israeli-American mathematician and a member of the United States National Academy of Sciences. Aumann received the Nobel Prize in Economics in 2005 for his work on conflict and cooperation through game-theory analysis. He shared the prize with Thomas Schelling.

Roger Cotes ['rouGq 'kouts] Ռոջեր Կոտե

Roger Cotes (1682–1716) was an English mathematician, known for working closely with Isaac Newton by proofreading the second edition of his famous book, the Principia, before publication. He also invented the quadrature formulas known as Newton–Cotes formulas and first introduced what is known today as Euler's formula. He was the first Plumian Professor at Cambridge University from 1707 until his death.

Simon Singh ['saimqn siN] Սիմոն Սինգ

Simon Singh (born 1 January 1964) is a British author who has specialized in writing about mathematical and scientific topics in an accessible manner. He is the maiden winner of the Lilavati Award. His written works include Fermat's Last Theorem (in the United States titled *Fermat's Enigma*: *The Epic Quest to Solve the World's Greatest Mathematical Problem*), *The Code Book* (about cryptography and its history), *Big Bang* (about the *Big Bang* theory and the origins of the universe) and *Trick or Treatment? Alternative Medicine on Trial* (about complementary and alternative medicine).

Sophie Germain	[sq'fi Gq'mein]	Սոֆի Ջերմեյն

Sophie Germain (1776 –1831) was a French mathematician, physicist, and philosopher. One of the pioneers of elasticity theory, she won the grand prize from the Paris Academy of Sciences for her essay on the subject. Her work on Fermat's Last Theorem provided a foundation for mathematicians exploring the subject for hundreds of years after.

Switzerland	['switsqlqnd]	Շվեյցարիա

Switzerland – a country in Europe. While still named the "Swiss Confederation" for historical reasons, modern Switzerland is a federal directorial republic consisting of 26 cantons, with Bern as the seat of the federal authorities, called Bundesstadt ("federal city"). The country is situated in Western and Central Europe.

Thales	['Teili:z]	Թալես

Thales (624–546 BC) was a pre-Socratic Greek and mathematician from Miletus in Asia Minor and one of the Seven Sages of Greece.Thales attempted to explain natural phenomena without reference to mythology. In mathematics, Thales used geometry to calculate the heights of pyramids and the distance of ships from the shore. He is the first to use deductive reasoning applied to geometry, by deriving four corollaries to Thales' Theorem.

The Maya civilization	[Dq 'majq"sivilai'zeiSn]	Մայա քաղաքա-կրթություն

The Maya civilization (2000 BC) developed by the Maya peoples, noted for the Maya hieroglyphic script, art, architecture and mathematical and astronomical systems.

Theodore von Kármán	['Tju:dq f0n 'ka:mqn]	Թեոդոր Ֆոն Կարման

Theodore von Kármán (1881–1963) a Hungarian-American mathematician, aerospace engineer and physicist who was active primarily in the fields of aeronautics and astronautics. He is responsible for many key advances in aerodynamics, notably his work on supersonic and hypersonic airflow characterization. He is regarded as the outstanding aerodynamic theoretician of the twentieth century.

Thomas Simpson	['t0mqs 'simpsqn]	Թոմաս Սիմփսոն

Thomas Simpson (1710–1761) was a British mathematician, inventor and eponym of Simpson’s rule to approximate definite integrals.

Vera Menchik	[verq 'menCik]	Վերա Մենչիկ

Vera Menchik (1906–1944) was a British-Russian chess player who gained renown as the world’s first women’s chess champion. She also competed in chess tournaments with some of the world’s leading male chess masters, defeating many of them, including future World Champion Max Euwe.

Viswanathan Anand	["vis'wanqTqn 'xnqnd]	Վիշվանաթան Անանդ

Viswanathan Anand (born in 11.12.1969) is an Indian chess Grandmaster and former World Chess Champion. In the World Chess Championship 2013 he lost to challenger Magnus Carlsen and he lost again to Carlsen in the World Chess Championship 2014.

Anand is one of nine players in history to pass 2800 Elo on the FIDE list. He occupied the number one position for 21 months, the 6th longest on record.

Wilhelm Gottfried Leibniz	['wilhelm 'g0tfrid 'laibnits]	Վիլհելմ Գոթֆրիդ Լայբնից

Wilhelm Gottfried Leibniz (1646–1716) was a German philosopher, mathematician and political adviser, important both as a metaphysician and as a logician and distinguished also for his independent invention of the differential and integral calculus.

Wilhelm Steinitz	['wilhelm 'Stainits]	Վիլհելմ Շտայնից

Wilhelm Steinitz (1836–1900) was an Austrian and later American chess player and the first undisputed world chess champion from 1886 to 1894. Steinitz lost his title to Emanuel Lasker in 1894 and also lost a rematch in 1896–97.

William Sharpe	['wiljqm'Sa:p]	Վիլյամ Շարփ

William Sharpe (born: June 16, 1934) is an American economist. He is the STANCO 25 Professor of Finance, Emeritus at Stanford University's Graduate School of Business, and the winner of the 1990 Nobel Memorial Prize in Economic Sciences. He is the author of the books "Investments, Portfolio Theory and Capital Markets" and is known for: Capital asset pricing model and Sharpe ratio.

Woolwich ['wu:liC] = ['wu:liG] Վուլվիչ

Woolwich – a historic town as well as an area of South East London within the Royal Borough of Greenwich. Woolwich became part of the London metropolitan area in the mid 19th century, although remaining part of Kent until 1889. In 1965, most of the Metropolitan Borough of Woolwich became part of Greenwich Borough.

Throughout the 18th, 19th and most of the 20th century, Woolwich was an important military and industrial town. It is a river crossing point, with the Woolwich Ferry and the Woolwich foot tunnel crossing to North Woolwich.

Woolwich is identified in the London Plan as one of 35 major centres in Greater London, with a potential to become a metropolitan centre.

Zurich= Zürich ['zjuqrik] Ցյուրիխ

Zurich – the largest city in Switzerland and the capital of the canton of Zurich. It is located in north-central Switzerland at the northwestern tip of Lake Zurich. Zurich is a hub for rail, road and air traffic. Both Zurich Airport and railway station are the largest and busiest in the country.

GUIDE TO MATHEMATICAL NOTATION[77]

Basic mathematical symbols

Symbol	Symbol Name	Meaning/definition	Example
$=$	equals sign	equality	$a = b$ 1) *a* equals *b* 2) *a* is equal to *b* 3) *a* is *b* $5 = 2 + 3$ 5 is equal to 2+3
$\neq$	not equal sign	inequality	$a \neq b$ 1) *a* isn't equal to *b* 2) *a* is not *b* $5 \neq 4$ 5 is not equal to 4
$\approx$	approximately equal	approximation	$\sin(0.01) \approx 0.01$ $x \approx y$ means *x* is approximately equal to *y*
$>$	strict inequality	greater than	$5 > 4$ 5 is greater than 4
$<$	strict inequality	less than	$4 < 5$ 4 is less than 5
$\geq$	inequality	greater than or equal to	$5 \geq 4$, $x \geq y$ means *x* is greater than or equal to *y*

$\leq$	inequality	less than or equal to	$4 \neq 5$, $x \leq y$ means x is less than or equal to y
( )	parentheses	calculate expression inside first	$2 \times (3+5) = 16$
[ ]	brackets	calculate expression inside first	$[(1+2) \times (1+5)] = 18$
$+$	plus sign	addition	$1+1=2$
$-$	minus sign	subtraction	$2-1=1$
Symbol	**Symbol Name**	**Meaning/definition**	**Example**
$\pm$	plus – minus	both plus and minus operations	$a \pm b$ a plus or minus b
$\mp$	minus – plus	both minus and plus operations	$a \mp b$ a minus or plus b
$*$	asterisk	multiplication	$2 * 3 = 6$
$\times$	times sign	multiplication	$2 \times 3 = 6$
$\cdot$	multiplication dot	multiplication	$2 \cdot 3 = 6$
$\div$	division sign / obelus	division	$6 \div 2 = 3$
$/$	division slash	division	$6/2=3$
—	horizontal line	division / fraction	$\frac{6}{2} = 3$
mod	modulo	remainder calculation	$7 \bmod 2 = 1$
.	period	decimal point, decimal separator	$2.56 = 2 + 56/100$
a^n	power	exponent the n^{th} power of a	$2^3 = 8$

$a \wedge b$	caret	exponent	$2 \wedge 3 = 8$
$\sqrt{a}$	square root	$\sqrt{a} \cdot \sqrt{a} = a$ square root of a	$\sqrt{9} = \pm 3$
$\sqrt[3]{a}$	cube root	$\sqrt[3]{a} \cdot \sqrt[3]{a} \cdot \sqrt[3]{a} = a$ cube root of a	$\sqrt[3]{8} = 2$
$\sqrt[4]{a}$	fourth root	$\sqrt[4]{a} \cdot \sqrt[4]{a} \cdot \sqrt[4]{a} \cdot \sqrt[4]{a} = a$ the fourth root of a	$\sqrt[4]{16} = \pm 2$
$\sqrt[n]{a}$	n^{th} root (radical)	n^{th} root of a	for $n = 3$, $\sqrt[n]{8} = 2$
%	percent	$1\% = 1/100$	$10\% \times 30 = 3$
‰	per-mille	$1‰ = 1/1000 = 0.1\%$	$10‰ \times 30 = 0.3$
ppm	per-million	$1\,ppm = 1/1000000$	$10\,ppm \times 30 = 0.0003$
ppb	per-billion	$1\,ppb = 1/1000000000$	$10\,ppb \times 30 = 3 \times 10^{-7}$
ppt	per-trillion	$10\,ppt = 10^{-12}$	$10\,ppt \times 30 = 3 \times 10^{-10}$

Geometry symbols

Symbol	Symbol Name	Meaning/definition	Example
$\angle$	angle	formed by two rays	$\angle ABC = 30^\circ$
$\measuredangle$	measured angle		$\measuredangle ABC = 30^\circ$
$\sphericalangle$	spherical angle		$\sphericalangle AOB = 30^\circ$
∟	right angle	$= 90^\circ$	$\alpha = 90^\circ$
°	degree	$1\,turn = 360^\circ$	$\alpha = 60^\circ$
deg	degree	$1\,turn = 360\deg$	$\alpha = 60\deg$
α'	prime	arcminute, $1^\circ = 60'$	$\alpha = 60^\circ 59'$

α''	double prime	arcsecond, $1^\circ = 60''$	$\alpha = 60^\circ 59' 59''$
$\overleftrightarrow{AB}$	line	infinite line	
$\overline{AB}$	line segment	line from point A to point B	
$\overrightarrow{AB}$	ray	line that start from point A	
$\overset{\frown}{AB}$	arc	arc from point A to point B	$\overset{\frown}{AB} = 60^\circ$
$\perp$	perpendicular	perpendicular lines (90° angle)	$\overline{AB} \perp \overline{BC}$
$\parallel$	parallel	parallel lines	$\overline{AB} \parallel \overline{BC}$
$\cong$	congruent to	equivalence of geometric shapes and size	$\Delta ABC \cong \Delta XYZ$
$\sim$	similarity	same shapes, not same size	$\Delta ABC \sim \Delta XYZ$
Δ	triangle	triangle shape	$\Delta ABC \cong \Delta BCD$
$\lvert x-y \rvert$	distance	distance between points x and y	$\lvert x-y \rvert = 5$
π	pi constant	$\pi = 3.141592654\ldots$ is the ratio between the circumference and diameter of a circle	$c = \pi \cdot d = 2 \cdot \pi \cdot r$
rad	radians	radians angle unit	$360^\circ = 2\pi \text{rad}$
c	radians	radians angle unit	$360^\circ = 2\pi^c$
grad	gradians / gons	grads angle unit	$360^\circ = 400 \text{grad}$
g	gradians / gons	grads angle unit	$360^\circ = 400^g$

Algebra symbols

Symbol	Symbol Name	Meaning/definition	Example
x	x variable	unknown value to find	when $2x = 4$, then $x = 2$
$\equiv$	equivalence	identical to	
$\triangleq$	equal by definition	equal by definition	
$:=$	equal by definition	equal by definition	
$\sim$	approximately equal	weak approximation	$11 \sim 10$
$\approx$	approximately equal	approximation	$\sin(0.01) \approx 0.01$
$\propto$	proportional to	proportional to	$y \propto x$ when $y = kx$, k constant
∞	lemniscate	infinity symbol	$x \to \infty$ x tends to infinity (x approaches infinity)
$\ll$	much less than	much less than	$1 \ll 1000000$
$\gg$	much greater than	much greater than	$1000000 \gg 1$
$(\,)$	parentheses	calculate expression inside first	$2*(3+5) = 16$
$[\,]$	brackets	calculate expression inside first	$[(1+2)*(1+5)] = 18$
$\{\,\}$	braces	set	

$\lfloor x \rfloor$	floor brackets	rounds number to lower integer	$\lfloor 4.3 \rfloor = 4$
$\lceil x \rceil$	ceiling brackets	rounds number to upper integer	$\lceil 4.3 \rceil = 5$
$x!$	exclamation mark	factorial	$4! = 1*2*3*4 = 24$
$\mid x \mid$	single vertical bar	absolute value	$\mid -5 \mid = 5$
$f(x)$	function of x	the value of the function f at x	$f(x) = 3x + 5$
$(f \circ g)$	function composition	$(f \circ g)(x) = f(g(x))$	$f(x) = 3x$, $g(x) = x - 1 \Rightarrow$ $(f \circ g)(x) = 3(x-1)$
(a,b)	open interval	$(a,b) = \{x \mid a < x < b\}$ the open interval from a to b	$x \in (2,6)$
$[a,b]$	closed interval	$[a,b] = \{x \mid a \le x \le b\}$ the closed interval from a to b	$x \in [2,6]$
log	logarithm	the power to which a number must be raised to get some other number	$\log_4 16$ logarithm base 4 of 16 $f(x) = \log_b x$ logarithmic function to the base b
Δ	delta	change / difference	$\Delta t = t_1 - t_0$
Δ	discriminant	$\Delta = b^2 - 4ac$	
Σ	sigma	summation – sum of all values in range of series	$\sum x_i = x_1 + x_2 + \ldots + x_n$

$\sum\sum$	sigma	double summation	$\sum_{j=1}^{2}\sum_{i=1}^{8} x_{i,j} =$ $= \sum_{i=1}^{8} x_{i,1} + \sum_{i=1}^{8} x_{i,2}$
Π	capital pi	product – product of all values in range of series	$\Pi x_i = x_1 \cdot x_2 \cdot \ldots \cdot x_n$
e	e constant / Euler's number	$e = 2.718281828$	$e = \lim (1+1/x)^x$, $x \to \infty$
γ	Euler – Mascheroni constant	$\gamma = 0.527721566\ldots$	
φ	golden ratio		

Linear Algebra Symbols

Symbol	Symbol Name	Meaning/definition	Example
$\cdot$	dot	scalar product	$a \cdot b$
$\times$	cross	vector product	$a \times b$
$A \otimes B$	tensor product	tensor product of A and B	$A \otimes B$
$\langle x, y \rangle$	inner product		
[ ]	brackets	matrix of numbers	
( )	parentheses	matrix of numbers	
$\lvert A \rvert$	determinant	determinant of matrix A	

$\det(A)$	determinant	determinant of matrix A	
$\|x\|$	double vertical bars	norm	
$\boldsymbol{A^T}$	transpose	matrix transpose	$\left(A^T\right)_{ij} = (A)_{ji}$
$A^\dagger$	Hermitian matrix	matrix conjugate transpose	$\left(A^\dagger\right)_{ij} = \left(\overline{A}\right)_{ji}$
A^*	Hermitian matrix	matrix conjugate transpose	$\left(A^*\right)_{ij} = \left(\overline{A}\right)_{ji}$
A^{-1}	inverse matrix	$AA^{-1} = I$	
$\mathrm{rank}(A)$	matrix rank	rank of matrix A	$\mathrm{rank}(A) = 3$
$\dim(U)$	dimension	dimension of matrix A	$rank(U) = 3$

Probability and Statistics Symbols

Symbol	Symbol Name	Meaning/definition	Example
$P(A)$	probability function	probability of event A	$P(A) = 0.5$
$P(A \cap B)$	probability of events intersection	probability that of events A and B	$P(A \cap B) = 0.5$
$P(A \cup B)$	probability of events union	probability that of events A or B	$P(A \cup B) = 0.5$
$P(A \mid B)$	conditional probability function	probability of event A given event B occurred	$P(A \mid B) = 0.3$
$f(x)$	probability density function (pdf)	$P(a \le x \le b) = \int f(x)dx$	
$F(x)$	cumulative distribution function (cdf)	$F(x) = P(X \le x)$	
μ	population mean	mean of population values	$\mu = 10$
$E(X)$	expectation value	expected value of random variable X	$E(X) = 10$
$E(X \mid Y)$	conditional expectation	expected value of random variable X given Y	$E(X \mid Y = 2) = 5$
$\text{var}(X)$	variance	variance of random variable X	$\text{var}(X) = 4$

σ^2	variance	variance of population values	$\sigma^2 = 4$
$\mathrm{std}(X)$	standard deviation	standard deviation of random variable X	$\mathrm{std}(X) = 2$
σ_x	standard deviation	standard deviation value of random variable X	$\sigma_x = 2$
$\tilde{\boldsymbol{x}}$	median	middle value of random variable X	$\tilde{x} = 5$
$\mathrm{cov}(X,Y)$	covariance	covariance of random variables X and Y	$\mathrm{cov}(X,Y) = 4$
$\mathrm{corr}(X,Y)$	correlation	correlation of random variables X and Y	$\mathrm{corr}(X,Y) = 0.6$
$\rho_{X,Y}$	correlation	correlation of random variables X and Y	$\rho_{X,Y} = 0.6$
$\sum$	summation	summation – sum of all values in range of series	$\sum_{i=1}^{4} x_i = x_1 + x_2 + x_3 + x_4$
$\sum\sum$	double summation	double summation	$\sum_{j=1}^{2}\sum_{i=1}^{8} x_{i,j} = \sum_{i=1}^{8} x_{i,1} + \sum_{i=1}^{8} x_{i,2}$
Mo	mode	value that occurs most frequently in population	
MR	mid-range	$MR = \dfrac{x_{\max} + x_{\min}}{2}$	
Md	sample median	half the population is below this value	
Q_1	lower / first quartile	25% of population are below this value	
Q_2	median / second quartile	50% of population are below this value = median of samples	

Q_3	upper / third quartile	75% of population are below this value	
$\overline{x}$	sample mean	average / arithmetic mean	$\overline{x} = \frac{2+5+9}{3} = 5.333$
s^2	sample variance	population samples variance estimator	$s^2 = 4$
s	sample standard deviation	population samples standard deviation estimator	$s = 2$
z_x	standard score	$z_x = \left(x - \overline{x}\right) / s_x$	
$X \sim$	distribution of X	distribution of random variable X	$X \sim N(0.3)$
$N(\mu, \sigma^2)$	normal distribution	Gaussian distribution	$X \sim N(0.3)$
$U(a, b)$	uniform distribution	equal probability in range a, b	$X \sim U(0.3)$
$\exp(\lambda)$	exponential distribution	$f(x) = \lambda e^{-\lambda x}$, $x \geq 0$	
$gamma(c, \lambda)$	gamma distribution	$f(x) = \frac{\lambda c x^{c-1} e^{-\lambda x}}{\Gamma(c)}$, $x \geq 0$	
$\chi^2(k)$	chi-square distribution	$f(x) = \frac{x^{k/2-1} e^{-x/2}}{2^{k/2} \Gamma(k/2)}$	
$F(k_1, k_2)$	F distribution		

$Bin(n,p)$	binomial distribution	$f(k) = {}_nC_k p^k (1-p)^{n-k}$	
$Poisson(\lambda)$	Poisson distribution	$f(k) = \frac{\lambda^k e^{-k}}{k!}$	
$Geom(p)$	geometric distribution	$f(k) = p(1-p)^k$	
$HG(N,K,n)$	hyper-geometric distribution		
$Bern(p)$	Bernoulli distribution		

Combinatorics Symbols

Symbol	Symbol Name	Meaning/definition	Example
$n!$	factorial	$n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n$	$5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$
${}_nP_k$	permutation	${}_nP_k = \frac{n!}{(n-k)!}$	${}_5P_3 = \frac{5!}{(5-3)!} = 60$
${}_nC_k$ $\binom{n}{k}$	combination	${}_nC_k = \binom{n}{k} = \frac{n!}{k!(n-k)!}$	${}_5C_3 = \frac{5!}{[3!(5-3)!]} = 10$

Set Theory Symbols

Symbol	Symbol Name	Meaning/definition	Example
$\{\}$	set	a collection of elements	$A = \{3,7,9,14\}$, $B = \{9,14,28\}$
$A \cap B$	intersection	objects that belong to set *A* and set *B*	$A \cap B = \{9,14\}$
$A \cup B$	union	objects that belong to set *A* or set *B*	$A \cup B = \{3,7,9,14,28\}$
$A \subseteq B$	subset	*A* is a subset of *B*, set *A* is included in set *B*	$\{9,14,28\} \subseteq \{9,14,28\}$
$A \subset B$	proper subset / strict subset	*A* is a subset of *B*, but *A* is not equal to *B*	$\{9,14\} \subset \{9,14,28\}$
$A \not\subset B$	not subset	set *A* is not a subset of set *B*	$\{9,66\} \not\subset \{9,14,28\}$
$A \supseteq B$	superset	*A* is a superset of *B*, set *A* includes set *B*	$\{9,14,28\} \supseteq \{9,14,28\}$
$A \supset B$	proper superset / strict superset	*A* is a superset of *B*, but *B* is not equal to *A*	$\{9,14,28\} \supset \{9,14\}$
$A \not\supset B$	not superset	set *A* is not a superset of set *B*	$\{9,14,28\} \not\supset \{9,66\}$
2^A	power set	all subsets of *A*	
$\mathcal{P}(A)$	power set	all subsets of *A*	
$A=B$	equality	both sets have the same members	$A = \{3,9,14\}$, $B = \{3,9,14\}$, $A=B$
A^c	complement	all the objects that do not belong to set *A*	

$A \setminus B$	relative complement	objects that belong to A and not to B	$A = \{3,9,14\}$, $B = \{1,2,3\}$, $A - B = \{9,14\}$
$A - B$	relative complement	objects that belong to A and not to B	$A = \{3,9,14\}$, $B = \{1,2,3\}$, $A - B = \{9,14\}$
$A \Delta B$	symmetric difference	objects that belong to A or B but not to their intersection	$A = \{3,9,14\}$, $B = \{1,2,3\}$, $A \Delta B = \{1,2,9,14\}$
$A \ominus B$	symmetric difference	objects that belong to A or B but not to their intersection	$A = \{3,9,14\}$, $B = \{1,2,3\}$, $A \ominus B = \{1,2,9,14\}$
$a \in A$	element of	set membership (a is an element of A)	$A = \{3,9,14\}$, $3 \in A$
$x \notin A$	not element of	no set membership (x is not an element of A)	$A = \{3,9,14\}$, $1 \notin A$
(a,b)	ordered pair	collection of 2 elements	
$A \times B$	cartesian product	set of all ordered pairs from A and B	
$\|A\|$	cardinality	the number of elements of set A	$A = \{3,9,14\}$, $\|A\| = 3$
$\# A$	cardinality	the number of elements of set A	$A = \{3,9,14\}$, $\# A = 3$
$\aleph_0$	aleph-null	infinite cardinality of natural numbers set	
$\aleph_1$	aleph-one	cardinality of countable ordinal numbers set	

$\varnothing$	empty set	$\varnothing = \{\}$	$C = \{\varnothing\}$
U	universal set	set of all possible values	
$\mathbb{N}_0$	natural numbers / whole numbers set (with zero)	$\mathbb{N}_0 = \{0,1,2,3,4,\ldots\}$	$0 \in \mathbb{N}_0$
$\mathbb{N}_1$	natural numbers / whole numbers set (without zero)	$\mathbb{N}_1 = \{0,1,2,3,4,\ldots\}$	$6 \in \mathbb{N}_1$
$\mathbb{Z}$	integer numbers set	$\mathbb{Z} = \{\ldots -3,-2,- -1,0,1,2,3,\ldots\}$	$-6 \in \mathbb{Z}$
$\mathbb{Q}$	rational numbers set	$\mathbb{Q} = \left\{ \begin{matrix} x \mid x = a/b, \\ a,b \in \mathbb{Z} \end{matrix} \right\}$	$2/6 \in \mathbb{Q}$
$\mathbb{R}$	real numbers set	$\mathbb{R} = \{x \mid -\infty < x < \infty\}$	$6.343434 \in \mathbb{R}$
$\mathbb{C}$	complex numbers set	$\mathbb{C} = \left\{ \begin{matrix} z \mid z = a+bi, \\ -\infty < a < \infty, \\ -\infty < b < \infty \end{matrix} \right\}$	$6+2i \in \mathbb{C}$

Logic Symbols

Symbol	Symbol Name	Meaning/definition	Example
$\cdot$	and	and	$x \cdot y$
^	caret / circumflex	and	x^y
&	ampersand	and	x & y
$+$	plus	or	$x + y$
$\vee$	reversed caret	or	$x \vee y$
$\mid$	vertical line	or	$x \mid y$
x'	single quote	not-negation	x'
$\overline{x}$	bar	not-negation	$\overline{x}$
$\neg$	not	not-negation	$\neg x$
!	exclamation mark	not-negation	$!x$
$\oplus$	circled plus / oplus	exclusive or – xor	$x \oplus y$
~	tilde	negation	$\sim x$
$\Rightarrow$	implies		
$\Leftrightarrow$ $\leftrightarrow$	equivalent	if and only if (iff)	
$\forall$	for all		
$\exists$	there exists		
$\nexists$	there does not exist		
$\therefore$	therefore		
$\because$	because / since		
$\ldots$		countinues unendingly, and so on up to, and so on indefinitely	

Calculus & Analysis Symbols

Symbol	Symbol Name	Meaning/definition	Example
$\lim\limits_{x \to x0} f(x)$	limit	limit value of a function *f* as *x* approaches 0	
ε	epsilon	represents a very small number, near zero	$\varepsilon \to 0$
e	*e* constant / Euler's number	$e = 2.718281828\ldots$	$e = \lim$, $\left(1+1/x\right)^x$, $x \to \infty$
y'	derivative	derivative – Lagrange's notation	$\left(3x^3\right)' = 9x^2$
y''	second derivative	derivative of derivative	$\left(3x^3\right)'' = 18x$
$y^{(n)}$	n^{th} derivative	*n* times derivation	$\left(3x^3\right)^{(3)} = 18$
$\dfrac{dy}{dx}$	derivative	derivative – Leibniz's notation the first derivative of *y* with respect to *x*	$\dfrac{d(3x^3)}{dx} = 9x^2$
$\dfrac{d^2 y}{dx^2}$	second derivative	derivative of derivative the second derivative of *y* with respect to *x*	$\dfrac{d^2(3x^3)}{dx^2} = 18x$
$\dfrac{d^n y}{dx^n}$	n^{th} derivative	*n* times derivation	
$\dot{y}$	time derivative	derivative by time – Newton's notation	

$\ddot{y}$	time second derivative	derivative of derivative	
$D_x y$	derivative	derivative – Euler's notation	
$D_x^2 y$	second derivative	derivative of derivative	
$\dfrac{\partial f(x,y)}{\partial x}$	partial derivative		$\dfrac{\partial(x^2+y^2)}{\partial x}=2x$
$\int$	integral	opposite to derivation	
$\iint$	double integral	integration of function of 2 variables	
$\iiint$	triple integral	integration of function of 3 variables	
$\int_a^b$		integral is to be taken between values *a* and *b* the definite integral from *a* to *b* the integral	
$\int 2xdx = x^2$		the integral of $2xdx$ is x^2	
$\oint$	closed contour / line integral		
$\oiint$	closed surface integral		
$\oiiint$	closed volume integral		

$[a,b]$	closed interval	$[a,b]=\{x \mid a \le x \le b\}$	
(a,b)	open interval	$(a,b)=\{x \mid a < x < b\}$	
i	imaginary unit	$i \equiv \sqrt{-1}$	$z = 3+2i$
$\mathbf{z}^*$	complex conjugate	$z = a+bi \rightarrow z^* = a-bi$	$z^* = 3-2i$
$\overline{z}$	complex conjugate	$z = a+bi \rightarrow \overline{z} = a-bi$	$\overline{z} = 3-2i$
∇	nabla / del	gradient / divergence operator	$\nabla f(x,y,z)$
$\vec{\boldsymbol{x}}$	vector		
$\widehat{\boldsymbol{x}}$	unit vector		
$x^* y$	convolution	$y(t)=x(t)^* h(t)$	
$\mathcal{L}$	Laplace transform	$F(s)=\mathcal{L}\{f(t)\}$	
$\mathcal{F}$	Fourier transform	$X(\omega)=\mathcal{F}\{f(t)\}$	
δ	delta function		
∞	lemniscate	infinity symbol	

Reading of whole numbers, fractions, equations, formulae

234	two hundred and thirty-five
	Addition
$2+3=5$	1) two plus three equals five 2) two plus three is equal to five 3) two and three is (are) five 4) two added to three makes five
	Subtraction
$8-4=4$	1) eight minus four equals four 2) the difference between four and eight is four
	Multiplication
$1\times1=1$ $2\times2=4$ $3\times3=9$	once one is one twice two is four three times three is nine
$5\times5=25$	$\text{5 multiplied by 5} \quad \left.\begin{array}{l}\text{is equals,}\\ \text{is equal to}\\ \text{are,}\\ \text{makes}\\ \text{make}\end{array}\right\} \quad 25$
$11\times7=77$ $a\times b=c$	11 multiplied by 7 equals 77 *a* multiplied by *b* equals *c*
	Division
$15\div3=5$	fifteen divided by three equals five
$16:4=4$	1) 16 divided by 4 equals 4 2) 16 divided by 4 is 4 3) the ratio of 16 to 4 is 4

Involution	
x^2	1) x square; x squared 2) x to the second power 3) x raised to the second power 4) the square of x 5) the second power of x
$5^2 = 25$	1) the second power of 5 is 25 2) 5 square is 25 3) 5 to the second power is 25 4) 5 raised to the second power is equal to 25
Involution	
y^3	1) y cube; y cubed 2) y to the third power 3) y raised to the third power 4) the cube of y 5) the third power of y 6) y to the third
4^3	four cubed
6^4	1) six to the fourth power 2) six to the power four
a^b	a to the power b
a'	a prime
a''	1) a second prime 2) a double prime 3) a twice dashed
a'''	a triple prime
a_m	1) a sub m 2) a m^{th}
a_1	1) a sub one 2) a first

a_2	1) *a* sub two 2) *a* second
f_c	1) *f* prime, sub *c* 2) *f* sub *c*, prime
a_1'	*a* first prime
a_2''	*a* second, second prime
Fractions	
$1/2$	one half, a half
$3\frac{1}{3}$	three and a third
$1/3$	one third
$\frac{4}{7}$	four-seventh
Decimal Fractions	
63.57	1) sixty-three point five seven 2) six three point five seven
0.2	1) 0 [ou] point two 2) naught point two 3) zero point two 4) point two
0.02	1) 0 [ou] point 0 [ou] two 2) naught point naught two 3) zero point zero two 4) point zero two
0.75	1) naught point seventy-five 2) point seven five

Ratio	
$20:5=16:4$	1) the ratio of 20 to 5 equals the ratio of 16 to 4 2) 20 is to 5 as 16 is to 4
$4:2$	the ratio of four to two
$a:b$	the ratio of *a* to *b*
Proportion	
$2:3=4:6$	1) two is three as four is to six 2) the ratio of two to three equals the ratio of four to six
$a:b=c:d$	*a* is to *b* as *c* is to *d*
$x=k/y$	1) *x* varies inversely as *y* 2) *x* is inversely proportional to *y*
Evolution	
$\sqrt{4}=2$	the square root of four is two
$\sqrt[3]{27}=3$	the cube root of twenty-seven is three
$\sqrt{a}$	the square root of *a*
$\sqrt[5]{a^2}$	the fifth root of *a* square

Equations	
$(a+b)(a-b)=a^2+b^2$	The product of the sum and difference of two quantities is equal to the difference of their squares.
$2+x+\sqrt{4}+x^2=10$	Two plus *x* plus the square root of four plus *x* squared is equal to ten.
Formulae	
$\frac{dx}{dy}$	first derivative of *x* with respect to *y*
$y=f(x)$	*y* is a function of *x*
$M=R_1x-P_1(x-a_1)-$ $-P_2(x-a_2)$	*M* is equal to *R* sub one multiplied by *x* minus *P* sub one, round brackets opened, *x* minus *a* sub one, round brackets closed, minus *P* sub two, round brackets opened, *x* minus *a* sub two, round brackets closed.
$\gamma=\frac{c'c}{ac'}$	gamma is equal to the ratio of the segment *c* prime *c* to the segment *ac* prime
$\varepsilon=\frac{B}{6.45}\times 2.54$	ε is equal to B divided by six point four five multiplied by two point five four

$\frac{a+b}{a-b}=\frac{c+d}{c-d}$	*a* plus *b* over a minus b is equal to c plus d over *c* minus *d*
$a^3=\log_e d$	*a* cubed is equal to the logarithm of *d* to the base *c*
$\frac{d^2y}{ds^2}+[t+b(s)]y=0$	the second derivative of *y* with respect to *s*, plus y times the quantity *t* plus *b* of *s*, is equal to zero
$x_{a-b}=c$	*x* sub a minus *b* is equal to *e*
$f(s)=k_{ab}$	*f* of s is equal to *k* sub *ab*
$\frac{\delta^2u}{\delta t^2}=0$	the second partial derivative of *u* with respect to *t* equals zero
$\varphi(z)=b\left[\left(2+\frac{z}{c_m}\right)^{\frac{m}{m-1}}-5\right]$	1) φ of *z* is equal to *b*, square brackets, parenthesis, *z* divided by *c* sub *m* plus 2, close parenthesis, to the power *m* over *m* minus 1, minus 5, close square brackets 2) φ of *z* is equal to be multiplied by the whole quantity; the quantity two plus *z* over c sub m, to the power *m* over *m* minus 1 minus 5

$\left\|\varphi_j(t_1)-\varphi_j(t_2)\right\| \le$ $\le M\left(t_1-\dfrac{B}{j}\right)-M\left(t_2-\dfrac{B}{j}\right)$	The absolute value of the quantity sub *j* of *t* one, minus sub *j* of t two is less than or equal to the absolute value of the quantity *M* of t_1 minus *B* over *j*, minus *M* of t_2 minus *B* over *j*.
Matrix	
$\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}$	matrix
$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$	a determinant
$\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$	determinant of the second order

Greek Alphabet Letters

Greek Symbol		Pronunciation	Greek Letter Name
Upper Case	Lower Case		
Α	α	['xlfq]	Alpha
Β	β	['beIt]; ['bJtq]	Beta
Γ	γ	['gxmq]	Gamma
Δ	δ	['deltq]	Delta
Ε	ε	['epsIlqn]	Epsilon
Ζ	ζ	['zJtq]; ['zeItq]	Zeta
Η	η	['eItq] – ['Jtq]	Eta
Θ	θ	['TeItq]; ['TJtq]	Theta
I	ι	[aI'outq]	Iota
Κ	κ	[kxpq]	Kappa
Λ	λ	['lxmbda]	Lambda
Μ	μ	['mJ]	Mu
Ν	ν	['nJ]	Nu
Ξ	ξ	['ksJ]; ['zQI]	Xi
Ο	o	[ou'maIkrqn]	Omicron
Π	π	['pJ]; ['paI]	Pi
Ρ	ρ	['ruo]	Rho
Σ	σ	['sIgmq]	Sigma
Τ	τ	['tou]; ['tL]	Tau
ϒ	υ	['jHpsIlqn]	Upsilon
Φ	φ	['fJ]; ['faI]	Phi
Χ	χ	['kJ]; ['kaI]	Chi
Ψ	ψ	['psJ]; ['saI]	Psi
Ω	ω	[ou'megq]; [ou'mJgq]	Omega

M I S C E L L A N E O U S

MATHEMATICS POETRY

"Practical Application"

He's teaching her arithmetic,
He said it was his mission,
He kissed her once, he kissed her twice
And said, "Now that's addition."

As he added smack by smack
In silent satisfaction,
She sweetly gave the kisses back
And said, "Now that's subtraction."

Then he kissed her, she kissed him,
Without an explanation,
And both together smiled and said,
"That's multiplication."

Then Dad appeared upon the scene and
Made a quick decision.
He kicked that kid three blocks away
And said, "That's long division!"[78]

* * *

One and one make two,
But if one and one should marry,
Isn't it queer-within a year
There's two and one two carry.

* * *

“IF”

If you can solve a literal equation
And rationalize denominator surds,
Do grouping factors (with a transformation)
And state the factor theorem in words;
If you can plot the graph of any function
And do a long division (with gaps),
Or square binomials without compunction
Or work cube roots with logs without mishaps.
If you possess a sound and clear-cut notion
Of interest sums with P and I unknown;
If you can find the speed of trains in motion,
Given some lengths and “passing-times” alone;
If you can play with R (both big and little)
And feel at home with l (or h) and Pi,
And learn by cancellation how to whittle
Your fractions down till they delight the eye.
If you can recognize the segment angles
Both at the center and circumference;
If you can spot equivalent triangles
And Friend Pythagoras (his power’s immense);
If you can see that equiangularity
And congruence are two things and not one,
You may pick up a mark or two in charity
And, what is more, you may squeeze through, my son.

* * *

“OLD MATHEMATICIANS NEVER DIE”

Old mathematicians never die; they just pass into another field.
They just tend to infinity,
They just lose some of their functions,
They just reduce to lowest terms,
Old mathematicians never die; they just disintegrate,
They just go off on a tangent,
They just get disarrayed,
They just get dissolved,
Old mathematicians never die; they just lose their identities,
They just tend to zero,
They just become angles,
They just become irrational,
Old mathematicians never die; their second derivative goes to zero.
Old mathematicians never die; their systems just become unsolvable.

MATH JOKES[79]

* * *

According to recent surveys, 51% of the people are in the majority.

* * *

It is proven that the celebration of birthdays is healthy. Statistics show that those people who celebrate the most birthdays become the oldest.

* * *

It is only two weeks into the term that, in a calculus class, a student raises his hand and asks, “Will we ever need this stuff in real life?”

The professor gently smiles at him and says, “Of course not - if your real life will consist of flipping hamburgers at MacDonald’s”.

* * *

Some engineers are trying to measure the height of a flag pole. They only have a measuring tape and are quite frustrated trying to keep the tape along the pole: It falls down all the time.

A mathematician comes along and asks what they are doing. They explain it to him.

“Well, that’s easy...”

He pulls the pole out of the ground, lays it down, and measures it easily.

After he has left, one of the engineers says: “That’s so typical of these mathematicians! What we need is the height - and he gives us the length!”

* * *

“Divide fourteen sugar cubes into three cups of coffee so that each cup has an odd number of sugar cubes in it.”

“That’s easy: one, one, and twelve.”

“But twelve isn’t odd!”

“It’s an odd number of cubes to put in a cup of coffee.”

* * *

Statistics Canada is hiring mathematicians. Three recent graduates are invited for an interview: one has a degree in pure mathematics, another one in applied math, and the third one obtained his B.Sc. in statistics.

All three are asked the same question: “What is one third plus two thirds?”

The pure mathematician: “It’s one.”

The applied mathematician takes out his pocket calculator, punches in the numbers, and replies: “It’s 0.999999999.”

The statistician: “What do you want it to be?”

* * *

i to π: "Be rational."

π to i: "Get real."

* * *

Question: What does the zero say to the eight?

Answer: Nice belt!

* * *

A lazy dog is a slow pup.

A slope up is an inclined plane.

An ink-lined plane is a sheet of writing paper.

Therefore, a lazy dog is a sheet of writing paper.

* * *

A father, angry because his teenage daughter had missed her curfew, said, "I told you to be home by a quarter of twelve."

The girl responded, "But my math teacher said that 1/4 of 12 is 3."

* * *

The lottery - a tax on people who are bad at math.

* * *

Question: "How does one insult mathematician?"

Answer: "Your brain is smaller than any $\varepsilon < 0$."

* * *

Question: "What do you get when you divide the circumference of a jack-o-lantern by its diameter?"

Answer: "Pumpkin pi"

* * *

Question: "What do you call a mathematician's bird that won't eat? "

Answer: "A poly "no meal"."

* * *

Question: "What happened to the plant in math class?"

Answer: "It grew square roots."

* * *

What is the shortest mathematicians joke?

Let epsilon be smaller than zero.

* * *

My geometry teacher was sometimes acute, sometimes obtuse, but always, he was right.

* * *

John: “Dad, will you do my math homework for me tonight?”

Dad: “No, son, it wouldn’t be right.”

John: “Well, you could try.”

* * *

Teacher: How much is half of 8?

Student: Up and down or across?

Teacher: What do you mean?

Student: Well, up and down it makes 3, but across the middle makes it 0.

* * *

An investment firm is hiring mathematicians. After the first round of interviews, three hopeful recent graduates - a pure mathematician, an applied mathematician, and a graduate in mathematical finance – are asked what starting salary they are expecting.

The pure mathematician: “Would \$30,000 be too much?”

The applied mathematician: “I think \$60,000 would be OK.”

The math finance person: “What about \$300,000?”

The personnel officer: “Do you know that we have a graduate in pure mathematics who is willing to do the same work for a tenth of what you are demanding?”

“Well, I thought of \$135,000 for me, \$135,000 for you – and \$30,000 for the pure mathematician who will do the work.”

* * *

"Students nowadays are so clueless," the math professor complains to a colleague. "Yesterday, a student came into my office and wanted to know if General Calculus was a Roman war hero..."

* * *

A guy gets on a bus and starts threatening everybody: "I'll integrate you! I'll differentiate you!" So everybody gets scared and runs away. Only one person stays. The guy comes up to him and says: "Aren't you scared, I'll integrate you, I'll differentiate you!" And the other guy says: "No, I am not scared, I am e^x."

* * *

Question: "How does a mathematician induce good behavior in her children?"

Answer: "I've told you n times, I've told you n+1 times..."

* * *

An engineer, physicist, and a mathematician were playing cards in a parlor. A fire breaks out. The engineer starts to calculate how much water it takes to put out the fire. The physicist figures out the best theory on how to put out the fire. The mathematician tries to prove the fire doesn't exist.

* * *

An engineer thinks that his equations are an approximation to reality. A physicist thinks reality is an approximation to his equations. A mathematician doesn't care.

* * *

A physicist, a biologist, and a mathematician are sitting on a bench across from a house. They watch as two people go into the house, and then a little later, three people walk out.

The physicist says, "The initial measurement was incorrect."

The biologist says, "They must have reproduced."

And the mathematician says, "If exactly one person enters that house, it will be empty."[80]

* * *

Two mathematicians were having dinner in a restaurant, arguing about the average mathematical knowledge of the American public. One mathematician claimed that this average was woefully inadequate, the other maintained that it was surprisingly high. “I’ll tell you what,” said the cynic, “ask that waitress a simple math question. If she gets it right, I’ll pick up dinner. If not, you do”. He then excused himself to visit the men’s room, and the other called the waitress over. “When my friend comes back,” he told her, “I’m going to ask you a question, and I want you to respond “one third x cubed”. There’s twenty bucks in it for you”. She agreed. The cynic returned from the bathroom and called the waitress over. “The food was wonderful, thank you,” the mathematician started. “Incidentally, do you know what the integral of x squared is?” The waitress looked pensive; almost pained. She looked around the room, at her feet, made gurgling noises, and finally said, “Um, one third x cubed?” So the cynic paid the check. The waitress wheeled around, walked a few paces away, looked back at the two men, and muttered under her breath, “...plus a constant.”[81]

* * *

A somewhat advanced society has figured how to package basic knowledge in pill form. A student, needing some learning, goes to the pharmacy and asks what kind of knowledge pills are available. The pharmacist says, “Here’s a pill for English literature.” The student takes the pill and swallows it and has new knowledge about English literature! “What else do you have?” asks the student. “Well, I have pills for art history, biology, and world history,” replies the pharmacist. The student asks for these, and swallows them and has new knowledge about those subjects. Then the student asks, “Do you have a pill for math?” The pharmacist says “Wait just a moment”, and goes back into the storeroom and brings back a whopper of a pill and plunks it on the counter. “I have to take that huge pill for math?”

inquires the student. The pharmacist replied “Well, you know math always was a little hard to swallow.”[82]

* * *

A physicist and a mathematician sitting in a faculty lounge. Suddenly, the coffee machine catches on fire. The physicist grabs a bucket and leaps towards the sink, fills the bucket with water and puts out the fire. The second day, the same two sit in the same lounge. Again, the coffee machine catches on fire. This time, the mathematician stands up, gets a bucket, hands the bucket to the physicist, thus reducing the problem to a previously solved one.[83]

* * *

A mathematician, an engineer and a physicist sat around a table discussing how to measure the volume of a cow.

The mathematician suggested the use of geometry and symmetry relationships of the cow, but his idea was rejected on the grounds of being too time consuming.

The engineer suggested placing the cow in a pool of water and measuring the change in the height of the water, but his idea was rejected on the grounds of impracticality.

“It’s easy,” said the physicist. “We’ll make an assumption that the cow is a small sphere, calculate the volume and then blow it up to the actual size.”[84]

* * *

A mathematician and a Wall Street broker went to races. The broker suggested to bet $10,000 on a horse. The mathematician was skeptical, saying that he wanted first to understand the rules, to look on horses, etc. The broker whispered that he knew a secret algorithm for the success, but he could not convince the mathematician.

“You are too theoretical,” he said and bet on a horse. Surely, that horse came first bringing him a lot of money. Triumphantly, he exclaimed: “I told you, I knew the secret!”

“What is your secret?” the mathematician asked.

“It is rather easy. I have two kids, three and five years old. I sum up their ages and I bet on number nine.”

“But three and five is eight,” the mathematician protested.

“I told you, you are too theoretical!” the broker replied, “Haven’t I just shown experimentally, that my calculation is correct! 3+5=9!”[85]

R E A L S T O R I E S

FROM THE LIVES OF GREAT SCIENTISTS

PAUL ERDÖS

Paul Erdös had his own particular language:

- **Supreme Fascist** = God (person who hides Erdös’s socks, glasses, Hungarian passport and kept the best equations to himself)
- **Boss** = woman
- **Slave** = man
- **Captured** = married
- **Liberated** = divorced
- **Recaptured** = remarried
- **Epsilon** = child (for the mathematical symbol)
- **To preach** = to give a math lecture
- **To exist** = to do math
- **To die** = to stop doing math
- **Trivial being** = someone who doesn’t do math
- **My brain is open** = I am ready to do math

* * *

Paul Erdồs had the habit of phoning fellow mathematicians over the whole world, no matter what time it was. He remembered the number of every mathematician, but did not know anybody's first name. The only person he called by his Christian name was Tom Trotter, whom he called Bill.

* * *

On one occasion, Paul Erdồs met a mathematician and asked him where he was from. "Vancouver", the mathematician replied. "Oh, then you must know my good friend Elliot Mendelson", Erdồs said. The reply was: "I AM your good friend Elliot Mendelson".

CARL GAUSS

Gauss's precocity

Gauss very early in life exhibited a remarkable cleverness with numbers, becoming a "wonder child" at the age of two. There are a couple of oft-told stories illustrating the boy's unusual ability.

One of the stories tells how on a Saturday evening Gauss's father was making out the weekly payroll for the laborers of the small bricklaying business that he operated in the summer. The father was quite unaware that his young three-year-old son Carl was following the calculations with critical attention, and so was surprised at the end of the computation to hear the little boy announce that the reckoning was wrong and that it should be so and so instead. A check of the figures showed that the boy was correct, and on subsequent Saturday evenings the youngster was propped up on a high stool so that he could assist with the accounts. Gauss enjoyed telling this story later in life, and used to joke that he could figure before he could talk.

* * *

The other story dates from Gauss's schooldays, whence was about ten years old. At the first meeting of the arithmetic class, Master Buttner asked the pupils to write down the numbers from 1 through

100 and add them. It was the custom that the pupils lay their slates, with their answers thereon, on the master's desk upon completion of the problem. Master Buttner had scarcely finished stating the exercise when young Gauss flung his slate on the desk. The other pupils toiled on for the rest of the hour while Carl sat with folded hands under the scornful and sarcastic gaze of the master. At the conclusion of the period, Master Buttner looked over the slates and discovered that Carl alone had the correct answer, and upon inquiry Carl was able to explain how he had arrived at his result. He said, "100 + 1 = 101, 99 + 2 = 101, 98 + 3 = 101, etc., and so we have as many "pairs" as there are in 100. Thus the answer is 50 x 101, or 5050."

Gauss and languages

Gauss mastered languages with great facility. This pursuit of languages became more than just a hobby; he would acquire a new language to test the plasticity of his mind as he grew older, and he considered the exercise as of value in keeping his mind young. At the age of sixty-two he started an intensive self-study of Russian. Within two years he read the language fluently and spoke it perfectly.

PIERRE -SIMON LAPLACE

Laplace seeks employment

When Laplace arrived as a young man in Paris seeking a professorship of mathematics, he submitted his recommendations by prominent people to d'Alembert, but was not received. Returning to his lodgings, Laplace wrote d'Alembert a brilliant letter on the general principles of mechanics. This opened the door, and d'Alembert replied: "Sir, you notice that I paid little attention to your recommendations. You don't need any; you have introduced yourself better." A few days later Laplace was appointed professor of mathematics at the Military School of Paris.

The Newton of France

Laplace's great five-volume work, the *Me'chanique ce'leste*

(1799-1825), earned him the title of “the Newton of France”. The work embraced all previous discoveries in this field along with Laplace’s own contributions, and marked the author as the unrivaled master in the subject. The work has been called the *Almagest* of modern times.

An unneeded hypothesis

When Napoleon teasingly remarked to Laplace that God is not mentioned in the *Me’chanique ce’leste,* Laplace replied, “Sir, I did not need that hypothesis.” When Napoleon later reported this reply to Lagrange, the latter remarked, “Ah, but that is a fine hypothesis. It explains so many things.”

The greatest mathematician in the world

Alexander van Humboldt once asked Laplace who was the greatest mathematician in Germany, and Laplace replied, “Pfaff.”

“But what about Gauss?” asked the astonished Humboldt.

“Gauss,” explained Laplace, “is the greatest mathematician in the world.”

Laplace’s “stepchildren”

Laplace did extend sincere generosity to beginners. A young man once read a scientific paper before a session of the French Academy at which Laplace was present. Afterwards, Laplace drew him aside and showed him the identical discovery in one of his own old but still unpublished manuscripts. Cautioning young man to secrecy, Laplace urged him to go ahead and publish his work.

Laplace used to say that young beginners in mathematical research were his stepchildren, and there are several instances in which he withheld publication of a discovery to allow a beginner the opportunity to publish first.

JOHN VON NEUMANN

A student cornered John von Neumann in the hallway:

Student: "Excuse me, Professor von Neumann, could you please help me with a calculus problem?"

John: "Okay, sonny, if it's real quick – I'm a busy man."

*Student: "*I'm having trouble with this integral."

John: "Let's have a look." After a short pause Neumann answered: "All right, sonny, the answer's two-pi over 5."

*Student: "*I know that, sir, the answer is in the back. I am having trouble deriving it, though."

John: "Okay, let me see it again. The answer's two-pi over 5."

Student (frustrated): "Uh, sir, I know the answer, I just don't see how to derive it."

John: "What do you want, sonny, I worked the problem in two different ways!"

* * *

Von Neumann and Norbert Wiener were both the subject of many dotty professor stories. Von Neumann supposedly had the habit of simply writing answers to homework assignments on the board (the method of solution being, of course, obvious) when he was asked how to solve problems. One time one of his students tried to get more helpful information by asking if there was another way to solve the problem. Von Neumann looked blank for a moment, thought, and then answered, "Yes".

ALBERT EINSTEIN

The story is that Albert Einstein's driver used to sit at the back of the hall during each of his lectures, and after a period of time, remarked to AE that he could probably give the lecture himself, having heard it several times. So at the next stop on the tour, AE & the driver switched places, with AE sitting at the back, in driver's uniform. The driver gave the lecture, flawlessly. At the end, a member of the audience asked a detailed question about some of the subject matter, upon which the lecturer replied, “Well, the answer to that question is quite simple, I bet that my driver, sitting up at the back, there, could answer it.”

* * *

Charlie Chaplin had invited Albert Einstein to the premiere of City Lights. When the public cheered them both, Chaplin remarked: “They cheer me because they all understand me, and they cheer you because no one understands you.”

* * *

Albert Einstein who fancied himself as a violinist, was rehearsing a Haydn string quartet. When he failed for the fourth time to get his entry in the second movement, the cellist looked up and said, “The problem with you, Albert, is that you simply can’t count.”

* * *

Einstein was attending a music salon in Germany before the Second World War, with the violinist S. Suzuki. Two Japanese women played a German piece of music and a woman in the audience exclaimed: “How wonderful! It sounds so German!” Einstein responded: “Madam, people are all the same.”

* * *

In 1946 a South African child, Tyffany Williams expressed in a letter her surprise that Einstein was still alive. He answered: “I have to apologize to you that I am still among the living. There will be a remedy for this, however”.

* * *

After the birth of his sister Maja, the two and a half year old Albert Einstein was told he would now have something to play with. After looking at the baby he complained: “Yes, but where are its wheels.”

* * *

Einstein never liked to dress well. When Einstein's Wife told him to dress properly when going to the office he argued: "Why should I? Everyone knows me there." When he was told to dress properly for his first big conference: "Why should I? No one knows me there."[86]

ISAAC NEWTON

The English mathematician John Wallis (1616-1703) was a friend of Isaac Newton. According to his diary, Newton once bragged to Wallis about his little dog Diamond. "My dog Diamond knows some mathematics. Today he proved two theorems before lunch."

"Your dog must be a genius," said Wallis.

"Oh I wouldn't go that far," replied Newton. "The first theorem had an error and the second had a pathological exception."[87]

PAUL DIRAC

During a lecture, Professor Dirac made a mistake in an equation he was writing on the blackboard. A courageous student raises his finger and says timidly: "Professor Dirac, I do not understand equation 2". Dirac continues writing without any reaction. The student supposes Dirac has not heard him and raises his finger again, and says, louder this time: "Professor Dirac, I do not understand equation 2". No reaction. Somebody on the first row decides to intervene and says: "Professor Dirac, that man is asking a question".

"Oh," Dirac replies, "I thought he was making a statement".[88]

FRIGYES RIESZ

Hungarian mathematician Frigyes Riesz needed two assistants for his lectures: one was reading aloud his (Riesz's) book, the second one was writing everything on the board, while Riesz was standing next to the board nodding.

ISAAC TODHUNTER

Maxwell asked the mathematician Isaac Todhunter whether he would like an experimental demonstration of conical refraction.

Todhunter answered: “No, I have been teaching it all my life, and I do not want to have my ideas upset”.[89]

ERNST EDUARD KUMMER

A German algebraist Ernst Eduard Kummer, was rather poor at arithmetic. Once he had to calculate 7 x 9.

Kummer said to himself: “Hmmm the product cannot be 61, because 61 is prime, it cannot be 65, because 65 is a multiple of 5, 67 is a prime, 69 is too big – only 63 is left”.[90]

QUOTES ON MATHEMATICS[91]

- "In mathematics you don't understand things, you just get used to them." *(John von Neumann)*
- "Mathematical discoveries, small or great are never born of spontaneous generation. They always presuppose a soil seeded with preliminary knowledge and well prepared by labour, both conscious and subconscious." (*Henri Poincaré)*
- "It is through science that we prove, but through intuition that we discover." (*Henri Poincaré)*
- "Geometry is not true, it is advantageous". (*Henri Poincaré)*
- Mathematics is the art of giving the same name to different things. *(Henri Poincaré)*
- The mathematician does not study pure mathematics because it is useful; he studies it because he delights in it and he delights in it because it is beautiful. *(Henri Poincaré)*
- "Mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true". *(Bertrand Russell)*
- "The true spirit of delight, the exaltation, the sense of being more than Man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as poetry." *(Bertrand Russell)*
- "I like Mathematics because it is not human and has nothing particular to do with this planet or with the whole accidental universe – because like Spinoza's God, it won't love us in return." *(Bertrand Russell)*
- Mathematics, rightly viewed, possesses not only truth, but supreme beauty - a beauty cold and austere, like that of sculpture. *(Bertrand Russell)*
- "A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas." *(G. H. Hardy)*
- "A surprising proportion of mathematicians are accomplished

musicians. Is it because music and mathematics share patterns that are beautiful?” (*Martin Gardner)*

- “A mathematician is a blind man in a dark room looking for a black cat which isn’t there.” *(Charles Darwin)*
- “God made the natural numbers. The others, were man-made.” *(Leopold Kronecker)*
- “The book of nature is written in the language of mathematics.” (*Galileo Galilei)*
- “Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country.” *(David Hilbert)*
- “The essence of mathematics lies in its freedom.” *(Georg Cantor)*
- “Thus, in a sense, mathematics has been most advanced by those who distinguished themselves by intuition rather than by rigorous proofs.” (*Felix Klein)*
- Everyone knows what a curve is, until he has studied enough mathematics to become confused through the countless number of possible exceptions. *(Felix Klein)*
- “God does not care about our mathematical difficulties. He integrates empirically.” (*Albert Einstein)*
- As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality. *(Albert Einstein)*
- Do not worry about your difficulties in Mathematics. I can assure you mine are still greater. (*Albert Einstein*)
- Pure mathematics is, in its way, the poetry of logical ideas. *(Albert Einstein)*
- “Mathematics is well and good but nature keeps us around by nose.” (*Albert Einstein)*
- “Life is good for only two things: discovering Mathematics and teaching Mathematics.” *(Siméon Poisson)*
- “For the things of this world cannot be made known without a knowledge of Mathematics.” *(Roger Bacon)*

- “All science requires Mathematics. The knowledge of mathematical things is almost innate in us. This is the easiest of sciences, a fact which is obvious in that no one’s brain rejects it; for laymen and people who are utterly illiterate know how to count and reckon.” *(Roger Bacon)*
- “A man whose mind has gone astray should study Mathematics.” *(Francis Bacon)*
- “I have hardly ever known a mathematician who was capable of reasoning.” *(Plato)*
- “Each problem that I solved became a rule which served afterwards to solve other problems.” *(Rene Descartes)*
- “The essence of Mathematics is not to make simple things complicated, but to make complicated things simple.” *(S. Gudder)*
- “The laws of nature are but the mathematical thoughts of God.” (*Euclid)*
- “But Mathematics is the sister, as well as the servant, of the arts and is touched with the same madness and genius.” *(Harold Marston Morse)*
- Mathematics are the result of mysterious powers which no one understands, and which the unconscious recognition of beauty must play an important part. Out of an infinity of designs a mathematician chooses one pattern for beauty’s sake and pulls it down to earth. *(Harold Marston Morse)*
- “Go down deep enough into anything and you will find mathematics.” *(Dean Schlicter)*
- “Arithmetic is numbers you squeeze from your head to you hand to your pencil to your paper till you get the answer.” *(Carl Sandburg)*
- “Arithmetic is where the answer is right and everything is nice and you can look out of the window and see the blue sky - or the answer is wrong and you have to start over and try again and see how it comes out this time.” *(Carl Sandburg)*

- “Arithmetic is where numbers fly like pigeons in and out of your head.” *(Carl Sandburg)*
- “In mathematics the art of proposing a question must be held of higher value than solving it.”
(Georg Ferdinand Ludwig Philipp Cantor)
- Pure mathematics is the world’s best game. It is more than chess, more of a gamble than poker, and lasts longer than Monopoly. It’s free. It can be played anywhere – Archimedes did it in bathtub.
(Richard J. Trudeau)
- I have only to touch mathematics, and I forget everything else on earth. (*Sofia Kovalevskaya)*
- Many who have had an opportunity of knowing any more about mathematics confuse it with arithmetic, and consider it an arid science. In reality, however, it is a science which requires a great amount of imagination. *(Sofia Kovalevskaya)*
- The laws of mathematics are not merely human inventions or creations. They simply “are”; they exist quite independently of the human intellect. The most that any (one) ... can do is to find that they are there and to take cognizance of them. *(Maurits Escher)*
- If equations are trains threading the landscape of numbers, then no train stops at pi. (*Richard Preston)*
- A mathematician is a scientist who can figure out anything except such simple things as squaring the circle and trisecting an angle.
(Evan Esar)

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